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Improved MDS-based Localization with Non-line-of-sight RF links

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Abstract The performance of indoor positioning techniques that use radio-frequency (RF) signals is usually degraded in non-line-of-sight (NLOS) environments. In this paper, we propose a technique for estimating NLOS biases and measurement noise in distances under multidimensional scaling (MDS) based positioning with fixed nodes. An ideal matrix of pairwise distance measurements exhibits a symmetry that allows to compute mobile node positions from those of fixed ones, and then recompute exactly the fixed node positions from the earlier computed mobile node positions. In a NLOS environment, this symmetry is lost; fixed node positions can not be reproduced exactly. This work exploits the error in the recomputation of fixed node positions for the correction of NLOS biases and noise in the pairwise distances. A constrained-optimization problem is formulated to estimate biases for each measured distance and final mobile node positions under the MDS scheme. A supplementary approach is presented for special cases where the number of mobile nodes is less than 3. Experimental results show that position errors can be reduced by up to 28% for a set up to 4 fixed and 3 mobile nodes. Simulations are used to further validate the results for larger deployments of nodes.

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1 Introduction

Under range-based radio positioning, the occurrence of multipath and attenuation effects caused by obstacles, occlusions or interference in the deployment region of radio positioning nodes contribute negatively to the performance of indoor positioning algorithms. A physical object lying between transmitter and receiver such that there is no clear visual line of sight between both will cause the transmitted signal 1) to be attenuated or diffracted if the obstacle is not completely blocking or 2) to travel a different path leading to anomalies at the receiving end. This would often cause the Received Signal Strength (RSS) to be weakened or cause the Time of Flight (ToF) to take longer, making the estimated distance to be longer than the actual length; these anomalies are classified in this work as *non-line-of-sight (NLOS) effects*. Figure 1 gives a pictorial description of a signal not having clear line-of-sight between anchor and tag.

Positioning with Multidimensional Scaling (MDS) places each node in a distance matrix into a 2 or 3 dimensional space such that the distance information of the derived configuration of nodes matches as closely possible the original distance matrix. In [6], a formulation in which the notion of anchors was included directly in the MDS algorithm was presented. This formulation was further extended in [12] for deployments where only pairwise distances between fixed nodes and mobile nodes, and fixed node positions are known, i.e., the distances between mobile nodes and their positions are unknown. Examples of such deployments can be found in scenarios such as:

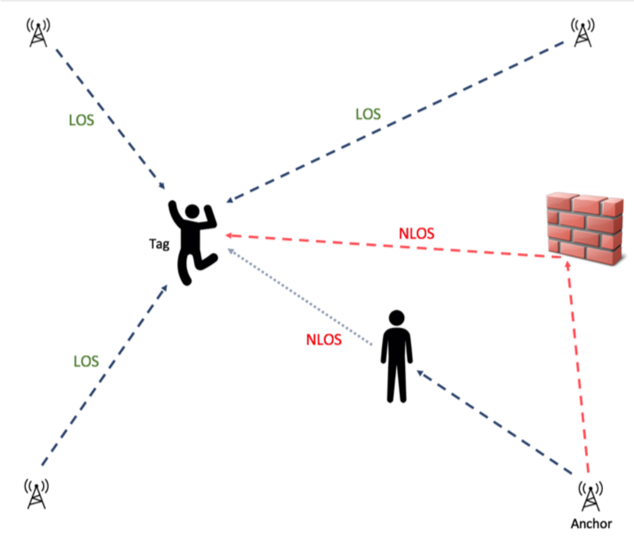


Fig. 1: NLOS effects from physical object lying between anchor and tags, i.e., the two considered nodes. The picture also shows examples of line-of-sight (LOS) situations.

1. localization schemes that use single/round-trip ToF between fixed and mobile nodes,
2. localization schemes where mobile nodes read Received Signal Strength Indication (RSSI) for signal transmissions from fixed nodes or the other way around.

Henceforth we will refer to mobile nodes as tags and fixed nodes as anchors for the sake of brevity.

The advantage of such deployments with missing tag-to-tag distances is, firstly, reduced architecture complexity and time required to collect all the required distance measurements in order to build a matrix (the matrix is sparse) and secondly, the elimination of potential NLOS biases and noise which may be present in the pairwise distances between tags (henceforth called tag within-sets distances). The latter will later on be shown to be vital for the performance of our proposed NLOS mitigation scheme.

In this work, we propose an NLOS mitigation technique under the MDS scheme. MDS has the advantage of being more scalable than other range-based localization methods in that all positions are estimated simultaneously. In our case, tag positions are computed given a matrix of pairwise distance measurements and known anchor positions. For a matrix of NLOS and noise free pairwise distances, tag positions can be computed from those of anchors, and the anchor positions can be recomputed exactly from those tags by swapping anchors with tags in the formulation proposed in [12]. This symmetry in anchor-tag and tag-anchor estimation applies for node deployments with at least 3 tags and 3 anchors, and a matrix of distance measurements free of NLOS bias and measurement noise. In real life applications, this is hardly the case, as distances are almost never accurate. Moreover, since the aforementioned symmetry is lost, tag positions estimated from the skewed matrix do not represent the exact tag positions. We provide an estimate on the goodness of this skewed matrix by recomputing anchor positions from the earlier computed tag positions, and taking the error with respect to the known anchor positions. The error in the tentative anchor positions will be proportional to the amount of noise and NLOS bias in the matrix of distance measurements. We derive this error analytically and we formulate a constrained optimization problem to find bias values that yield the least error on tentative anchor positions. Since the tag within-sets distances are not available, the support for computation of tentative anchor positions is provided only by anchor-tag pairwise distances (henceforth called between-sets distances), therefore making the error a better reflection of between-sets NLOS biases and noise.

