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An empirical analysis of the global input–output network and its evolution

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ABSTRACT

This paper studies the global production network using a general equilibrium model calibrated on world input–output data. The analysis of propagation of idiosyncratic productivity shocks in the calibrated model allows to define a model-based network centrality measure. Such measure is used to investigate the topology of the global input–output network in 2014 and its evolution from 2000 to 2014. We find that new influential sectors have emerged over time. Moreover, we show that the global production system has evolved to become more sensitive to idiosyncratic productivity shocks and that this result is related to the increase of the intermediate input intensity of production.

1. Introduction

The importance of firms' production network in affecting micro and macro economic behavior has been stressed by past catastrophic events [1]:

> When flood waters rose in Thailand, Wal-Mart stores in Japan ran out of mouthwash(...). A string of natural disasters has exposed a vulnerability in global supply chains. How do you set up a network that is compact enough to be efficient but spreads widely enough so that no single unexpected event can knock it out? Japan's earthquake and tsunami in March caused auto parts shortages worldwide. Thailand's recent flooding shut down some of the world's largest hard drive makers, which could cut personal computer shipments by as much as 20 percent in the first quarter of 2012 (Reuters 2011).[1]

Similarly, during the recent financial crisis, there has been much discussion about firms considered *too big to fail*. As reported by Carvalho [1], the bail-out of General Motors was perceived as necessary to avoid disruptions in the supply chain of the American automotive industry. The idea that sectoral interdependencies, arising from the input–output structure, have an important influence on aggregate economic behavior has a long history in economics, see e.g., Long and Plosser [2]. More recently it has been revived by Horvath [3,4] and Acemoglu et al. [5] who showed that the topology of the input–output network has a crucial role in determining the aggregate behavior of the system. If the input–output network is significantly asymmetric, i.e. if relatively few sectors play a predominant role as suppliers, then idiosyncratic shocks give rise to aggregate fluctuations. When the organization of production is dominated by a small number of *hubs* supplying inputs to many different sectors, disruptions in these critical nodes can affect the global production system, determining

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1 Retrieved on 26 March 2015 from http://www.reuters.com/article/2011/11/13/us-apec-disaster-idUSTRE7AC12Q20111113.

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losses in production and welfare [5]. Understanding the structure of the production network, and in particular determining which sectors act as hubs in the network, is important to understand the origin of aggregate fluctuations [see also 6,7] and to inform policymakers on how to prepare for, and recover from, adverse shocks hitting the production network.

In complex network theory, these key sectors are identified by applying measures of node centrality. The idea of centrality was initially proposed in the context of social systems, where a relation between the location of a subject in the social network and its influence on group processes was assumed. Various measures of centrality have been proposed in network theory such as the number of neighbors of a node (degree centrality), which is a *local centrality* measure, or measures based on the spectral properties of the graph [see 8]. Spectral centrality measures include the eigenvector centrality [9,10], Katz's centrality [11], PageRank [12], hub and authority centralities [13]. These measures are *global centrality* measures and provide information on the position of each node relative to all other nodes. Centrality analysis has been recognized to provide important insights into economic phenomena. For example, Fagiolo et al. [14,15] and [16,17] investigate the topology of the world trade network and its evolution. Kali and Reyes [18,19] studied how the position in the trade network, specified by the number of links of a country (node degree), has substantial implications for economic growth and a good potential in predicting episodes of financial contagion. Schiavo et al. [20] showed that node centrality measures, such as the degree/strength and the random betweenness centrality of the nodes, applied to trade and financial networks, may help to account for the evolution of international economic integration. Chinazzi et al. [21] showed that econometric models, fed with international financial network measures, provide useful information about the aggregate performance of different countries.

In this paper we study the topology of the production network by analyzing the propagation of productivity shocks through the input–output network. In particular, we build a simple general equilibrium model along the lines of Long and Plosser [2] and Acemoglu et al. [5], and calibrate the model using world input–output data ranging from 2000 to 2014. The calibrated model allows to define a centrality measure for each sector related to the impact of a productivity shock to that sector on global production. This centrality measure is used to study the structure of the production network and its evolution over time. Moreover, we study the fragility of the network by measuring the impact of random productivity shocks on global production. In particular, we assume that the probability of a productivity shock hitting a sector is uniform across sectors and we define *fragility* as the expected impact of a shock on real GDP. The system is more fragile when a shock to a random sector has greater influence on real GDP.[2]

Our work is related to the stream of literature developing empirical measures to assess the role of different sectors in production using input–output tables [e.g. 22–24]. Our topology measures are grounded on a calibrated general equilibrium model allowing us to have a simple and direct economic interpretation of the results. Our work is also related to the stream of literature focusing on the empirical analysis of input–output networks and their effects on aggregate outcomes [e.g., 4,5,25–27] and in particular it is related to Acemoglu et al. [28], who study the propagation of shocks through the US input–output network. But differently from Acemoglu et al. [28] we exploit a dataset recording the world input–output flows, allowing us to study the propagation of shocks at a global level and to investigate the worldwide interactions between production sectors. Finally, this paper is also related to the literature investigating systems' fragility in presence of networks. In the context of the world trade network, Foti et al. [29] study the effect of exogenous shocks on nodes and links on economic outcomes and He and Deem [30] apply evolutionary theory to study the reaction of the trade network to recessionary shocks. In their analysis, Foti et al. [29] and He and Deem [30] do not use an explicit economic model. In our study, we exploit the calibrated general equilibrium model to investigate the effects of random shocks hitting the system. Using an explicit economic model allows to study the reaction of the economic system using a formal definition of shocks, i.e., changes to sectors' productivity, and a micro-founded reaction of agents to shocks.

Our results can be summarized as follows. Sectors' centrality is heterogeneous and highly asymmetrical, i.e. few sectors have very strong impact on global production. From 2000 to 2014 the topology of the world production network has changed. Sectors that were peripheral in 2000 have become very influential in 2014. Moreover, we aggregate sectors' centralities by country and find that the structure of the production network at the country level has evolved from a one-star to a multi-star structure. Finally, we compute the fragility of the production network from 2000 to 2014 and find that it has increased. The reason for the increased fragility is that the relative importance of intermediate goods in the production process has increased. The intuition is that when intermediate goods are relatively important, production is more sensitive to shocks to intermediate goods production.

The paper is organized as follows. In Section 2, we describe a simple economic model with an input–output structure and calibrate it using the WIOT 2014 data. In Section 3 we analyze the influence of each sector on global production costs and in Section 4 we define our centrality measure and analyze the impact of each sector on global real GDP. In Section 5, we exploit the yearly WIOT data from 2000 to 2014, to explore how the influence of sectors and how the network fragility has evolved over time. Finally, Section 6 concludes.

[2] A disclaimer is due to correctly interpret our results. In order to build and calibrate the model we impose a set of simplifying assumptions. However, we would like to stress that the aim of this paper is build a simple, theoretically based, empirical measure of sector centrality. The aim of the calibrated model is to approximate and measure the short term relations between sectors, rather than fully describe the production dynamics. The centrality measure should not be interpreted literally as the effect of a productivity shock on world GDP, but as a measure of relative importance of sectors in global production. Similarly, the fragility index should not be interpreted literally as the expected impact of a random shock on real GDP, but a measure of how the fragility of the production network has evolved over time.

2. The model

In this section we describe a model in the spirit of Long and Plosser [2] and Acemoglu et al. [5]. Technological and preference parameters will be calibrated to reproduce the observed nominal trade flows in 2014.

We assume that the economy is composed by N sectors. Each sector is represented by a representative firm with technology[3]:

$$x_i = z_i l_i^{\alpha_i} \prod_{j=1}^{N} x_{ij}^{(1-\alpha_i)w_{ij}} , \tag{1}$$

where x_i is the output of the ith sector, z_i is the total factor productivity, l_i is the labor employed by sector i, α_i is the share of labor in sector i, $0 < \alpha_i < 1$, x_{ij} is the amount of good j used in the production of i and $(1 - \alpha_i)w_{ij}$ is the share of input j in the production of i. We assume that $w_{ij} \geq 0$ and constant return to scale, i.e. $\sum_j w_{ij} = 1$. The values of w_{ij} determine the technological network in the economic system. If $w_{ij} > 0$ there is a link with weight w_{ij} between sector i and sector j, i.e. sector i is using good j to produce and there is a flow of goods from sector j to sector i. If $w_{ij} = 0$ no link is present since sector i is not using good j for production. The matrix $\mathcal{W}$ containing all the elements w_{ij} is an adjacency matrix representing a *directed* and *weighted* network, where nodes represent sectors and links represent the technological relationships between sectors.

The household sector is represented by one representative individual which supplies inelastically $\bar{l}$ units of labor and maximizes the following utility function:

$$\mathcal{U} = \prod_{i=1}^{N} q_i^{c_i} , \tag{2}$$

where q_i is the quantity of good i consumed by the household, c_i is a preference parameter for good i, $c_i \geq 0$ and $\sum_i c_i = 1$, under the budget constraint:

$$\sum_i q_i p_i \leq \bar{l}\eta \tag{3}$$

where $\bar{l}$ is the inelastic labor supply and η is the unitary nominal salary, p_i is the price of good i. The resulting demand function for good i is:

$$q_i = \frac{\bar{l}\eta c_i}{p_i} . \tag{4}$$

The differences with the model described in Acemoglu et al. [5] are that we consider heterogeneous labor shares, α_i and heterogeneous preference parameters, c_i. This is necessary to calibrate the model. Moreover, as we will discuss in Sections 4 and 5, these parameters have a crucial role in determining the properties of the economic system.

2.1. Equilibrium

The system is in equilibrium when all markets are cleared, households maximize their utility and firms maximize their profits. We assume that in each sector there is an infinite number of perfectly competitive firms. This implies that the price set in each sector by the representative firm equals the marginal cost. We assume that the labor supply is inelastically equal to $\bar{l} = 1$. Optimal demand for inputs derived from the maximization of profits are the following:

$$x_{ij} = \frac{(1-\alpha_i)w_{ij} p_i x_i}{p_j} \tag{5}$$

$$l_i = \frac{\alpha_i p_i x_i}{\eta} \tag{6}$$

Substituting Eq. (5) and Eq. (6) in the production function and taking the logs we obtain:

$$\alpha_i \log \eta = \log(z_i) + \alpha_i \log \alpha_i + \alpha_i \log p_i + (1-\alpha_i)\log p_i + \\ + (1-\alpha_i)\log(1-\alpha_i) - (1-\alpha_i)\sum_j w_{ij} \log p_j + (1-\alpha_i)\sum_j w_{i,j} \log w_{ij}$$

setting $A_i \equiv \alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + (1-\alpha_i)\sum_j w_{i,j} \log w_{ij}$ we can write the optimal price of sector i as:

$$\log p_i = -\log(z_i) + (1-\alpha_i)\sum_j w_{ij} \log p_j - A_i + \alpha_i \log \eta$$

[3] In the following we will use interchangeably *firm* and *sector* since in our context they can be used as synonyms.