We verify the accuracy improvements of the proposed approach by experiments which indicate significant reductions in positioning root-mean-square error (RMSE) by up to 28% for a setup of 4 anchors and 3 tags. Simulations are used to further demonstrate accuracy improvements for deployments involving an area with larger dimensions, and higher number of anchors and tags.

1.1 Organization of this paper

The remainder of this paper is organized as follows. An overview of related works is provided in Section 2. Section 3 briefly introduces MDS and a formulation that includes the notion of anchors in summarized mathematical details. In section 4, the proposed NLOS removal technique is presented. Section 5 shows experimental results that demonstrate the improved positioning accuracy of the proposed technique while subsection 5.1 shows simulation results for larger numbers of node deployments. And finally, Section 6 concludes the paper.

2 Related works

The complete removal of NLOS effects and measurement noise may be impossible due to incomplete knowledge about the environment and the nature of obstacles. However, the removal of some amount of NLOS bias and noise from distance measurements has been investigated under various localization contexts that use RF. The more common NLOS mitigation techniques [4,8–11,21,24,25] use the propagation channel model or channel statistics from historical data to distinguish between LOS/NLOS signals, in which NLOS ones are detected by identifying some anomalies in the signal property with differentiated values for direct and non-direct paths. Other methods for distinguishing NLOS signals from LOS ones examine the fitness of the various distance measurements under a specified cost function and may employ a priori probabilities about the distribution of NLOS-prone nodes in the deployment area. Methods described in [22] and [23] are applicable to localization algorithms that compute positions on a per tag basis, unlike MDS that computes all positions jointly. The work in [23] presented comprehensive approaches where new distances and bias values are estimated from the original distance measurements via a minimization problem whose performance can be improved by a priori probabilities. A Sequential Quadratic Programming (SQP) based formulation and a Maximum Likelihood Estimation (MLE) where both introduced. Bias was estimated on a per anchor basis with the advantage of reducing the complexity of the minimization problem, and the disadvantage of penalizing the accuracy of NLOS bias estimation. Moreover, the minimization problem was formulated per tag, so that a separate minimization is performed for each tag, leading to likely scaling issues. In a similar related work [18], the authors tune the elements of the covariance matrix R of a Biased Extended Kalman Filter (BEKF) with respect to NLOS identification; matrix elements are increased if NLOS is detected or decreased otherwise. Since their approach is used alongside trilateration, it also does not scale well. Furthermore, their approach works considerably better for setups with a few number of NLOS distances compared to LOS ones. When NLOS measurements are relatively large, a scarcity of LOS measurements may lead to estimation failure.

This work expands on previous results obtained in [13] where this constrained-optimization problem for mitigating NLOS bias and noise under the MDS scheme using recomputed anchor positions was first proposed. This work adds practical experiments in order to validate our earlier claims in a real life scenario with a setup of nanoLoc [1] transmitters and receivers that use round-trip time of flight (RT-ToF) for ranging. The experiments present the advantage of testing the proposed method in an uncontrolled environment to allow a better understanding of the localization performance gains and possible drawbacks. We improve on the earlier proposed method by adding a per-distance NLOS bias and noise estimation using SQP.

3 Node Positioning with Multidimensional Scaling

MDS is a non-linear dimensionality reduction technique for visually expressing the similarity of objects in a dataset [3]. The dataset is a matrix of pairwise distances between the objects and MDS places the objects in an embedding with lower dimensions, usually 2 or 3 for effective visualization purposes, assigning each object coordinates by which similarities are easy to visualize while the pairwise-distances are preserved. The original and derived distance matrices can be used to formulate a cost function. If the cost is a residual sum of squares, it is called *stress*, denoted with $\sigma(\boldsymbol{X})$, where $\boldsymbol{X}$ is the configuration of all node positions. The stress $\sigma(\boldsymbol{X})$ is defined by:

$$\sigma(\boldsymbol{X}) = \sum_{i<j\leq n} w_{ij}(d_{ij}(\boldsymbol{X}) - \delta_{ij})^2 \tag{1}$$

$$w_{ij} = \begin{cases} 1, \text{ if } \delta_{ij} \text{ is known} \\ 0, \text{ if } \delta_{ij} \text{ is missing} \end{cases} \tag{2}$$

where $d_{ij}(\boldsymbol{X})$ is the distance matrix for the derived configuration, δ_{ij} is the original distance matrix and w_{ij} are weights that assign importance to individual distance measurements. Alternative weighting schemes can be used as long as $w_{ij} \geq 0$ [3] and typically, the weight matrix $\boldsymbol{W}$ is symmetric and non-negative, and less often hollow.

In order to minimize the stress $\sigma(\boldsymbol{X})$ in Equation 1, a procedure called *stress majorization* is generally applied. Moreover, since $\sigma(\boldsymbol{X})$ can be an arbitrarily complicated function, a convex function τ which bounds σ from above and touches its surface at a support point $\boldsymbol{Z}$ can be minimized in place of σ using an iterative procedure called *Scaling by MAjorizing a COmplicated Function* (SMACOF) [14]. This iterative minimization procedure terminates when the stress value converges, i.e., the difference between values from the previous and current iteration fall beneath a threshold ε. Formally, the convergence is verified by the following condition:

$$\sigma(\boldsymbol{X}^{(k-1)}) - \sigma(\boldsymbol{X}^{(k)}) < \varepsilon \tag{3}$$

where configuration $\boldsymbol{X}$ is the parameter that minimizes the function τ and is given by:

$$\hat{\boldsymbol{X}} = \min_{\boldsymbol{X}} \tau(\boldsymbol{X},\boldsymbol{Z}) = \boldsymbol{V}^{+}\boldsymbol{B}(\boldsymbol{X})\boldsymbol{Z} \tag{4}$$

The SMACOF iterative procedure, which has been shown to monotonically decrease the stress [14, 16], can be rewritten as:

$$\boldsymbol{X}^{(k)} = \boldsymbol{V}^{+}\boldsymbol{B}(\boldsymbol{X})^{(k-1)}\boldsymbol{X}^{(k-1)} \tag{5}$$

Equation 5 is known as the *Guttman transform*. The matrix $\boldsymbol{V}^{+}$ is the Moore-Penrose pseudoinverse of matrix $\boldsymbol{V}$ which is used since $\boldsymbol{V}$ is not full-rank, and the matrices $\boldsymbol{V}$ and $\boldsymbol{B}$ are matrices with elements v_{ij} and b_{ij} defined by:

$$v_{ij} = \begin{cases} -w_{ij} & i \neq j \\ -\sum_{j=1,j\neq i}^{N} v_{ij} & i = j \end{cases} \tag{6a}$$

$$b_{ij} = \begin{cases} -\frac{w_{ij}\delta_{ij}}{d_{ij}(\boldsymbol{X})} & d_{ij}(\boldsymbol{X}) \neq 0, i \neq j \\ 0 & d_{ij}(\boldsymbol{X}) = 0, i \neq j \\ -\sum_{j=1,j\neq i}^{N} b_{ij} & i = j \end{cases} \tag{6b}$$

where N is the total number of nodes in the deployment region.

3.1 MDS with Anchors

In previous MDS formulations, positions of anchors and tags are computed, and then a post-alignment or roto-translation is performed to adjust tag positions with respect to the known anchor positions [19,20]. This approach works relatively well when the configuration of computed anchor positions correspond exactly to the known ones. Otherwise, small errors are propagated to the tag positions by the subsequent roto-translation. To eliminate these errors, [6] introduced a MDS formulation with a SMACOF procedure in which tag positions $\boldsymbol{X}_t \subset \boldsymbol{X}$ are computed iteratively with respect to those of anchors $\boldsymbol{X}_a \subset \boldsymbol{X}$. Tag positions were derived analytically as with Equation 5 by minimizing the function τ with respect to tag positions only, yielding the following relationship:

$$\boldsymbol{X}_t = \boldsymbol{V}_{11}^{-1}(\boldsymbol{B}_{11}\boldsymbol{Z}_t + (\boldsymbol{B}_{12} - \boldsymbol{V}_{12})\boldsymbol{X}_a) \tag{7}$$

where $\boldsymbol{V}_{11}$, $\boldsymbol{V}_{12}$, $\boldsymbol{B}_{11}$ and $\boldsymbol{B}_{12}$ are submatrices of the $\boldsymbol{V}$ and $\boldsymbol{B}$ matrices and their sizes depend on the number of anchors n and number of tags m.

3.2 MDS with Anchors and missing Tag interactions

In the previous formulation of SMACOF with anchors, anchor within-sets, anchor-tag between-sets and tag within-sets distances are all assumed to be available. However, for such cases where tag interactions are not available, [12] showed that Equation 7 can be reduced to a more simplified form that neglects tag within-sets distances. This simplified form is given by:

$$\boldsymbol{X}_t = \frac{\boldsymbol{B}_{11}\boldsymbol{Z}_t + (\boldsymbol{B}_{12} + \boldsymbol{11}_{m,n})\boldsymbol{X}_a}{n} \tag{8}$$

where $\boldsymbol{11}_{m,n}$ is a matrix of ones with shape $m \times n$. The formulation is similar to the original in that tag positions are computed iteratively until a stationary support $\boldsymbol{Z}_t$ is reached, which overall corresponds to a convergence of the stress.

4 Mitigation of NLOS effects and measurement noise

In this section, we propose a heuristic for the removal of NLOS in a matrix of pairwise distances. This heuristic is based on the knowledge of the symmetric property of ideal distance matrices, which allows to compute tag positions from those of anchors, and recompute the same anchor positions from the earlier computed tag positions.

In the presence of NLOS effects, the symmetry of the pairwise distance matrix is lost. However, the ensuing asymmetry allows to mitigate NLOS effects by minimizing the error on the recomputation of anchors. In Figure 2, we show

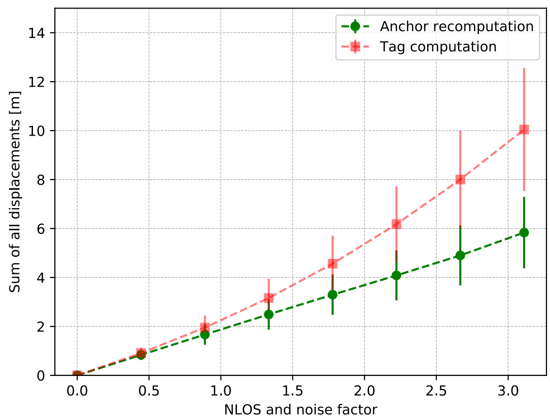


Fig. 2: Correlation between position errors for tags and recomputed anchors.