Letting $\hat{\mathcal{W}} = ((1-\alpha)\mathbf{1}') \circ \mathcal{W}$, where $\circ$ is the Hadamard product operator, the optimal price in vectorial form is:

$$\log p = -\log(z) + \hat{\mathcal{W}} \log p - A + \alpha \log(\eta)$$

where symbols without subscript i denote $N \times 1$ column vectors. The vector of equilibrium prices is:

$$\log p = (I - \hat{\mathcal{W}})^{-1}(-\log(z) - A + \alpha \log \eta) \tag{7}$$

Using the market clearing conditions we compute the equilibrium quantities. In particular the market clearing condition for good i implies that total production equals the demand for consumption goods and inputs:

$$x_i = \frac{\eta c_i}{p_i} + \frac{1}{p_i} \sum_j (1-\alpha_j) w_{ji} p_j x_j$$

where we have used $\bar{l} = 1$. Multiplying both sides by p_i we obtain:

$$p_i x_i = \eta c_i + \sum_j (1-\alpha_j) w_{ji} p_j x_j$$

and

$$s_i = \eta c_i + \sum_j (1-\alpha_j) w_{ji} s_j$$

where $s_i \equiv p_i x_i$ is the equilibrium value of sales for sector i, and in vectorial form:

$$s = \eta c + \hat{\mathcal{W}}' s$$

Thus, the vector of equilibrium sales is the following:

$$s' = \eta c' [I - \hat{\mathcal{W}}]^{-1} \tag{8}$$

Using the equilibrium vector of prices, the equilibrium production is $x_i = s_i / p_i \ \forall i$. Choosing to set the nominal wage as *numeraire*, we can compute all equilibrium values in the model.

2.2. Data and calibration

To calibrate the model we use the World Input–Output Table (WIOT). WIOT is a database providing the input–output tables for 44 countries and 56 sectors.[4] and a set of final consumption sectors for each country, covering the period from 2000 to 2014 [31]. The input–output table records the nominal flows of goods between each sector in each country and all other sectors in the database, at current prices. We aggregate the final consumption sectors into one global sector.[5] In Appendix B we explore and describe the topology of the world input–output network, reporting the commonly used centrality measures computed on the input–output network in 2014.

We set the number of firms, N, equal to the number of sectors in WIOT 2014. Each sector i corresponds to a given country-sector pair observed in the dataset. To simplify the notation we do not use the country-sector notation. However, to interpret the results it is important to stress that a sector i actually represents a sector (s, c), where s is a specific sector, and c is a specific country. The subscript i can be interpreted as the sector's unique identifying number. Appendix A provides two tables reporting all countries and all sectors in the dataset and describes how to map each sector i to the unique corresponding observed country-sector pair.

We assume that the entries of WIOT 2014 are equilibrium sales. Perfect competitive equilibrium implies that profits are zero in equilibrium and that the nominal GDP coincides with the nominal wage η. We use the nominal flows recorded in WIOT 2014 to set the technological parameters α_i, w_{ij} and the preference parameters c_i. We normalize the total factor productivity to 1 for every sector. The calibrated model reproduces the observed nominal input–output matrix and provides the equilibrium prices and the equilibrium quantities consistent with the observed input–output matrix.[6]

In the paper we use a calibrated economic model to investigate the topology of the global input–output network. By using the calibrated model, we shock the total factor productivity of each sector and measure the influence of the shock on the production costs and on global real GDP. The influence of the shock on the economy provides a centrality measure for each sector. It is important to stress that this centrality measure is mediated by the economic model. The definition of *shock* depends on the production network topology, but also on the model, and the effect of the shock is mediated by the assumptions about the technology of firms, the utility of households and the of the agents. By comparing this centrality measure with the traditional centrality measures described in Appendix, we will highlight how the model changes the interpretation of the data. Indeed, the aim of this paper is to perform a network analysis using the observed

4 Countries and sectors present in the database are reported in Appendix A.

5 For further details on the data, see Timmer et al. [31,32] and Appendix C.

6 For details on calibration see Appendix C.

global production network and characterizing the relations between sectors using an economic model. Similarly to Foti et al. [29], who study the stability of the international trade network to exogenous perturbations to links and nodes. We also study the stability of the production network, but we ground our analysis on a calibrated general equilibrium model and measure the impact of productivity shocks on real GDP.

In the context of an empirical evaluation of the global input–output relations, this model makes a rather unsatisfactory assumption: the absence of endogenous financial shocks. This hypothesis offers simplicity and analytical tractability, but may overlook some important effects of the shocks on the network. The financial side of the economy [see 33,34] is, indeed, not explicitly consider by the model since our focus is on the real economy and on the complex supply chains which led to the creation of products. The financial side of the economy, in our framework, works as an additional source of shock on the real economy.

3. On the dynamics of price: the spread of a shock

We determine sector centrality by measuring the effect of a shock to a sector on global real GDP. We perform a preliminary analysis focusing on the effect of a shock to productivity of sector j on the equilibrium prices of all sectors in the network. To this aim we compute the derivative of the equilibrium prices with respect to z_j.

Cobb–Douglas production and utility functions imply that the elasticity of demand of good i with respect to price p_i is 1. Therefore, studying the effect of a shock on prices is equivalent to study the effect of a shock on quantities. Two further important assumptions are implied by the Cobb–Douglas production and utility functions. The first assumption is that a shock does not cause movements of workers across production nodes. As described by Eq. (6), labor demand of sector j depends on nominal production $p_j x_j$, labor share α_j, and nominal wage η, which are all constant to a shock to productivity. The labor share employed in each sector is constant to productivity shocks. The second assumption is that the shock on productivity does not affect the topology of the input–output network. We are not taking into account creations or destructions of links. Labor mobility and changes in the input–output structure are likely to play a role in the reaction of firms to a productivity shock in the medium and long run. In this paper we measure the effect of a shock in the short run, therefore the assumptions of constant technological network and labor immobility are not too restrictive.

The assumption of perfect competition implies that a change in prices is equivalent to a change in costs. By computing the derivative of prices with respect to $\log z_j$, we determine the effect of a productivity shock to sector j on the marginal costs of all sectors in the production system. Recall the equation of the equilibrium log price vector in Eq. (7):

$$\log p = (I - \hat{\mathcal{W}})^{-1}(-\log z - A + \alpha \log \eta)$$

Denoting the Leontief inverse as $\bar{\mathcal{W}} \equiv (I - \hat{\mathcal{W}})^{-1}$ and $\varepsilon \equiv \log z$, we can write the derivative of the logarithm of equilibrium prices with respect to ε_j as:

$$\frac{\partial \log p}{\partial \varepsilon_j} = \bar{\mathcal{W}}_{\bullet j}(-1) \tag{9}$$

where $\bar{\mathcal{W}}_{\bullet j}$ is column j of the Leontief inverse. Eq. (9) describes the spreading of a shock in the network. The effect of a shock to sector j depends on the technological relationship between sector j and all the other sectors in the network *and* on the optimal reaction of firms to the change in the price of their inputs. As expected, a positive productivity shock to sector j will lead to a decrease of the price of sectors which are directly or indirectly linked to sector j.

3.1. Spreading of a shock

Using the model calibrated with the WIOT 2014 and computing Eq. (9), we obtain the direct and indirect effect of a shock to each sector j on the price (and production cost) of each sector i, i.e. $\partial \log p_i / \partial \varepsilon_j \forall i, j$, in 2014. The result is the $N \times N$ matrix shown in Fig. 1, where the element (i, j) is the derivative of the price of sector i with respect to a shock to sector j. Sectors are arranged in a Country-Sector order.[7] Lighter colors are associated with a greater (absolute) derivative. Fig. 1 reveals a clear community structure at the country level shown by the lighter block main diagonal, meaning that shocks mainly propagate inside each country. Inter-country relationships are weaker, with some important exceptions. The lighter columns correspond to sectors with a strong impact on world production costs. It is interesting to note that these columns are clustered together, showing that globally important sectors are concentrated in few globally important countries.

The economic model allows to transform the empirical matrix of nominal trade flows into a matrix of relations taking into account the technological links and the optimal behavior of the firms. A negative shock to sector j implies an increase

[7] Assume that the input–output table is composed by countries A and B and sectors 1 and 2. The first column of the *network of effects* matrix would display the derivative with respect to a shock to sector A1, the second column the derivative with respect to a shock to sector A2, and so forth. The element (2,1) is the derivative of price A2 with respect to a shock on sector A1. In this case the element on the first row–first column shows the effect of a shock to *Crop and animal production, hunting and related service activities* in Australia on the sector *Crop and animal production, hunting and related service activities* in Australia. The element on the second row–first column shows the effect of a shock to *Crop and animal production, hunting and related service activities* in Australia on *Forestry and logging* in Australia. Sectors ID and corresponding country and sector can be read in Appendix A.

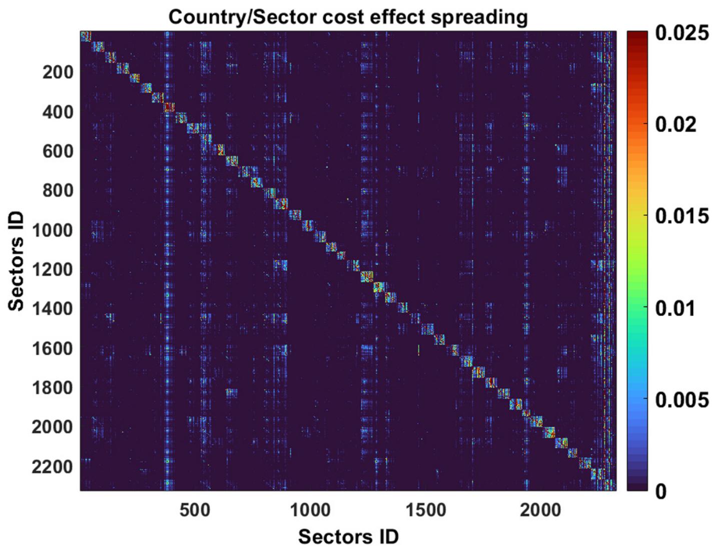

Fig. 1. Price derivatives computed with 2014 data. The colors show the value of derivatives. The element (i, j) represents the derivative of the price of sector i with respect to a shock to sector j. Sectors ID and corresponding country-sector can be read in Appendix A.

in the price of good j. Downstream firms react to the increase in cost of input j by optimally substituting it with the other available inputs, dampening but not removing the impact of the shock. The value of $\partial \log p_i/\partial \varepsilon_j$ takes into account the overall effect of a shock to sector j on the price of sector i. In order to have a deeper understanding of the global influence of each sector on the cost of the other sectors, we compute a *cost effect index*. We build the cost effect index of sector j as follows:

$$\iota_j = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\partial \log p_i}{\partial \varepsilon_j}\right| \tag{10}$$

The cost effect index of sector j represents the average effect of a productivity shock to sector j on all sectors of the world, and it approximates the average percentage change of prices (and costs) caused by the shock. It is important to stress that the cost effect index is not weighted by the size of the sectors. The contribution of a 2% change in the cost of *Forestry and logging* in Denmark and a 2% change in the cost of *Constructions* in China to the cost effect index is the same. In other words, if a sector influences only *Forestry and logging* in Denmark, and another sector influences only *Constructions* in China by the same amount, they will have the same cost effect index. This is because the cost effect index measures how a shock spreads through the production system, and how a shock hits prices around the globe.

The cost effect index and its distribution are shown in Fig. 2. The distribution follows approximately a power-law with exponent $\gamma_\iota = 3$ as reported in the right panel of Fig. 2. The distribution of the cost effect index is skewed, with few important sectors and a large number of less important sectors. In particular Mining and Quarrying in Rest of the World has a huge effect on global production costs because it provides most of the raw materials to world manufacturers. Mining and Quarrying in Rest of the World is upstream to most of world sectors, therefore, a shock to its productivity influences strongly almost all sectors in the world economy.

The cost effect index is important to analyze how the world production is organized and how shocks spread through the network, but it is not suited for the study of the influence of sectors on global production. The reason is that the cost effect index does not take into account the size of sectors. To provide a centrality measure capable of capturing the influence of sectors on global production, we introduce in the next section an index measuring the impact of shocks on global real GDP and on global welfare.

4. The effect of a shock on global production

Having studied the effect of a shock on production costs, we are now ready to study the effect of a shock on real GDP and on global welfare. Let us start by defining the price index in the world economy:

$$\mathcal{P} = \prod_{i=1}^{N}\left(\frac{p_i}{c_i}\right)^{c_i}$$

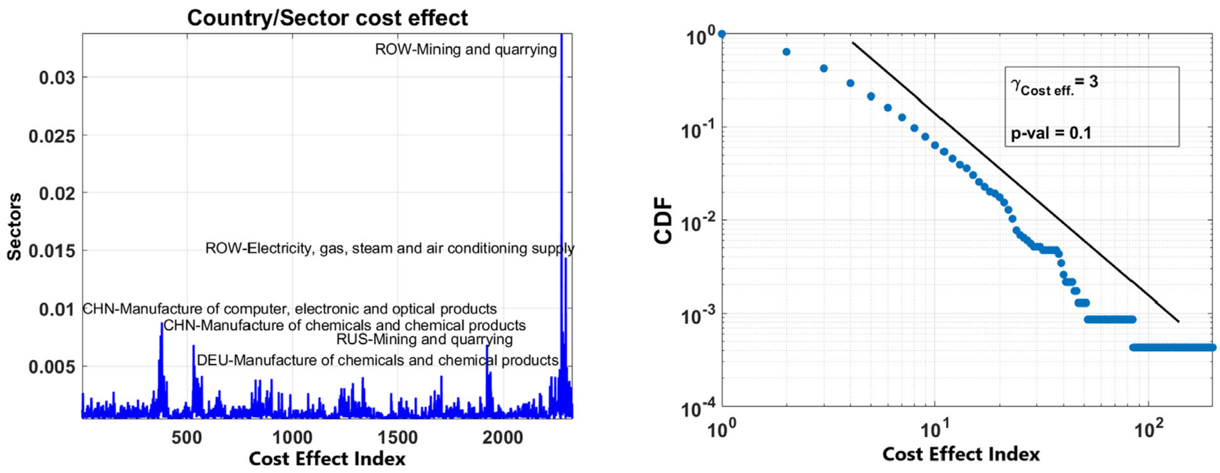

Fig. 2. The cost effect index in 2014. **Left**: Cost effect index of each sector. The *x*-axis reports the ID of the 2464 sectors in the WIOD dataset. The *y*-axis shows the cost effect index. **Right**: Log–log plot of the cumulative distribution function and maximum likelihood power-law fit for the cost effect index distribution.

where c_i is set equal to the share of nominal GDP consumed in each sector in the WIOT database. In a perfect competitive equilibrium, where total labor $\bar{l} = 1$, the nominal GDP coincides with the nominal wage η. Thus, the real GDP is:

$$y = \frac{\eta}{\mathcal{P}}$$

Taking the logarithm of real GDP and computing the derivative with respect to a productivity shock to a generic sector j we obtain:

$$\frac{\partial \log y}{\partial \varepsilon_j} = -\sum_i c_i \frac{\partial \log p_i}{\partial \varepsilon_j} \tag{11}$$

It is worth noticing that, by construction, Eq. (11) describes also the effect of the shock on households' welfare. In fact, it is easy to show that the derivative of the utility function is equivalent to the derivative of real GDP.[8]

Eq. (11) measures the *influence* of a shock to sector j on real GDP, and we use it to define the centrality measure of sector j, which we will call *influence index*:

$$\varphi_j \equiv \left|\frac{\partial \log y}{\partial \varepsilon_j}\right| = \left|\sum_i c_i \frac{\partial \log p_i}{\partial \varepsilon_j}\right| \tag{13}$$

The influence index of sector j is the absolute value of the weighted sum of the effect on all prices of a shock to sector j. The weights are the parameters c_i, which are the preference parameters in the utility function, and the share of global income consumed on the good produced by sector i. The influence index takes into account both the spreading of the shock, summarized by effect of the shock on global prices, and the characteristics of the sectors hit by the shock. A sector j has a high influence index if a shock on its productivity has a strong effect on prices of sectors producing highly consumed goods.

Substituting Eq. (9) in Eq. (13) and using vectors, we can rewrite the influence of sector j as

$$\varphi_j = c'\bar{\mathcal{W}}_{\bullet j} \tag{14}$$

and write the column vector φ, whose elements are the influence index of each sector, φ_j, as:

$$\varphi = c'\bar{\mathcal{W}} \equiv c'[I - \hat{\mathcal{W}}]^{-1} \tag{15}$$

8 The logarithm of the utility function is:

$$\log \mathcal{U} = \sum_i c_i \log q_i$$

where q_i is the quantity of good i consumed by the representative consumer. By substituting $q_i = \frac{\bar{l}\eta c_i}{p_i}$, and computing the derivative with respect to z_j, we obtain

$$\frac{\partial \log \mathcal{U}}{\partial \varepsilon_j} = -\sum_i c_i \frac{\partial \log p_i}{\partial \varepsilon_j}. \tag{12}$$

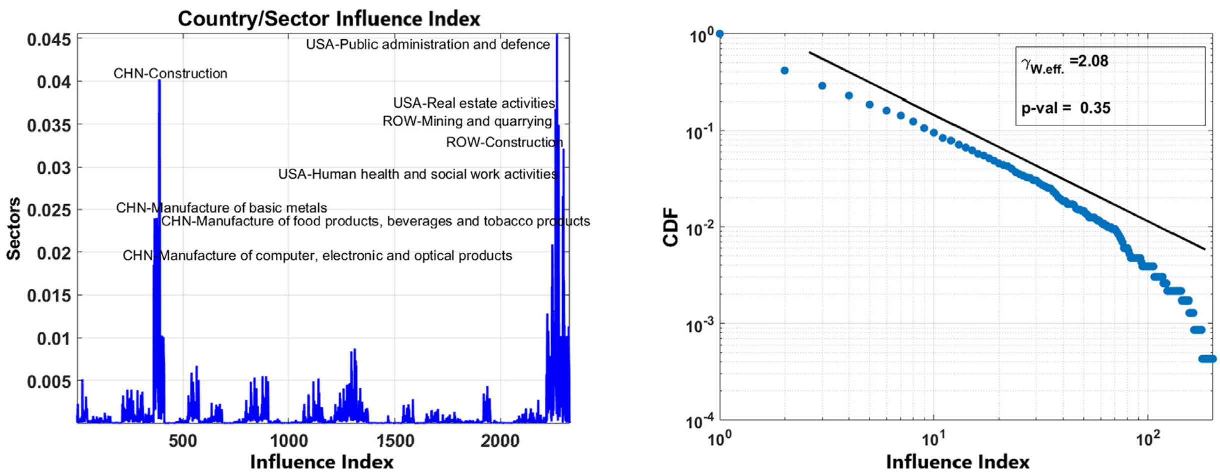

Fig. 3. GDP effect index. **Left**: the *x*-axis reports the sectors, the *y*-axis reports the GDP effect index. **Right**: Log–log plot of cumulative distribution function and maximum likelihood power-law fit for the GDP effect index distribution.

Using Eq. (8) we can conclude that $\varphi = s/\eta$, which are the *Domar weights* defined in Domar [35] and Hulten [36], and used among others, in Carvalho and Gabaix [6]. The influence index of each sector is proportional to total sales.

The *influence index* for each sector is shown in the left panel of Fig. 3, and its distribution is shown in the right panel of Fig. 3. The influence index is distributed as a power-law with exponent $\gamma_\varphi = 2.08$ and the Gini coefficient which measure the influence index dispersion takes value of $G = 0.8621$ meaning that few sectors owning a huge effect on world real GDP coexist with many small sectors with very little effect on world real GDP. The influence index is a centrality measure explicitly considering the behavior of firms and consumers and is related to the Bonacich centrality measure. To understand if the influence index is adding information with respect to the *traditional* centrality measures, we compare it with the output PageRank (blue circles) and the out-strength (green diamonds) in Fig. 4 computed on the input–output network. The influence index and the output PageRank[9] have a correlation of 0.66, and the influence index and the out-strength have a correlation of 0.81. The PageRank and out-strength measures explain imperfectly the importance of sectors in terms of influence on global real GDP. Some sectors are characterized by a small PageRank and a small out-strength but have a huge impact on real GDP. The interpretation is the following. The importance of a sector depends both on the position in the network as input supplier and on the proximity to the consumption sector. Using the model we are evaluating the sectors not only by their size in the input–output network, but also by their importance in the production of consumption goods, which is the final aim of the whole production system. In this regard, there are sectors which are highly influential for their position as input suppliers, such as "ROW- Mining and Quarrying", and other sectors which are highly influential for their role as consumption good producers, such as "USA-Public Administration". Moreover, the model allows to take into account the short-term reaction of firms to shocks, and the incentive of firms to switch from more expensive to cheaper inputs, according to their production technology. Neither the PageRank score nor the out-strength can represent the consequences of such behavior. The model allows to take into account the economic relationship between sectors and to select an appropriate centrality measure.