how the errors on the recomputation of anchor positions grows monotonically with increasing NLOS and measurement noise. The data for the plots was generated from simulations where NLOS effects and measurement noise are added to a distance matrix using the ToF error model described in [2] and tag positions are computed using the skewed distance matrix alongside anchor positions. The NLOS effects and noise were amplified using multiplicative factors, with a factor of zero indicating zero NLOS effects and noise. At each simulation, we take the sum of the errors on the tag positions and sum of the errors on the recomputation of anchors, using the earlier computed tags as anchors. The median values and interquartile range (IQR) values for both error sums are plotted. The plots indicate that tag position errors are lower bounded by the anchor position errors, so that all of the NLOS effects and measurement noise are not completely traceable from anchor recomputation errors. This confirms that complete removal of NLOS effects and measurement noise is not possible, generally and under our proposed scheme in particular. We observe that there are no errors on anchor recomputation at zero NLOS and noise.

The tentative anchors positions $\tilde{\boldsymbol{X}}_a$ are recomputed by swapping anchors with tags in Equation 7, so that anchors are now recomputed from tag positions $\boldsymbol{X}_t$ using the anchor-tag between-sets distances as support. This yields a new equation of the following form:

$$\tilde{\boldsymbol{X}}_a = \frac{\boldsymbol{B}_{22}\boldsymbol{Z}_a + (\boldsymbol{B}_{21} + \boldsymbol{11}_{n,m})\boldsymbol{X}_t}{m} \tag{9}$$

From Equation 9, anchors positions are directly computable from those of tags if the number of tags $m \geq 3$, where 3 is the minimum number of fixed nodes sufficient for the computation of unique anchor positions. We present a supplementary approach in subsection 4.2 for special cases where the number of tags $m = 1$ and $m = 2$.

The error in the computation of the tentative anchor positions is the sum of the displacements between each recomputed position in the tentative anchor configuration $\tilde{\boldsymbol{X}}_a$ and its corresponding exact position in the known anchor configuration $\boldsymbol{X}_a$. This error e_a is expressed mathematically as:

$$e_a = \sum_{i=1}^{n} \|\tilde{\boldsymbol{x}}_{a(i)} - \boldsymbol{x}_{a(i)}\| \tag{10}$$

where $\tilde{\boldsymbol{x}}_{a(i)}$ and $\boldsymbol{x}_{a(i)}$ are the tentative and known positions of the i^{th} anchor respectively.

Since $\tilde{\boldsymbol{X}}_a$ is computed from a non-ideal matrix of distance measurements containing some unknown NLOS bias and measurement noise, then the equation for $\tilde{\boldsymbol{X}}_a$ can be rewritten as:

$$\tilde{\boldsymbol{X}}_a = \texttt{MDS}(\boldsymbol{X}_t,\ \boldsymbol{\delta} + \boldsymbol{b}). \tag{11}$$

where MDS indicates the MDS algorithm as a callable procedure that takes parameters $\boldsymbol{X}_t,\ \boldsymbol{\delta},\ \boldsymbol{b}$ and returns a configuration of positions. We make no attempt to distinguish NLOS effects from measurement noise in this work, so that both are estimated jointly and cumulated in the matrix $\boldsymbol{b}$. Henceforth, we will refer to both NLOS bias and measurement noise as simply NLOS bias.

Equation 11 allows to also rewrite the error e_a as function of NLOS bias in a similar form:

$$e_a = \sum_{i=1}^{n} \|\texttt{MDS}(\boldsymbol{X}_t,\ \boldsymbol{b})_{(i)} - \boldsymbol{x}_{a(i)}\| \tag{12}$$

where the matrix of measured distances $\boldsymbol{\delta}$ has been dropped since its elements are constant.

The optimization problem for finding tag positions and NLOS bias values that minimize error e_a is now written as:

$$\hat{\boldsymbol{\theta}}_t = \min_{\boldsymbol{\theta}}\ e_a(\boldsymbol{X}_t,\ \boldsymbol{b}), \qquad \boldsymbol{\theta} = [\boldsymbol{X}_t\ \boldsymbol{b}]^T \tag{13}$$

The tag positions $\boldsymbol{X}_t$ are unconstrained since they are explicitly computed by Equation 8. However, the cumulated NLOS bias $b_{ij} \in \boldsymbol{b}$ are constrained so that their corresponding values for any of the distances are finite and positive (or zero). This condition can be formally expressed as:

$$b^L \leq b_{ij} \leq b^U, \quad \forall b_{ij} \in \boldsymbol{b}_\theta \tag{14a}$$

$$b_{ij} = 0, \quad \forall b_{ij} \notin \boldsymbol{b}_\theta \tag{14b}$$

where $\boldsymbol{b}_\theta \subset \boldsymbol{b}$ is the set of all anchor-tag between-sets biases. The lower bound b^L can be set as $b^L = 0$ for ideal distances while the upper bound b^U can be based on information regarding the geometrical layout of the deployment region, as proposed in [23]. The values for anchor within-sets NLOS biases are set to 0 since all anchor positions are known and their pairwise distances are ideal while those for tag within-sets are equally set to 0 since within-sets distances for tags are not available.