In Fig. 5 we aggregate the influence index by country. The influence of a country on world production is simply computed as the average influence index of its sectors. By construction, the measure of *country influence* can also be interpreted as the expected effect on the global real GDP of a shock hitting a random sector of a country. It is interesting to note that the ranking is highly correlated with, but different from, the GDP ranking. The reason is that the size of a country is an important determinant of influence, but it is not the only one. The position in the input–output network plays a crucial role and this is the reason why China, being an important input supplier, has a greater influence on global production than the USA.[10]

5. The evolution of the global input–output network

We exploit the WIOT data from 2000 to 2014 to study the evolution of the global production system. Using the input–output data we re-calibrate the model for each year. This allows to calibrate the technological parameters and household's

[9] See Appendix B.3 for a formal definition of PageRank.

[10] The ranking by nominal GDP in 2014 can be found in the IMF database http://www.imf.org/external/datamapper/NGDPD@WEO/OEMDC/ADVEC/WEOWORLD. The ranking (nominal GDP in US dollars) is the following: United States (17.39 trillion), China (10.53 trillion), Japan (4.85 trillion), Germany (3.89 trillion), United Kingdom (3 trillion), France (2.84 trillion), Brazil (2.46 trillion), Italy (2.16 trillion), Russian Federation (2.06 trillion), India (2.03 trillion).

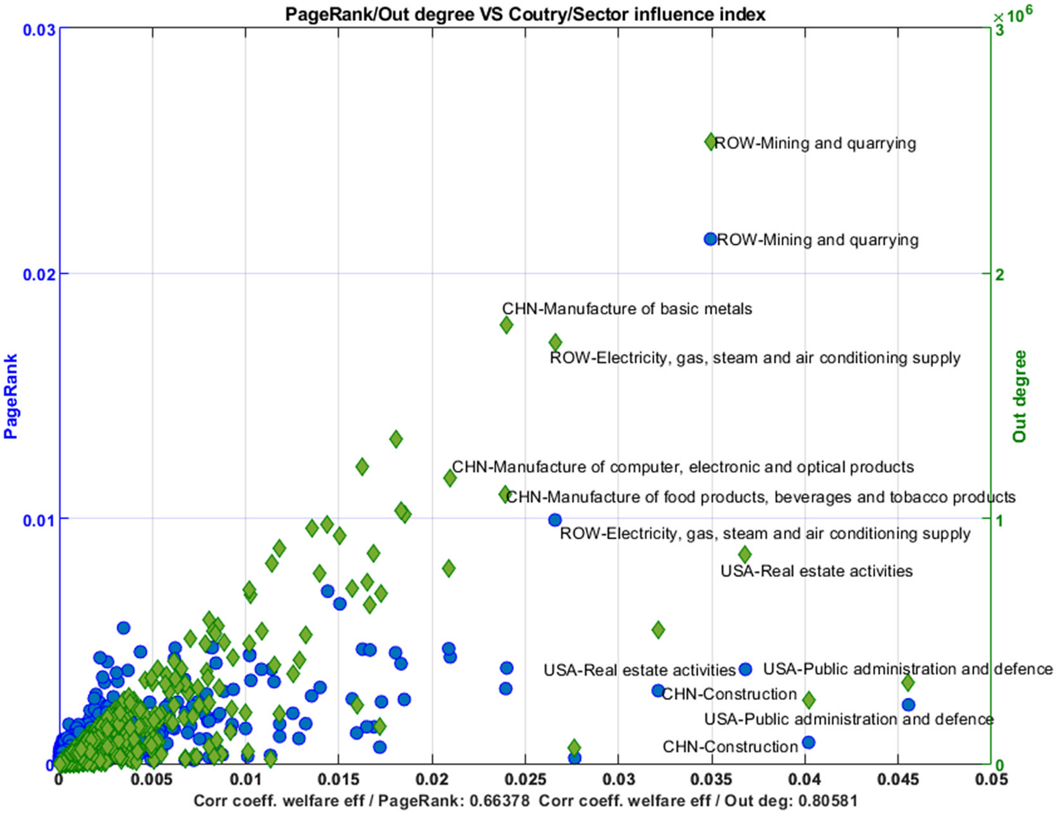

Fig. 4. Influence index plotted against out-strength (green diamonds) and out-PageRank score (blue circles). Below the figure we report the correlation coefficients between the measures.

preferences in each year and track the evolution of the sectoral influence index in time. We will denote the influence index of sector j in year t with φ_{jt}.

We show that the characteristics of the global production network have changed greatly from 2000 to 2014. In particular, in Section 5.1 we study how the influence of sectors has evolved leading to the emergence of new influential sectors, and in Section 5.2 we define a fragility index of the production system and study its evolution over time.

5.1. The rise of China: New sectors at the center of the network

The input–output network has evolved over the years leading to the emergence of new globally important sectors. To analyze how the role of sectors in the production network has changed over time, we study the change in the relative influence index from 2000 to 2014. To correctly compare the influence index of sectors across years, we must normalize the influence index and compute the relative influence index as:

$$\bar{\varphi}_{jt} = \frac{\varphi_{jt}}{\sum_j \varphi_{jt}} \tag{16}$$

The value of $\bar{\varphi}_{jt}$ describes the relative importance of sector j in year t. To analyze the change in the relative importance of sectors, we simply compute the difference between the normalized influence index in 2014 and in 2000 for each sector: $\Delta\bar{\varphi}_j = \bar{\varphi}_{j,2014} - \bar{\varphi}_{j,2000}$. The results are shown in Fig. 6. Some sectors that were peripheral in 2000, became central in 2014. The most spectacular change occurred in China, whose production system has gained an unprecedented centrality in the global production system. On the contrary, US and Japanese sectors have a smaller influence index in 2014 with respect to 2000. The change in the organization of world production is evident also from Fig. 7 where we show the 10 most influential countries in each year. The country relative influence index is computed as the average relative influence of the sectors belonging to each country. In Fig. 7 we also highlight the contribution to countries' influence of sectors belonging to different ISIC Rev.4 classification. The first evident result is that from 2000 to 2014 global production has evolved from a one star system to a multi-star system. In 2000 the influence of the US economy was largely and globally dominant. Over time, the relative influence of USA and of Japan have declined and the relative influence of China has increased. The second important result is the clear increase of the influence of all Chinese sectors, and especially of sectors belonging

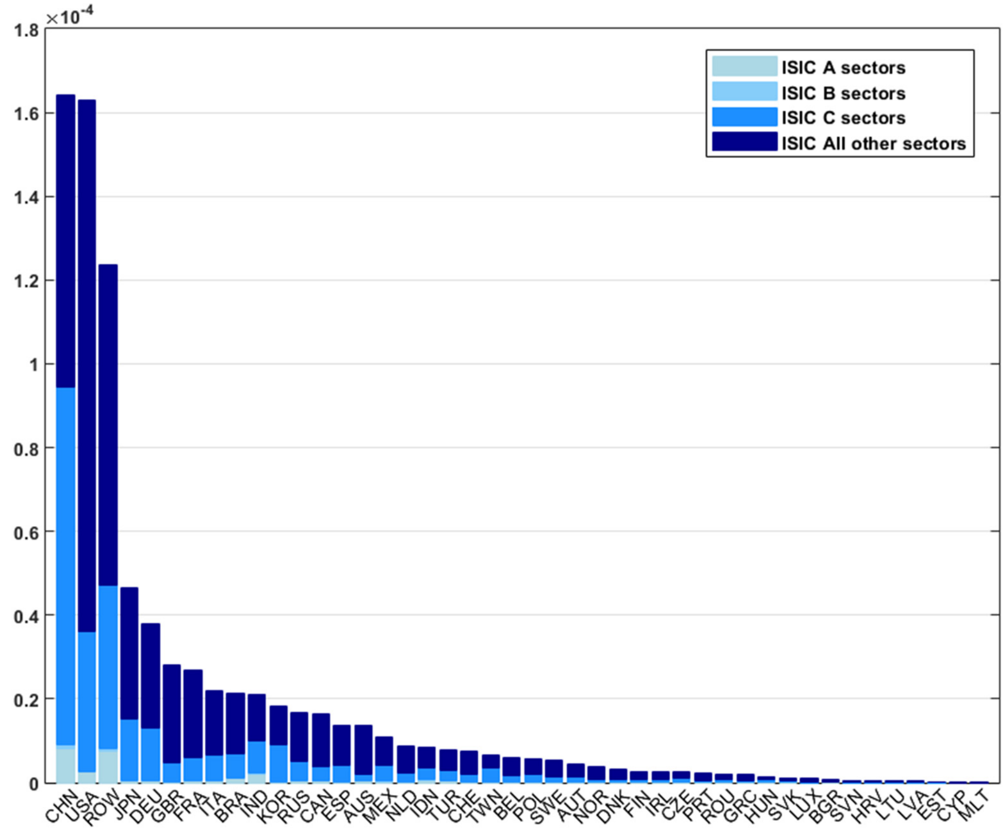

Fig. 5. The influence index aggregated by country (data WIOT 2014). The colors represent the contribution of different sets of sectors to the aggregated influence vector. ISIC A sectors are the primary sectors, i.e., sectors 1 and 2 in Table A.1; ISIC B sectors are Mining and quarrying sectors, i.e., sector 3 in Table A.1; ISIC C are the manufacturing sectors, i.e., sectors 5 to 23 in Table A.1; ISIC all other sectors are energy supply and services, i.e., sectors 24 to 56 in Table A.1.

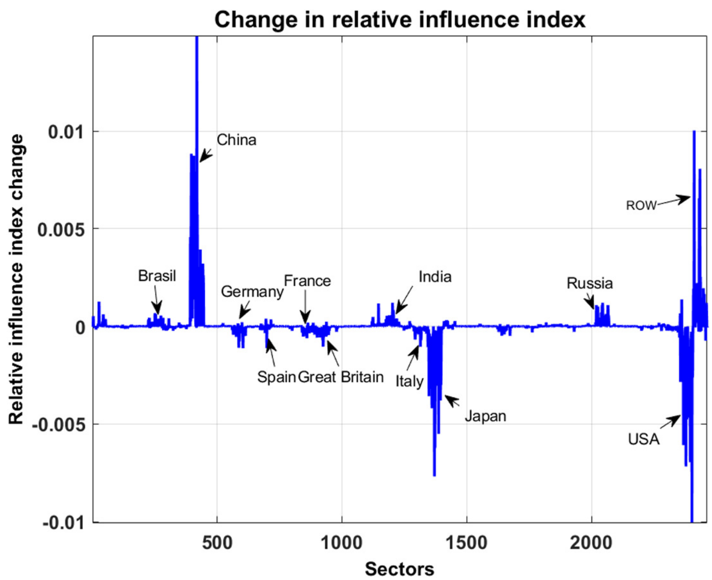

Fig. 6. Total change in sectors' relative importance from 2000 to 2014.