4.1 Node deployments with at least 3 tags

When the number of tags is at least 3, i.e., $m > 3$, the tentative anchor positions can be computed by swapping anchors with tags described in Equation 9. The error function e_a is nonlinear with respect to $\boldsymbol{b}$ (the partial $\boldsymbol{B}$ matrices $\boldsymbol{B}_{21}$ and $\boldsymbol{B}_{22}$ are updated with $\boldsymbol{\delta} + \boldsymbol{b}$ instead of $\boldsymbol{\delta}$), it is therefore also non-linear with respect to the parameter vector $\boldsymbol{\theta}$.

We propose the application of Sequential Quadratic Programming (SQP) to solve the constrained nonlinear problem. Due to the complexity of the SQP algorithm and the positive semi-definiteness of the square matrix $\boldsymbol{b}$, we introduce a preprocessing algorithm (see Algorithm 1) to trim a constant NLOS bias from the matrix $\boldsymbol{b}$. The algorithm mitigates a constant NLOS bias value b_c if all biases $b_{ij} \in \boldsymbol{b}_\theta$ satisfy the condition $|b_{ij}| \geq |b_c|$. From Algorithm 1, the bias value b_c is initialized to zero and incremented with $step$, repeating the computation of error e_a at each iteration until it no longer decreases. It is important to set a value of $step$ that is sufficiently small to allow the algorithm to reach the minimum as close possible, without overshooting too quickly. The algorithm performs most effectively when all between-sets distances are affected by approximately the same amount of NLOS bias. Otherwise, it will trim off small measurement noise or return after the first iteration if any of the distances is unbiased.

Regarding the complexity of the MDS-SMACOF invocations applied in Algorithm 1, one of the ways we speed up the convergence of SMACOF is to initialize only the first computation for $\boldsymbol{X}_t^{(0)}$ with a random array. Subsequent $\boldsymbol{X}_t^{(k)}$ computations are initialized with $\boldsymbol{X}_t^{(0)}$ or $\boldsymbol{X}_t^{(k-1)}$. The same applies to SMACOF computations for $\tilde{\boldsymbol{X}}_a^{(k)}$, which are initialized with $\boldsymbol{X}_a$ or $\tilde{\boldsymbol{X}}_a^{(k-1)}$. These initializations generally make SMACOF converge in constant time $\mathcal{O}(1)$.

The SQP algorithm as shown in Algorithm 2 is then run on the constrained nonlinear problem defined by Equations 13 and 14. The implementation of the optimization algorithm makes use of Python-SciPy's SLSQP (Sequential Least Squares Programming) solver. The solver applies the Hans-Powell quasi-Newton method [17] for the derivative of the Lagrangian associated with the minimization problem in Equation 13 with a Broyden-Fletcher-Goldfarb-Shanno (BFGS) update. All the elements b_{ij} of this matrix are initialized with the constant bias value b_c. Afterwards, a matrix

Algorithm 1 Trimming NLOS bias from distances

Input: $\boldsymbol{X}_a,\ b^U,\ step$
Output: $\boldsymbol{X}_t,\ b_c$
Initialization : $b_c \to 0$
$\boldsymbol{X}_t^{(0)} = \mathtt{MDS}(\boldsymbol{X}_a,\ b_c)$
$\tilde{\boldsymbol{X}}_a^{(0)} = \mathtt{MDS}(\boldsymbol{X}_t^{(0)},\ b_c)$
compute $e_a^{(0)}$
$k \to 0$
while $k = 0$ **or** $e_a^{(k)} < e_a^{(k-1)}$ **do**
  $b_c = b_c + step$
  $k = k + 1$
  if $(b_c \geq b^U)$ **then**
    break
  end if
  $\boldsymbol{X}_t^{(k)} = \mathtt{MDS}(\boldsymbol{X}_a,\ b_c)$
  $\tilde{\boldsymbol{X}}_a^{(k)} = \mathtt{MDS}(\boldsymbol{X}_t^{(k)},\ b_c)$
  compute $e_a^{(k)}$
end while
$b_c = b_c - step$ {reverse last update}
$\hat{\boldsymbol{X}}_t = \mathtt{MDS}(\boldsymbol{X}_a,\ b_c)$

$\boldsymbol{U}$ composed by uniformly distributed values in $[0,1)$ and with the same dimensions as $\boldsymbol{b}_\theta$ is added to $\boldsymbol{b}_\theta$. This is done to apply small perturbations to the initial values, which is known to provide better results than initializing all elements with the same value. The function f_e updates the value of e_a at each iteration within the SQP procedure while tol specifies the tolerance of the stopping criterion.

Algorithm 2 NLOS bias mitigation by SQP

Input: $\boldsymbol{X}_a,\ b_c,\ tol$
Output: $\boldsymbol{X}_t,\ \boldsymbol{b}_\theta$
Initialization : $\boldsymbol{b}_\theta^{(0)} \to b_c + \boldsymbol{U}[0,1)$
$\boldsymbol{X}_t^{(0)} = \mathtt{MDS}(\boldsymbol{X}_a,\ \boldsymbol{b}_\theta^{(0)})$
$\boldsymbol{b}_\theta = \mathtt{SQP}(f_e,\ \boldsymbol{b}_\theta^{(0)},\ \boldsymbol{X}_t^{(0)},\ \boldsymbol{X}_a,\ tol)$
$\hat{\boldsymbol{X}}_t = \mathtt{MDS}(\boldsymbol{X}_a,\ \boldsymbol{b}_\theta)$

4.2 Node deployments with less than 3 tags

When the number of tags is $m = 1$ or $m = 2$, Equation 9 can not be applied directly for the computation of tentative anchor positions since at least 3 fixed nodes are required in order to produce unique solutions.

To overcome the lack of sufficient tags, the tag configuration is padded with some anchors from the anchor configuration so the number of tags makes up to 3. For example, in the case there are only two tags, we pad with one anchor. To ensure that all anchors participate in the padding and an error value can still be taken from tentative anchor positions, we apply a modified form of the jackknifing technique adopted in [15]. Anchors are sampled without replacements or ordering, taking $3-m$ anchors at each sampling step. Afterwards, the sampled anchors are added to the tag configuration so that the number of tags plus the anchor(s) becomes equal to 3. Tentative positions for the anchors left in the configuration after the samples have been acquired are computed using the augmented tag configuration. This process of sampling/jackknifing and computing of tentative positions is repeated for all possible combinations of anchors in the anchor configuration. Errors are computed and stored, and all the errors are later summed into a final tentative anchor positions error e_a as described by the following equation:

$$e_a = \sum_{i=1}^{s} e_{a(i)}(\boldsymbol{X}_{t(i)}^{aug},\ \boldsymbol{b}) \tag{15}$$

The i^{th} augmented tag configuration $\boldsymbol{X}_{t(i)}^{aug}$ is defined by:

$$\boldsymbol{X}_{t(i)}^{aug} = \left[\boldsymbol{X}_t^T | \boldsymbol{X}_{a(i)}^T\right]^T \tag{16}$$

where $\boldsymbol{X}_{a(i)}$ is the i^{th} anchor sample(s) of size $(3-m)\times 2$ for 2D or $(3-m)\times 3$ for 3D positioning. The number s appearing in Equation 15 is the number of unordered combinations without replacement possible with the given number of anchors and tags and is defined by:

$$s = \binom{n}{3-m} \tag{17}$$

For instance, in a setup of 4 anchors and 1 tag, 6 anchor combinations are possible, allowing for 6 different augmented tag configurations.

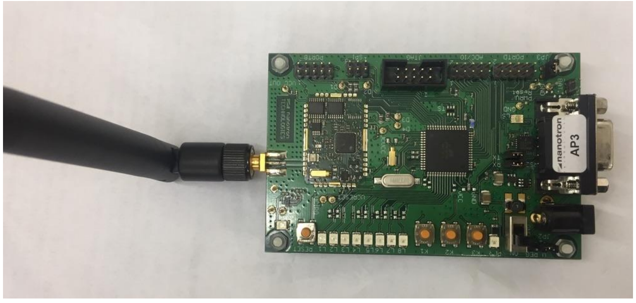
As with the previous scenario with at least 3 tags, NLOS biases are first trimmed using Algorithm 1 after which the final bias values are computed using Algorithm 2.

5 Experimental Results

The experiments were performed using nanoLoc transceiver nodes in the laboratory environment shown in Figure 3b. The environment was not adjusted whatsoever for the sake of the localization exercise. This had the benefit of allowing RF interference from the multiple wireless devices in the environment and possible occlusions and reflections from nearby walls and objects.

A nanoLoc node is shown in Figure 3a. The nodes use RT-ToF for estimating the ranges that are collected by the tags and sent via a base-station to a computer where, in turn, the distance matrix is constructed and node positions are calculated.

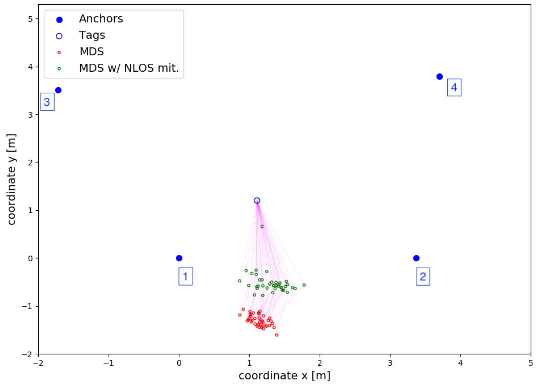
The nanoLoc nodes are mounted on tripods, as shown in Figure 3b. There are 4 anchors and a varying number of tags, from 1 to 4. The tags are kept stationary and an obstacle (shown in yellow in Figure 3b) is placed directly in front of anchor 3. This obstacle causes the distances between anchor 3 and all the tags to be approximately doubled. Variations



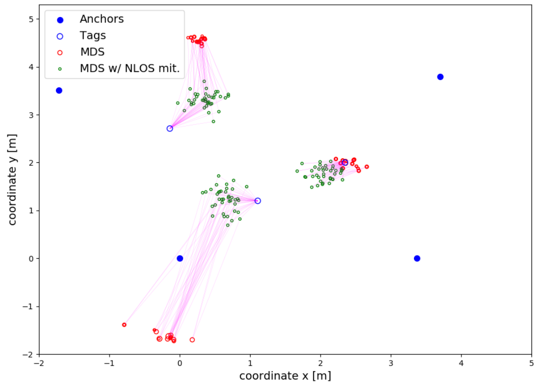
(a) The nanoLoc node used in the experiments.

(b) Deployment of 4 anchors (blue) and 3 tags (red) with an artifical obstacle (yellow).

Fig. 3: Localization setup in a laboratory setting.



(a)



(b)

Fig. 4: Positioning for 1 (a) and 3 (b) stationary nanoLoc tags with 40 data points per tag.

in the tag positions in Figure 4b are directly explainable by small perturbations in the distance measurements taken by the transceivers leading to random hops within the MDS algorithm. A technique for dampening these hops with respect to the MDS scheme has been discussed in [7]. The method takes into account the velocity of the nodes and applies MDS over a number of distance matrices contiguous in time. Usually, the method is applied for 2 contiguous matrices.