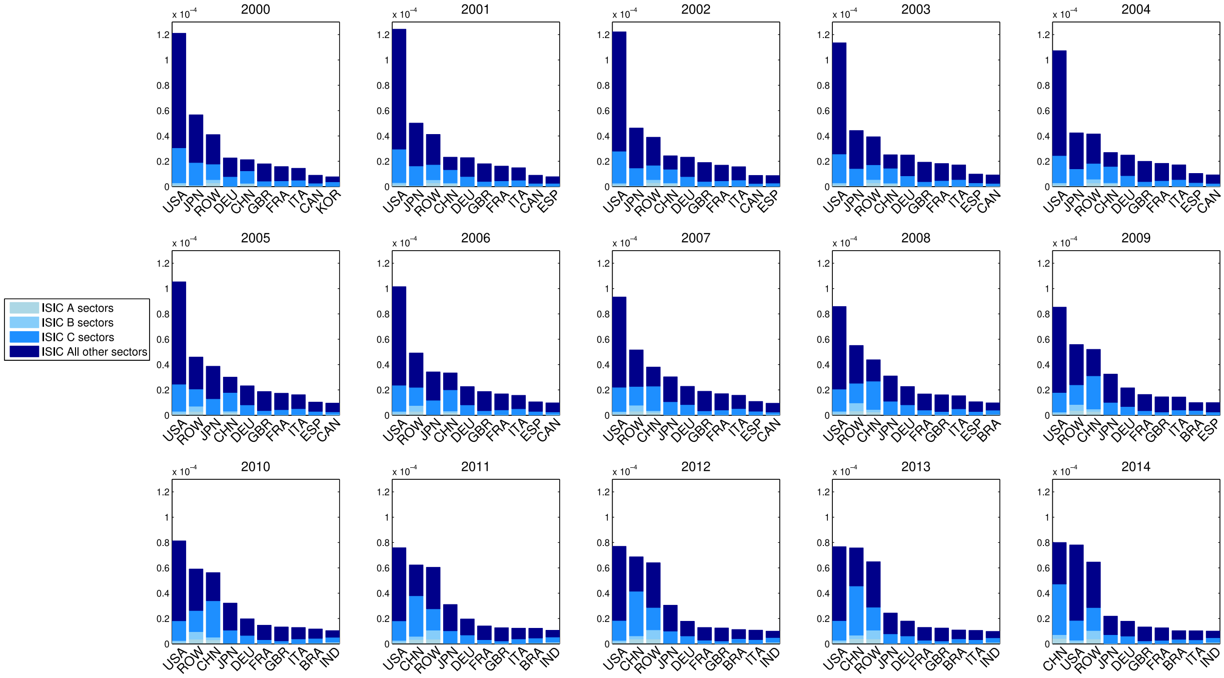

Fig. 7. Evolution of the 10 most influential country from 2000 to 2014. For comparison reasons the influence index has been normalized by its sum over all sectors. From a one-star to a multi star production system. The ISIC classifications are the following. ISIC A includes Agriculture, forestry and fishing. ISIC B includes Mining and quarrying. ISIC C includes Manufacturing. All other sectors includes all other sectors. The classification used here is the ISIC Rev. 4. Details can found on https://unstats.un.org/unsd/cr/registry/regcst.asp?Cl=27.

to ISIC C and to ISIC all-other sectors. The Chinese production system has increased its global importance due to an increase of global influence of its manufacturing and services sectors. To analyze this general pattern in world production organization, we aggregate all sectors in primary, mining, secondary and all-other sectors[11] and compute the aggregate change in the influence index in China and in the OECD countries. The results are shown in the left panel of Fig. 8. As already shown in Fig. 7, sectors belonging to Manufacturing, i.e. ISIC C, had a prominent role in the increase of the global importance of the Chinese economy. In general, from 2000 to 2014 world production organization has evolved leading to an increase of the importance of the Chinese production system and a decrease of the importance of OECD countries production systems.

To further highlight the *Chinese centralization* in world production and to determine which sectors have been leading the increase of the Chinese influence, we rank all sectors (from the highest to the lowest) with respect to the influence index in each year, and we analyze the evolution of the ranking of the 10 most central sectors in 2014. The left panel of Fig. 8 shows that some of the most central sectors in 2014 were peripheral sectors in 2000. This is particularly true for "CHN-Manufacture of computer, electronic and optical products", growing from the 105th position in 2000 to the 10th in 2014. Similar dynamics was experienced by "CHN - Manufacture of food products, beverages and tobacco products", "CHN-Manufacture of basic metals", climbing the ranking of most influential global sectors, respectively from the 93rd to the 9th position and from 86th to the 8th position.

5.2. Fragility of the input–output network

The input–output network transmits shocks across sectors and the properties of the input–output network determine how shocks spread through the production nodes. Different network topology can imply very different reactions of the economic system to random shocks. To understand how the production system has evolved over time, we define a measure of *fragility*. In particular, we define fragility as the vulnerability of the system to economic shocks. Therefore, in our definition, a production system is more fragile when shocks have greater impact on global real GDP. To measure fragility we compute the expected reaction of the economy to a productivity shock hitting a random sector. If the probability of the shock to hit is uniform across sectors and equal to $1/N$, then we can compute the expected reaction of

[11] Following the ISIC Rev.4 classification we include in the primary sector all sectors classified A, in the mining sector all sectors classified B, in the secondary sector all sectors classified C, in the tertiary all the remaining sectors.

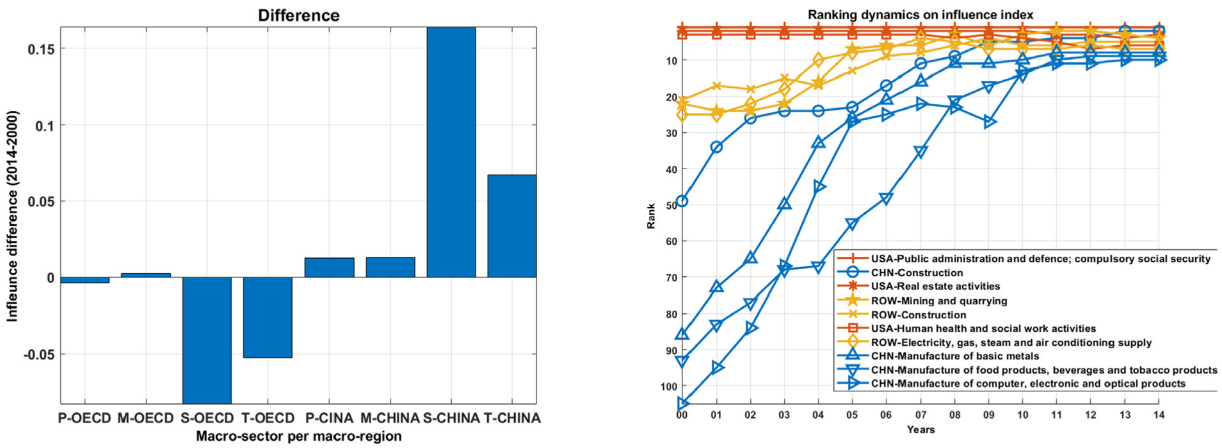

Fig. 8. **Left**: Time evolution of the ranking of the 10 sectors with the highest influence index in 2014. The x-axis displays the years, the y-axis the position in the ranking. Some of the most influential sectors in 2014, were non-influential sectors in 2000. **Right**: The change in cost effect index from 2000 to 2014 for *primary* (P), *mining* (M), *secondary* (S) and *all-other* (T) macro sectors for OECD and China.

the real GDP as:

$$\Phi_t = \frac{1}{N}\sum_j \varphi_{jt} \tag{17}$$

where φ_{jt} is defined in Eq. (13) and t is the time subscript. In the left panel of Fig. 9 we show the evolution of the fragility index Φ from 2000 to 2014. An increase of Φ_t should be interpreted as an increase of the fragility of the production system. In fact, when Φ_t is high, the expected effect of a shock on real GDP is higher. The evidence suggests that the fragility of the world input–output network has increased over time: from 2000 to 2014 the production system has evolved toward a more fragile configuration. To explain what happened, let us analyze in detail the evolution of Φ_t. Using the definition of Φ_t and Eq. (13) we can write:

$$\Phi_t = \frac{1}{N}\sum_j \varphi_{jt} = \frac{1}{N}\sum_j \sum_i c_{it}\frac{\partial \log p_{it}}{\partial \varepsilon_{jt}} = \frac{1}{N}\sum_j \sum_i c_{it}\bar{w}_{ijt}$$

It is convenient to recall that $\bar{w}_{ijt}$ is the element (i, j) of the Leontief inverse defined as $\bar{\mathcal{W}}_t \equiv (I - \hat{\mathcal{W}}_t)^{-1}$ and c_{it} denotes the consumption share of sector i in year t. The value of Φ_t depends on the interaction between the consumption shares and the elements of the Leontief inverse. It is difficult to write an analytical expression for $\bar{w}_{ijt}$. However, it is possible to give an intuition about its behavior in time using a simplification. For the sake of the illustration, let us assume that both labor shares and consumption shares are homogeneous across sectors, i.e., $\alpha_{it} = \bar{\alpha}$ and $c_{it} = \bar{c}$. In this simplified case, the size of Φ_t is proportional to the sum of the elements of the Leontief inverse, and we can write $\Phi_t = (1/N)\bar{c}\sum_j\sum_i \bar{w}_{ijt}$. The Leontief inverse can be rewritten as $\bar{\mathcal{W}}_t = (I - (1-\bar{\alpha})W)^{-1}$ or equivalently as:

$$\bar{W} = (I - (1-\bar{\alpha})W)^{-1} = I + (1-\bar{\alpha})W + (1-\bar{\alpha})^2W^2 + \cdots \tag{18}$$

From Eq. (18), it is clear that the labor share has a direct and negative impact on $\bar{W}$ and, in turn, on Φ_t. The interpretation is the following. If sectors are strongly connected to each other, the transmission of idiosyncratic shocks is stronger. On the contrary, if production is carried out using mainly labor, the connections between sectors are weaker and shocks spread less. In this regard, it is useful to think about the limit situation in which $\bar{\alpha} = 1$. In this extreme case firms are isolated (since production is carried out using only labor) and a shock to any sector does not affect the other sectors in the economy. On the contrary, when $\bar{\alpha} < 1$ sectors use intermediate inputs in the production process and shocks can spread through the input–output network. In our setting, with heterogeneous α_{it} and c_{it}, the size of Φ_t depends non-linearly on the distribution of α_{it} and c_{it} among the sector, but in general the size of Φ_t is positively related to the importance of intermediate goods in production. This point is easier to see if we use the definition of influence index related to the Domar weights. Using Eq. (15) we can write:

$$\Phi_t = \frac{1}{N}\sum_j \varphi_{jt} = \frac{1}{N}\sum_j \frac{s_{jt}}{\eta_t}$$

where s_j is total nominal production of sector j and η_t is the nominal GDP in period t. The fragility index is the average Domar weight in the economy. As the sum of Domar weight increases, i.e. as the amount of total production relative to GDP increases, the fragility of the production system to a random shock increases. The value of $1/\eta_t \sum_j s_{jt}$ is the ratio

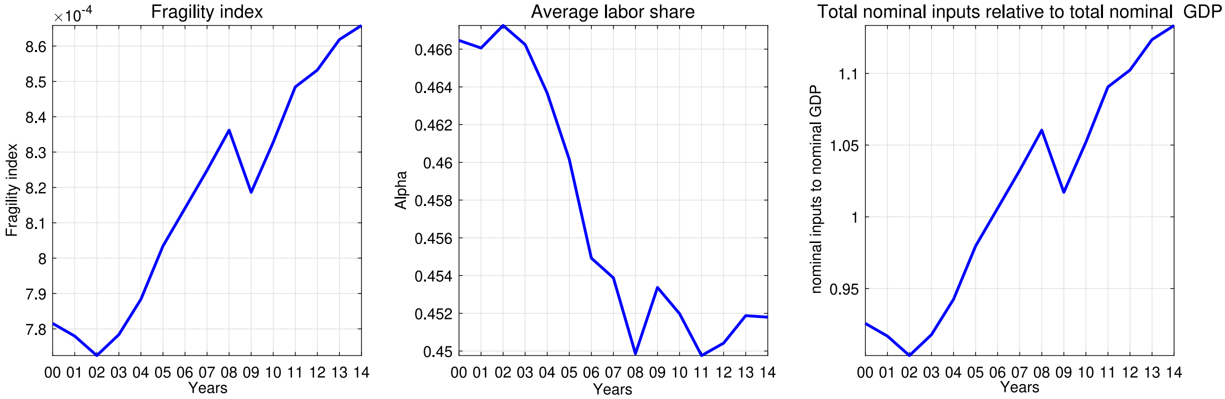

Fig. 9. Left: The evolution of the fragility index, Eq. (17). **Center**: The evolution of the average labor share. **Right**: The evolution of the ratio between the total nominal intermediate goods and nominal GDP.

between total nominal production and nominal GDP. This ratio depends on the intermediate goods intensity in global production, i.e. on the ratio between total nominal inputs and nominal GDP, which in turn depends on average labor share and on the topology of the input–output network. Fig. 9 shows the fragility index (left panel), the average labor share (central panel) and the ratio between total nominal inputs and nominal GDP (right panel). The right panel shows that the relative importance of intermediate goods in global production has increased over time. Sectors and countries have strengthen the links to each other. The central panel of Fig. 9 shows the average value of α_i in each year. Average labor share has decreased over time. The production technology has evolved over time, reducing the labor share and increasing the share of intermediate goods. As the relative importance of intermediate inputs increases, shocks spread wider on the network and have stronger effects on global production.

6. Conclusions

We study the topology of the production network by analyzing the propagation of productivity shocks through the input–output network. We build a simple general equilibrium model along the lines of Long and Plosser [2] and Acemoglu et al. [5], and calibrate the model using world input–output data ranging from 2000 to 2014. The calibrated model allows to define a centrality measure for each sector related to the impact of a productivity shock to that sector on global production. We call this centrality measure influence index and use it to study the structure of the production network and its evolution over time. By defining the nodes as firms and the links as flows of inputs, we can study the properties of the network with a proper theoretical framework.

We find that sectors' centrality is heterogeneous and asymmetric. Moreover, we find that the topology of the production network has changed from 2000 to 2014 both when looking at sector level and at country level. In fact, sectors that were in the periphery of the system in 2000 became very influential in 2014. Moreover, we aggregate sectors' centrality by country and find that the structure of the production network at the country level has evolved from a one-star to a multi-star structure. The evolution of the production system is strongly related to the *Chinese centralization*. From 2000 to 2014 the Chinese production system has gained an unprecedented centrality, and this is evident by investigating the evolution of the influence index both at the sector level and at country level.

In order to analyze the evolution of the production network, we also study how the reaction of the system in response to productivity shocks has changed over time. In particular we measure the reaction of real GDP to a shock to a random sector, and call this measure *fragility*. We find that the fragility of the production network has increased over time and show that this can be explained by an increase of the intermediate good intensity in global production.

CRediT authorship contribution statement

Jakob Grazzini: Conceptualization, Methodology, Investigation, Visualization, Writing – original draft, Writing – review & editing. **Alessandro Spelta:** Software, Data curation, Investigation, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Sectors and countries in WIOT

Table A.2 show identification numbers (ID) of sectors in each country. For example, the IDs from 1289 to 1344 represent Italian sectors. Using Table A.1 it is possible to determine the sector-country pair. For example sector 1289 is sector 1 in Table A.1 located in Italy, therefore sector $i = 1289$ is *Crop and animal production, hunting and related service activities* in Italy. Similarly, sector $i = 2360$ is sector 8 in Table A.1 located in the United States, therefore $i = 2360$ is *Manufacture,of paper and paper products* in the US. Each sector is numbered from 1 to 2464, and each number can be linked to the specific country and sector.

Appendix B. The empirical world input–output network

B.1. Data

The World Input–Output Table (WIOT) is a database providing the input–output tables for 43 countries[12] and 56 industries and a set of final consumption sectors for each country, covering the period from 2000 to 2014 [32]. The input–output table records the nominal flows of goods between each sector in each country and all other sectors in the dataset, at current prices. We aggregate the final consumption sectors in one global sector.[13]

B.2. Empirical analysis of the WIOT network

The WIOT has a network representation in which a generic element $W_{i,j}$ represents the nominal amount of goods j used as input by sector i, with $i, j = 1, \ldots, N$, where N is the number of sectors. The nodes are sectors and the links are the nominal flow of goods between sectors.[14] The WIOT network is therefore a directed and weighted network. In order to understand the *importance* of a node in the global production system, the literature on networks provides a set of centrality measures. The simplest measure is the degree centrality, which is defined as the total number of links attached to a node. Since the input–output network is directed and weighted, we use the directed-weighted version of degree centrality. We compute the weighted in-degree (or in-strength) and the weighted out-degree (out-strength) of each node. The in-strength is the sum of all incoming flows of goods, i.e. the sum of the nominal inputs used by a sector. The out-strength is the sum of the outflow of goods, i.e. the sum of the nominal inputs produced by a sector. Formally:

$$k^w_{in,i} = \sum_{j=1}^{N} W_{i,j} \quad k^w_{out,i} = \sum_{j=1}^{N} W_{j,i}$$

where $k^w_{in,i}$ is the in-strength of node i (i.e. the sum of elements of row i), and $k^w_{out,i}$ is the out-strength of node i (i.e. the sum of elements of column i). The degree centrality is a *local* measure of centrality, because it does not include information about the network position of the neighbors of a node, nor about the network position of the neighbors of its neighbors, etc. A well-known global centrality measure is the core of Google's PageRank algorithm [12]. The idea behind PageRank is that a node is systemically important if its neighbors are important *and/or* the neighbors of the neighbors are important (for a formal definition of the PageRank see Appendix B.3).

In Fig. 10 we show the tail distributions[15] of nodes' in-strength and out-strength scores and nodes' input PageRank and output PageRank. To have a complete topological description of the flow of goods in the world production network, we analyze the share of each sector in global consumption. We define c_i as the share of global nominal GDP consumed in each sector, and show its tail distribution in Fig. 10.

In order to further characterize the tail distributions, we test whether the empirical distributions are consistent with the hypothesis that they are drawn from a power law distribution. A power law distribution in general is defined as:

$$P(x > \bar{x}) \propto \left(\frac{x}{x_{min}}\right)^{-\gamma}$$

[12] 28 EU countries and 15 other major countries in the world and a model for the rest of the world.

[13] For further details on the data, see Timmer et al. [32] and Appendix C.

[14] In order to analyze the WIOT network we look only at the flows of goods between industries, ignoring the flows of goods toward the final consumption sector. In this way we obtain a square matrix.

[15] The tail distribution of x is defined as the probability $P(x > \bar{x})$.

Table A.1
The list of sectors in WIOT, with their ISIC Rev.4 code and name.