Table 1 shows the mean and variance of RMSE values for vanilla MDS and proposed method. The RMSEs at 2 tags and 3 tags are not correlated as a different version of the NLOS mitigation technique was applied for both scenarios, i.e., Algorithms 1 and 2 respectively. We notice that in Figure 5a, around RMSE values of $3m$ on the Y-axis, the NLOS bias mitigation results are occasionally slightly worse than the original. We believe this is due to approximation errors in Algorithms 1 and/or 2 and can be corrected by setting the *step* in Algorithm 1 to a smaller value or reducing the tolerance value *tol* in Algorithm 2. Decreasing the value of these parameters to provide fractional improvements in accuracy increases the time it will take for both algorithms to converge. The increase in RMSE in Figure 5a is due to a shift of the barrier closer to anchor 3. Human movements within the deployment region also create sudden spikes or dips. The histogram and empirical cumulative distribution

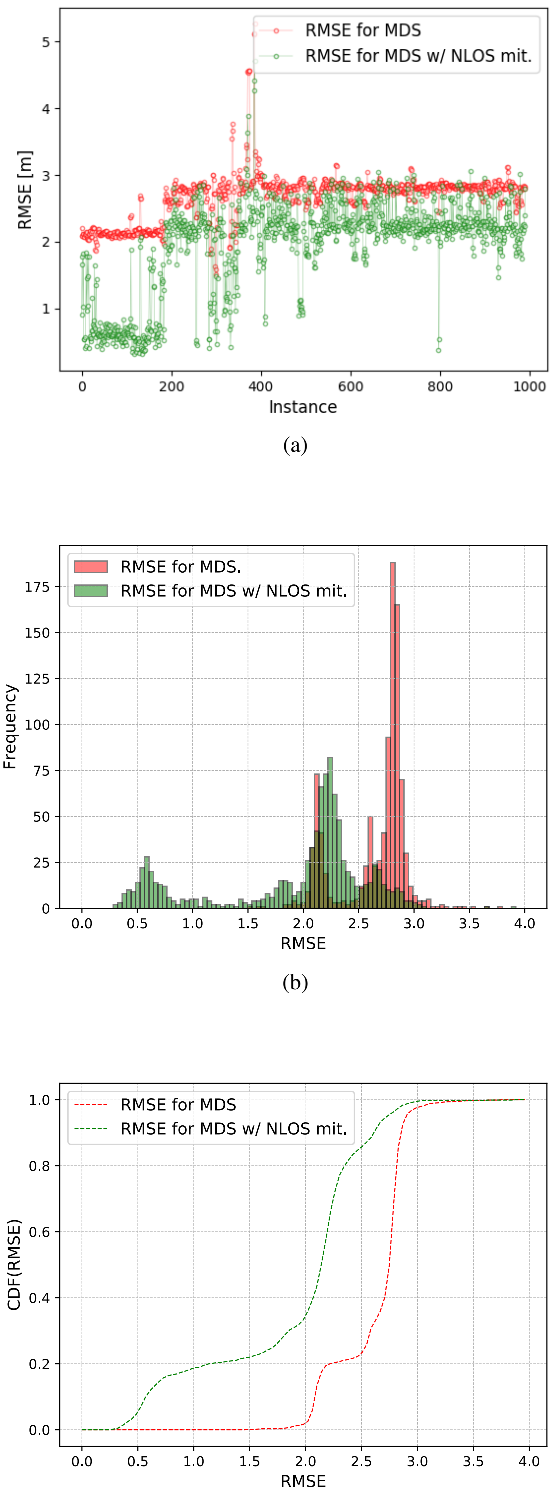
Table 1: Mean and variance of RMSEs for different nanoLoc tag counts.

No. of tags	MDS	MDS w/ NLOS mit.
	mean/var. (m)	
1	2.51/0.01	1.76/0.05
2	3.71/0.01	3.18/0.02
3	2.67/0.13	1.94/0.52
4	2.84/0.03	2.24/0.01

function (CDF) for the RMSEs are reported in Figures 5b and 5c.

5.1 Further Simulations

To verify the performance of our proposed method compared to vanilla MDS on a larger setup, simulations were performed to allow scaling up of the number of nodes and



(a)

(b)

(c)

Fig. 5: a) RMSEs; b) histogram of RMSEs; and c) CDF of RMSEs for 4 nanoLoc anchors and 3 tags with ≈ 1000 runs.

an expansion of the area of the deployment region. We initialize a rectangular $35m \times 25m$ simulated area with varying number of anchors and tags at random positions. Distance matrices with elements δ_{ij} are constructed from the pairwise distance between all the nodes and then tag within-sets distances are marked as unknown. The simulation ranging dynamics were modeled according to range estimation accuracy data provided in [2,5], where NLOS bias is reported to approximate an exponential distribution while the measurement noise is modeled as a zero mean Gaussian distribution. For ToF, the exponential distribution for NLOS bias $b_{ij} = Exp(\lambda)$ has scale $\lambda = 0.08\delta_{ij}$ and the noise $n_{ij} = \mathcal{N}(0, \sigma^2)$ has a standard deviation $\sigma = 0.02\delta_{ij}$ [2]. These values allow to inject the original distance δ_{ij} with some randomized NLOS and noise value so that the true distance d_{ij} is now related to δ_{ij} by:

$$\delta_{ij} = d_{ij}(\boldsymbol{X}) + b_{ij} + n_{ij} \tag{18}$$

Simulations were repeated for setups of 3 to 8 anchors and 3 to 22 tags. NLOS/noise multipliers are set in the range from 0 to 1. Anchors were placed at the four corners and midway between the four corners of the simulated area and tags were always initialized randomly within the rectangular bounds defined by the anchors.