	ISIC Rev.4	Sector name
1	A01	Crop and animal production, hunting and related service activities
2	A02	Forestry, and logging
3	A03	Fishing, and aquaculture
4	B	Mining, and quarrying
5	C10–C12	Manufacture, of food products, beverages and tobacco products
6	C13–C15	Manufacture, of textiles, wearing apparel and leather products
7	C16	Manufacture, of wood and of products of wood and cork, except furniture;
8	C17	Manufacture, of paper and paper products
9	C18	Printing and reproduction of recorded media
10	C19	Manufacture of coke and refined petroleum products
11	C20	Manufacture, of chemicals and chemical products
12	C21	Manufacture, of basic pharmaceutical products and pharmaceutical preparations
13	C22	Manufacture of rubber and plastic products
14	C23	Manufacture, of other non-metallic mineral products
15	C24	Manufacture, of basic metals
16	C25	Manufacture, of fabricated metal products, except machinery and equipment
17	C26	Manufacture, of computer, electronic and optical products
18	C27	Manufacture, of electrical equipment
19	C28	Manufacture, of machinery and equipment n.e.c.
20	C29	Manufacture, of motor vehicles, trailers and semi-trailers
21	C30	Manufacture of other transport equipment
22	C31_C32	Manufacture, of furniture; other manufacturing
23	C33	Repair, and installation of machinery and equipment
24	D35	Electricity, gas, steam and air conditioning supply
25	E36	Water collection, treatment and supply
26	E37–E39	Sewerage; waste collection, treatment and disposal activities; materials recovery;
27	F	Construction
28	G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
29	G46	Wholesale trade, except of motor vehicles and motorcycles
30	G47	Retail trade, except of motor vehicles and motorcycles
31	H49	Land transport and transport via pipelines
32	H50	Water transport
33	H51	Air transport
34	H52	Warehousing and support activities for transportation
35	H53	Postal, and courier activities
36	I	Accommodation and food service activities
37	J58	Publishing activities
38	J59_J60	Motion picture, video and television programme production, sound recording and music publishing activities
39	J61	Telecommunications
40	J62_J63	Computer, programming, consultancy and related activities; information service, activities
41	K64	Financial service activities, except insurance and pension funding
42	K65	Insurance, reinsurance and pension funding, except compulsory social security
43	K66	Activities, auxiliary to financial services and insurance activities
44	L68	Real, estate activities
45	M69_M70	Legal, and accounting activities; activities of head offices; management consultancy, activities
46	M71	Architectural and engineering activities; technical testing and analysis
47	M72	Scientific, research and development
48	M73	Advertising, and market research
49	M74_M75	Other, professional, scientific and technical activities; veterinary activities
50	N	Administrative, and support service activities
51	O84	Public, administration and defence; compulsory social security
52	P85	Education
53	Q	Human, health and social work activities
54	R_S	Other, service activities
55	T	Activities, of households as employers; undifferentiated goods- and services-producing, activities of households for own use
56	U	Activities, of extraterritorial organizations and bodies, use

We estimate the parameters γ for each empirical distribution and test for power law hypothesis following Virkar et al. [37]. The parameter γ is the *exponent* or *scaling parameter* [38]. The results of the estimation and test procedure are shown in Fig. 10. According to Clauset et al. [38] the test cannot be rejected if the p-value (reported in Fig. 10) is greater than 0.1. Moreover, even if the goodness-of-fit test is useful to demonstrate that the power-law model is plausible, it does not determine whether it is more plausible than alternatives. To answer this question, we adopt the likelihood ratio test of Virkar et al. [37] considering four alternative distributions, the exponential, the log-normal and the stretched

Table A.2
Countries in WIOT and the ID of sectors.

Country	Sectors ID
Australia	1–56
Austria	57–112
Belgio	113–168
Bulgaria	169–224
Brasil	225–280
Canada	281–336
Switzerland	337–392
China	393–448
Cyprus	449–504
Czechia	505–560
Germany	561–616
Denmark	617–672
Spain	673–728
Estonia	729–784
Finland	785–840
France	841–896
Great Britain	897–952
Greece	953–1008
Croatia	1009–1064
Hungary	1065–1120
Indonesia	1121–1176
India	1177–1232
Ireland	1233–1288
Italy	1289–1344
Japan	1345–1400
South Korea	1401–1456
Lithuania	1457–1512
Luxemburg	1513–1568
Latvia	1569–1624
Mexico	1625–1680
Malta	1681–1736
The Netherlands	1737–1792
Norway	1793–1848
Poland	1849–1904
Portugal	1905–1960
Romania	1961–2016
Russia	2017–2072
Slovakia	2073–2128
Slovenia	2129–2184
Sweden	2185–2240
Turkey	2241–2296
Taiwan	2297–2352
USA	2353–2408
Rest of the World	2409–2464

exponential (Weibull) distribution, plus a power-law distribution with exponential cutoff. Given a pair of parametric models A and B for which we may compute the likelihood of the data, the model with the larger likelihood is a better fit. Using the ratio of the two likelihoods we are able to discriminate between alternative distribution.

Similarly to Acemoglu et al. [5] we found that the centrality measures associated to the outgoing links are power law distributed while the in-strength, the input PageRank and the consumption distributions follow a log-normal behavior. All this distributions are characterized by heavy-tails: there are few very important nodes playing the major role in the input–output network. Moreover, the power law distribution of the out-strength and of the output PageRank is usually interpreted in the network analysis literature as an evidence in favor of a *robust-yet fragile* feature [see39]. Robust-yet fragile feature means that a network is highly robust against the random removal of nodes, but it is fragile to the specific removal of the most highly connected nodes. If an important node on the network suffers an attack, the whole network is affected. In the economic literature, an input–output network with heavy tails implies that idiosyncratic shocks can influence the whole economic system. The important nodes have a crucial role in shaping the aggregate behavior.

B.3. PageRank algorithm

The PageRank computes the probability that a random walker will land on a given node. Suppose that each unit of input in the system moves according to a Markov process defined by an $N \times N$ transition probability matrix $\mathbf{p} = [\mathbf{p}]_{ij}$. Under a regularity condition (ergodicity of $\mathbf{p}$), there exists a real, positive vector $\boldsymbol{\pi}_{in} = [\boldsymbol{\pi}_{in}]_i$, $i = 1, \ldots, N$ such that $\boldsymbol{\pi}_{in} = \mathbf{p}\boldsymbol{\pi}_{in}$ and $\sum \boldsymbol{\pi}_{in}(i) = 1$. This is the PageRank vector. When $\mathbf{p}$ is not ergodic, one typically assumes that with some

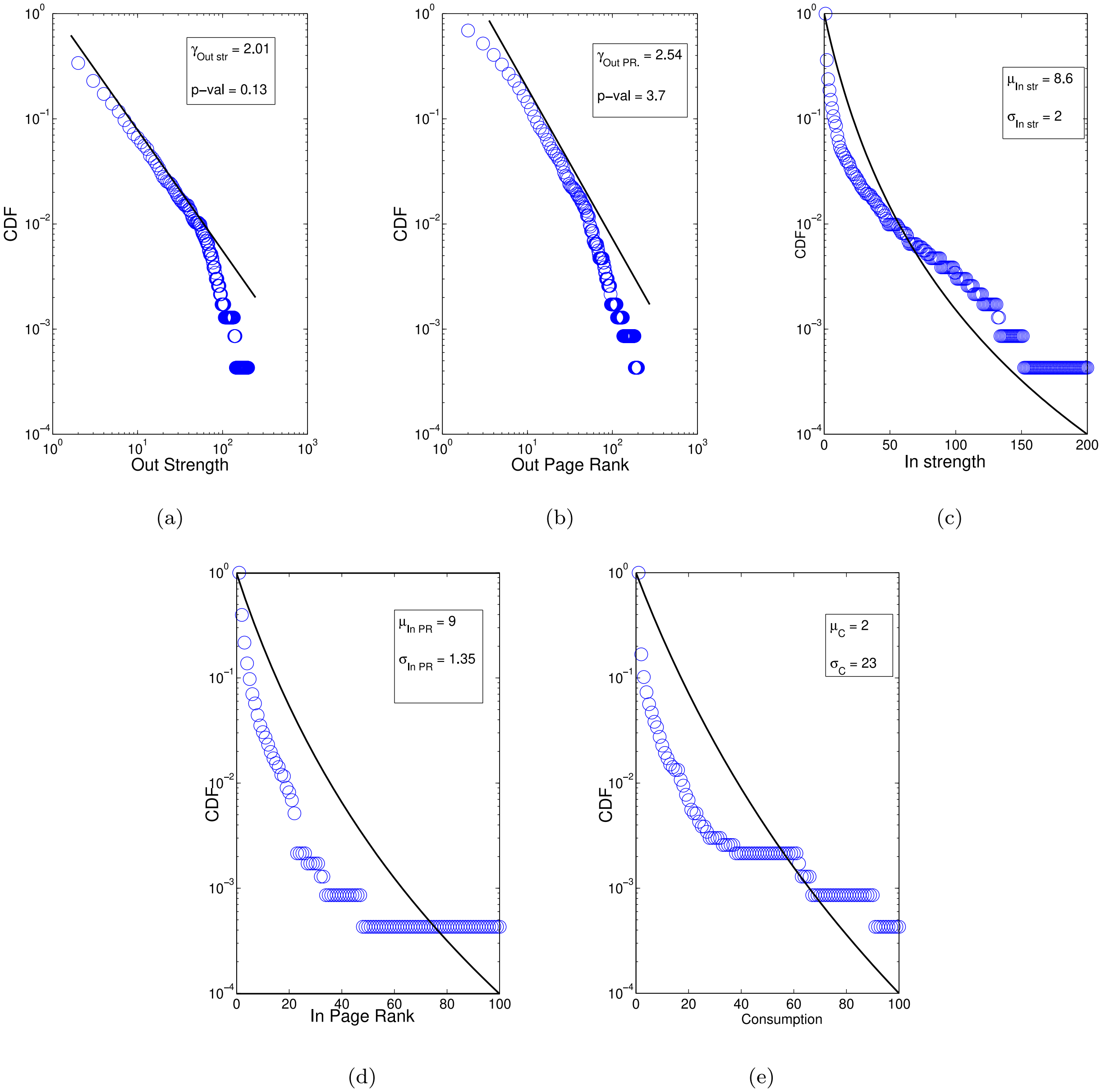

Fig. 10. Tail distribution functions and maximum likelihood fit for the out-strength (**a**), out-Page Rank (**b**), in-strength (**c**), in-Page Rank (**d**) and of consumption distribution (**e**). Notice that the out-strength and output PageRank follow a power law and are shown in log–log, the other distributions follow a log-normal behavior and are reported in lin-log scale. In the boxes we report the estimated coefficients and for the power-law distributions also the p-value.

small probabilities a unity of input moves from any i to any j, so $\boldsymbol{\pi}_{in}$ exists. The input PageRank is formally defined as

$$\boldsymbol{\pi}_{in} = \varepsilon \left(\mathbf{W\Phi} + \mathbf{fd}'_{out} \right) \boldsymbol{\pi}_{in} + (1-\varepsilon)\,\mathbf{f}$$

The parameter $\varepsilon \in (0, 1)$ is a damping parameter that determines the relative importance of the matrix $\left(\mathbf{W\Phi} + \mathbf{fd}'_{out} \right)$ and the teleportation distribution $\mathbf{f}$. $\mathbf{W}$ is the weighted adjacency matrix and $\boldsymbol{\Phi}$ is a diagonal matrix with elements $\boldsymbol{\Phi}_{ii} = \min\left(\frac{1}{k_{out,i}}, 1 \right)$. The second component is $\mathbf{fd}'_{out}$ where $\mathbf{d}_{out}$ is a column vector with elements $\mathbf{d}_{out,i} = 1$ if $k_{out,i} = 0$ and otherwise 0. The vector $\mathbf{d}_{out}$ identifies those individuals that have no outgoing links and avoids that the random walker 'gets stuck' on a dead-end node. Furthermore, not all nodes in the network are necessarily directly connected to one another. Therefore, the PageRank is adjusted again so that with probability $(1-\varepsilon)$ the walker is allowed to jump to any other node in the network according to $\mathbf{f}$. This is the reason why the vector $\mathbf{f}$ is called the teleportation distribution.