Figures 6a, 6b, and 6c show the results for the simulations where nodes and ToF dynamics were randomly initialized 500 times for each variation in the setup. The error in the position estimates is given by the RMSE of the computed tag positions with respect to the true tag positions. The median and IQR values for the 500 runs at each simulation are shown in the plots. From Figure 6a, the positioning RMSEs decrease as more anchors are added to the setup; tags were kept constant at 22 for this simulation set. This is because increasing the number of anchors increases the average magnitude of the error of tentative anchor positions, thereby allowing for a more robust inference on NLOS biases from the error. From Figure 6b, the number of anchors is kept constant at 4 while the number of tags is varied. This has the effect of increasing RMSE as tag count increases since information is lost as only 4 tentative anchor positions are reproduced from a relatively higher number of tags, with the NLOS mitigation approach always having a lower median value. In Figure 6c, we multiply the randomized b_{ij} and n_{ij} values by factors ranging from 0 to 1. Both the MDS and MDS with NLOS mitigation approaches produce the precise tag positions when the multiplying factor is 0. As the factor is increased, the RMSE of our NLOS mitigation approach grows at a rate 0.7 that of vanilla MDS.

6 Conclusion

This paper presented a NLOS mitigation technique under the MDS scheme which is an extension of the work in [13]

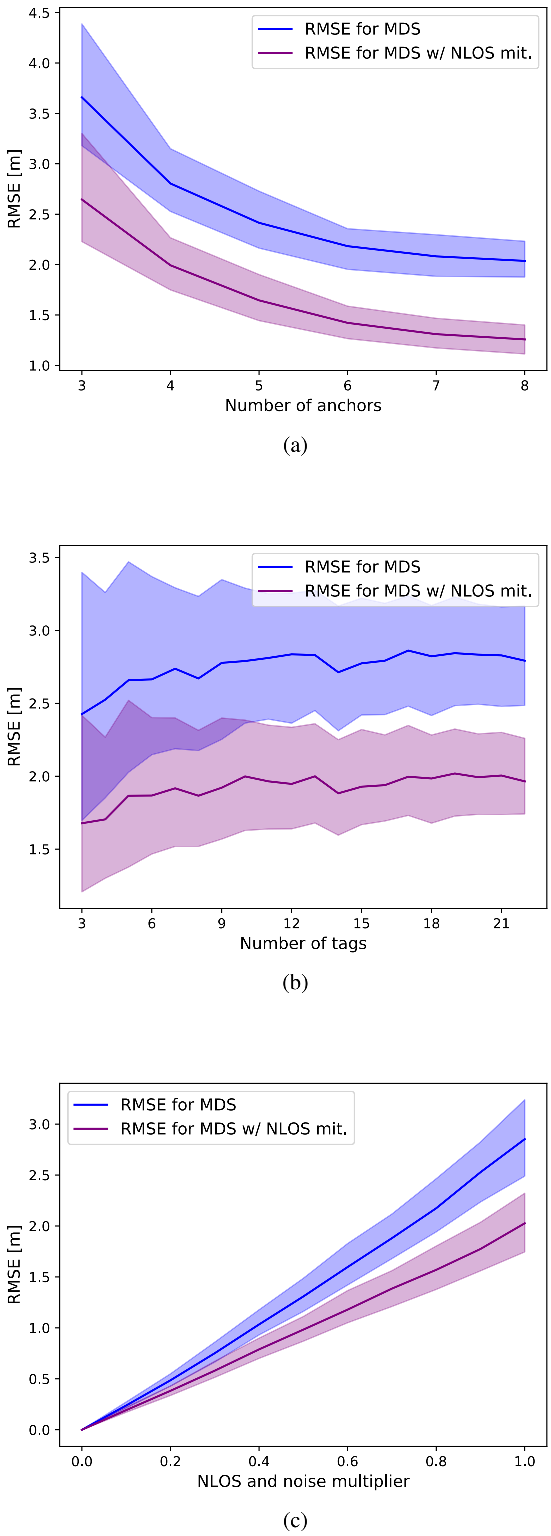


Fig. 6: Medians and IQRs for RMSEs under a) varying number of anchors; b) varying number of tags; and c) varying NLOS and noise multipliers.

where the technique was first introduced. This formed the basis for the more elaborate discussion on NLOS mitigation using recomputed anchor positions. An ideal matrix of pairwise distances is known to exhibit a symmetry that allows to recompute exact anchor positions from those tags using MDS. In a NLOS environment, this symmetry does not hold so that the ensuing asymmetry can be exploited, as done in this work, to estimate NLOS bias values for each of the pairwise distances. We presented a minimization problem where the error on anchor recomputation is presented as cost, and tag positions and NLOS bias are parameters to be optimized. An algorithm for the fast and global trimming of NLOS biases was first introduced, after which a SQP solution which computes per distance NLOS bias values was presented. The performance improvements of the proposed NLOS mitigation technique was validated by experiments with nanoLoc anchors and tags, and further on with simulations allowing for larger node deployments. Generally, results showed that positioning RMSEs can be decreased by up to 28% for a setup of 4 nanoLoc anchors and 3 tags.

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