Table C.3
Input–output table.

Sectors	Sector 1	Sector 2	Sector n	Value input	Value added	Value production
Sector 1	$W_{1,1}$	$W_{1,2}$	$W_{1,n}$	$\sum_{j=1}^{n} W_{1,j}$	va_1	Y_1
Sector 2	$W_{2,1}$					
Sector n	$W_{n,1}$					
Value intermediate	$\sum_{i=1}^{n} W_{i,1}$					
Final consumption	D_1					
Value use	$D_1 + \sum_{i=1}^{n} W_{i,1}$					

Since the matrix **W** is asymmetric; as with the case of the in- and out-degree, it is possible to derive two measures of centrality by looking at the transpose of **W**. The output PageRank is formally defined as follows:

$$\boldsymbol{\pi}_{out} = \varepsilon \left(\mathbf{W}' \boldsymbol{\Psi} + \mathbf{f} \mathbf{d}'_{in} \right) \boldsymbol{\pi}_{out} + (1 - \varepsilon)\, \mathbf{f}$$

now $\boldsymbol{\Psi}_{ii} = \min\left(\frac{1}{k_{in,i}}, 1\right)$ and $\mathbf{d}_{in,i} = 1$ if $k_{in,i} = 0$ and 0 otherwise.

Appendix C. Data and calibration

C.1. WIOT

The World Input–Output Table (WIOT) is a database providing the input–output tables for 43 countries (28 EU countries and 15 other major countries in the world and a model for the rest-of-the-world) and 56 industries and a set of final consumption sectors for each country, covering the period from 2000 to 2014. We aggregate the final consumption sectors in one global sector and transpose the original WIOT data to make the input–output table consistent with the notation used in the model described in Section 2. WIOT provides data on the flow of inputs between sectors, the value of consumption and the value added for each sector. Column i provides the nominal flow of goods from sector i to the other sectors and the flow of goods from sector i to the final consumption sector. The sum of column i provides the total use of good i. Row i provides the inputs flowing into sector i and the value added produced by sector i. The sum of nominal intermediate inputs used by sector i plus the value added in sector i, i.e. the sum over row i, gives the total value of production of sector i. The sum of the value added column gives the nominal GDP. A general representation of the input–output data is shown in Table C.3.

C.2. Calibration

The model described in Section 2 assumes market clearing and perfectly competitive markets. The former assumption implies that total nominal production is equal to total nominal demand. The latter assumption implies zero profits, and that value added in each sector is equal to the wage bill. Assuming that total labor is 1, the nominal GDP is equal to the nominal salary. We assume a Cobb–Douglas production function:

$$x_i = z_i l_i^{\alpha_i} \prod_{j=1}^{N} x_{ij}^{(1-\alpha_i)w_{ij}} \tag{19}$$

where α_i is the optimal share of labor, and $(1-\alpha_i)w_{ij}$ is the optimal share of input j. The model is therefore reproducing the WIOT data by setting α_i equal to the observed share of labor:

$$\alpha_i = \frac{va_i}{Y_i} \quad \forall i \tag{20}$$

where va_i is the value added in sector i, and Y_i is the total nominal production of sector i. Noting that $(1-\alpha_i)w_{ij}$ is the share of input j, we can set:

$$w_{i,j} = \frac{W_{ij}}{\sum_j W_{ij}} \quad \forall i \tag{21}$$

the weight of sector j in the production of good i is determined as the share of input j on the total use of intermediate inputs by sector i. Assuming an inelastic total supply of labor equal to 1 implies that total nominal GDP is equal to the nominal wage. Market clearing implies that nominal wage is equal to nominal consumption. In the data this assumption does not hold due to saving. However, we force market clearing and we set the nominal wage, η, equal to the observed sum of nominal consumption:

$$\eta = \sum_i C_i \tag{22}$$

Table C.4
Input–output table from the model.

Sectors	Sector 1	Sector 2	Sector n	Value input	VA	Value production
Sector 1	$p_1 x_{1,1}$	$p_2 x_{1,2}$	$p_n x_{1,n}$	$\sum_{j=1}^{n} p_j x_{1,j}$	$l_1 \eta$	$p_1 x_1$
Sector 2	$p_1 x_{2,1}$					
Sector n	$p_1 x_{n,1}$					
Value intermediate	$\sum_{i=1}^{n} p_1 x_{i,1}$					
Final consumption	$p_1 d_1$					
Value use	$p_1(d_1 + \sum_{i=1}^{n} p_1 x_{i,1})$					

where C_i is the nominal consumption sector i. Households have a Cobb–Douglas utility function:

$$\mathcal{U} = \prod_{j=1}^{N} x_i^{c_i} \tag{23}$$

Given the Cobb–Douglas consumption function, the share of budget spent on good i is c_i. We set the utility parameter equal to:

$$c_i = \frac{C_i}{\eta} \tag{24}$$

where C_i is nominal consumption in sector i. By construction $\sum_i c_i = 1$.

There are (very) few sectors which have negative value added in the data[16] that cannot be handled in our model. To solve the problem we set the value added of these sectors to zero. Since these sectors are very small, this assumption does not affect significantly the output of the model. The average percentage difference between the observed nominal input–output flows and the theoretical nominal input–output flows produced by the calibrated model is in the order of 10^{-5} in 2014. The calibrated model reproduces the observed nominal flow of goods. A general representation of output of the model is shown in Table C.4. Where $p_i x_{i,j}$ is the equilibrium nominal flow of goods from sector j to sector i (p_i is the equilibrium price of good i and $x_{i,j}$ is the equilibrium quantity of good j used by sector i). The value of final consumption of good i is given by $p_i d_i$, where d_i is the demand for goods i by the household. Total nominal production is given by $p_i x_i$. The labor hired by sector i is l_i. The model uses nominal values to determine prices and quantities.

C.3. Downstream propagation of technology shocks

Shocks propagate only from upstream firms to downstream firms. We show this property of the model in the following.

Proposition C.1. *If the economic system is characterized by Cobb–Douglas production functions and utility functions, supply shocks propagate only downstream.*

To simplify the discussion, consider an economic system characterized by a directed and weighted input–output matrix, W, *without circular links*. This network topology allows to clearly determine the upstream–downstream relations. In this simplified environment, it easy to show how shocks spread through the production network. A network without circular links, can be represented by a lower-triangular adjacency matrix:

$$W = \begin{pmatrix} w_{1,1} & 0 & \cdots & 0 \\ w_{2,1} & w_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ w_{N,1} & w_{N,2} & \cdots & w_{N,N} \end{pmatrix}$$

where the element $w_{i,j}$ represents a input–output link from node j to node i. Note that node 1 supplies input to all nodes, node 2 supplies inputs to all nodes but node 1, and so on. This implies that node 1 is upstream to all nodes and node 2 is upstream to all nodes but node 1. Node N supplies intermediate goods only to it self. For the properties of triangular matrices, the matrix $\bar{W} = (I - W)^{-1}$ is still a lower triangular matrix:

$$\bar{W} = \begin{pmatrix} \bar{w}_{1,1} & 0 & \cdots & 0 \\ \bar{w}_{2,1} & \bar{w}_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{w}_{N,1} & \bar{w}_{N,2} & \cdots & \bar{w}_{N,N} \end{pmatrix}$$

[16] The sector with negative value added in each year are the following. In year 2000 Malta-Mining and Quarrying,; in year 2011 Latvia-Air Transport and Malta-Mining and Quarrying; in year 2012 Malta-Mining and Quarrying; in year 2013 Luxemburg-Water Transport and Luxemburg-Postal and courier activities; in year 2014 Luxemburg-Water Transport.

From Eq. (9), we know that the derivative of the price of sector i with respect to a productivity shock on sector j is positive only if the element $\bar{w}_{i,j}$ is positive. This implies that the price derivative is positive only if sector i is downstream with respect to sector j. Upstream sectors are unaffected: supply shocks propagate only downstream. The result can be easily generalized to the case of a more realistic input–output network, where circular links are present. A shock to sector j affects other sectors only through its supply of inputs, i.e., it affects only sectors which are directly or indirectly downstream to sector j. This is the first useful information provided by the model with respect to a pure empirical study of the input–output network. Therefore, according to the model, the only relevant network measures in the input–output network are the ones regarding the position of sectors as input suppliers. Only outgoing links are relevant to measure the importance of a sector in the economic system. This result depends crucially on the assumption of Cobb–Douglas production function and is important in order to interpret the results.

Appendix D. Derivation of the Gini coefficient

In order to better describe the inequality among the influence index, we derive the Lorenz curve and the Gini coefficient in terms of the power law exponent associated with φ_j [see 40]. We first define the complementary CDF associated to such a distribution $P_<(\varphi) = 1-(P_>(\varphi))$ which correspond the fraction of country-sectors with GDP effect index no bigger than φ. Then we compute the Lorenz curve $L(P_<)$ as:

$$L(P_<) = \frac{\int_0^{P_<} \varphi\left(P'_<\right) \mathrm{d}P'_<}{\int_0^1 \varphi\left(P'_<\right) \mathrm{d}P'_<}$$

where $\varphi(P_<)$ is the inverse of the complementary CDF and it gives the maximum value of the GDP effect index . The numerator is the country-sector subpopulation's average GDP effect index. The denominator is the average GDP effect index of the whole population. For power law distributions, assuming a finite mean (i.e. the exponent must be greater than one), the quantile function can be defined as:

$$\varphi(P_<) = \frac{\varphi_{\min}}{(1-P_<)^{1/(\gamma_\varphi-1)}}$$

where γ_φ is the power law coefficient associated to the distribution of φ_j. So the Lorenz curve is computed as:

$$L(P_<) = 1-(1-P_<)^{1-1/(\gamma_\varphi-1)}$$

The Gini coefficient, which measures the deviation of the Lorenz curve from perfect equality is twice of the area between the Lorenz curve and the equi-distribution line, i.e.

$$G = 1-2\int_0^1 L(P_<)\,\mathrm{d}P_<$$

Then, for power law distributions, the Gini coefficient reads as:

$$G = \frac{1}{2(\gamma_\varphi-1)-1}$$

Thus, the Gini coefficient, which measures the statistical dispersion of GDP index distribution among country sector, takes value of $G = 0.8621$ meaning that the few sectors owning a huge effect on world real GDP coexist with many small sectors with very little effect on world real GDP.

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