



# Inequality, openness, and growth through creative destruction

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## ABSTRACT

We examine how inequality and openness interact in shaping the long-run growth prospects of developing countries. To this end, we develop a Schumpeterian growth model with heterogeneous households and non-homothetic preferences for quality. We show that inequality affects growth very differently in an open economy as opposed to a closed economy: If the economy is close to the technological frontier, the positive demand effect of inequality on growth found in closed-economy models may be amplified by international competition. In countries with a larger distance to the technology frontier, however, rich households satisfy their demand for high quality via importing, and the effect of inequality on growth is smaller than in a closed economy and may even be negative. In such case trade gives rise to the endogenous emergence of a ‘dual economy’ where some domestic sectors are highly innovative while others are lagging behind.

## 1. Introduction

In recent years, few topics have been debated more than the rise in income inequality and countries’ openness to trade. Many academic and political debates are around winners and losers from international trade in terms of income. With respect to economic growth, there is a well-developed literature on how income inequality affects growth, and a large body of work examining the relation between openness and economic growth.<sup>1</sup> However, much less is known about how income inequality and trade openness interact in shaping a country’s long-run prosperity. This is the focus of the present paper. This question is of particular importance for developing countries which can exhibit large income inequality and have often been under pressure by industrialized countries or international organizations to open their economies for international trade.

To examine how inequality and openness interact in shaping long-run economic growth, we consider a Schumpeterian growth model with heterogeneous households and non-homothetic preferences for quality. So far, the literature has used this type of framework to analyze the effects of inequality on growth in closed economies. The innovation of this paper is that we consider an open economy and show why and how the effects change when allowing for international trade. In particular, we show how the positive ef-

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<sup>1</sup> Several channels of how income inequality affects growth have been put forward in the literature: Inequality might affect growth via differential propensities to save between income groups (Kaldor, 1955), via credit constraints that limit the ability of poor households to invest in the built-up of their human capital (Galor and Zeira, 1993; Galor and Moav, 2004), or via their impact on the political process and hence institutions (Alesina and Rodrik, 1994; Persson and Tabellini, 1994; Gersbach et al., 2019). Our focus will be on a demand channel as in e.g. Matsuyama (2002); Foellmi and Zweimüller (2006); Foellmi et al. (2014); Latzer (2018). See the literature discussion at the end of this section for a discussion of literature on growth in open economies.

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fect of inequality on growth found for closed economies can turn negative in an open economy that is not at the technological frontier. The key reason behind this negative relationship is that rich households can satisfy their demand for high quality via importing.

Our theoretical model considers two types of households: rich and poor. Households spend their income on a homogeneous good and a continuum of differentiated goods. Richer households demand more of the homogeneous good and higher qualities of the differentiated goods. Production of quality of a differentiated good is constrained by the set of available blueprints for quality versions of that good. Firms can earn a patent on higher quality versions by investing in R&D and increasing the upper bound on quality for a specific differentiated good. The decision problem of an innovating firm is key to characterizing the equilibrium in the economy.<sup>2</sup> Depending on parameter values, all monopolists either pool households or separate rich from poor households. In the latter case, non-homothetic preferences over quality give rise to multi-quality firms, analogous to Latzer (2018).

A higher variance of the income distribution, keeping the skewness and average income constant, implies making the rich richer and the poor poorer while keeping their respective shares in the population constant. The increase in the income of the rich increases their willingness to pay for quality, making it more lucrative for firms to innovate to serve this demand. This leads to the robust comparative statics result that in a separating equilibrium, an increase in the variance of the income distribution has an unambiguously positive effect on growth in closed economies.

The key point of our paper is that this relationship between inequality and growth may be very different when allowing for international trade. For this purpose, we develop a small open economy (SOE) variant of our model by adding trade subject to an iceberg trade cost with a technologically advanced rest of the world (ROW). In essence, this implies that if domestic firms in the SOE want to sell innovative high qualities to rich households, they need to outbid import competition for high qualities.<sup>3</sup> We then identify three scenarios with respect to the effects of inequality and foreign competition on growth in an SOE that is technologically lagging:

First, if inequality is low, the high quality demand by the rich part of the population is only slightly above the domestic technological frontier, and for these quality levels, the trade costs effectively shield the domestic firms from international competition, leading to the same equilibrium as in the closed economy.

Second, for higher but still moderate inequality, the high quality demand is further above the domestic technological frontier. Innovating domestic firms now have to face up to the international competition, but are still competitive. In this case, outside competition leads to higher domestic innovation and higher growth. The reason for this positive growth effect of foreign competition is that it induces innovating domestic firms to lower their price. This triggers interesting equilibrium effects on innovation: The lower price for the high quality versions of all differentiated goods allows rich households to economize on their spending for the differentiated goods. The associated income effect induces them to increase their consumption of the homogeneous good and, in turn, this boosts their demand for quality due to the complementarity between the homogeneous good and the quality of the differentiated goods. This positive *demand effect* lifts innovation incentives above the respective level in the closed economy.

Third, if inequality is large, the quality demanded by high income earners is substantially above the domestic technological frontier, and it is no longer profitable for all domestic firms to compete with the technologically advanced foreign firms to serve the rich. Consequently, rich households start satisfying their demand for high quality by importing *some* of the differentiated goods, and the SOE exports the homogeneous good and/or low qualities of the differentiated goods in turn. The key observation is that this has a direct negative *business stealing effect* on innovation and growth. This effect becomes larger as inequality increases further, and domestic firms in fewer and fewer differentiated good sectors innovate to serve high qualities to the rich. Interestingly, our work thus also shows how a ‘*dual economy*’—with some innovative and some lagging sectors—endogenously arises in developing countries that are open to trade even in an ex-ante perfectly symmetric set-up. The basic intuition is that the domestic population is not rich enough to satisfy all of its demand for quality by importing pricey high qualities from abroad.

Our set-up further gives rise to a novel ‘*Dutch disease*’-type negative effect of windfall gains on growth in developing countries: Windfall gains increase demand for quality by rich households vis-à-vis the domestic level of technology, and therefore they make it harder for domestic innovators to compete with foreign high-quality providers.

In sum, our work shows how inequality, trade openness, and distance to frontier interact in shaping a country’s growth prospects, and this has important implications for the dynamic gains from trade and for policy. We show that firms in countries sufficiently close to the frontier are better positioned to profitably outbid foreign competitors, increasing the scope for stimulating growth via trade liberalizations. On the contrary, in an unequal SOE that lags behind the world’s technological frontier, rich households satisfy their demand for high quality via importing. This negative business stealing effect implies that, ceteris paribus, growth is lower than it would be in a closed economy. Supplementary material S3 shows that this theoretical prediction can speak to basic correlations in the data.<sup>4</sup>

<sup>2</sup> We show that this decision problem boils down to a problem of optimal non-linear monopoly pricing over quality, but with three key differences when compared to the textbook case (e.g. Bolton and Dewatripont (2005)): First, there is an endogenous upper bound on quality. Second, the shape of a consumer’s payoff function from one specific differentiated good depends on the full equilibrium. Third, in the open economy, foreign competition introduces a second set of individual rationality constraints. The costly quality upgrading implies that firms may find it optimal to pool rich and poor households if differences in income and / or the population share of the rich are small. The dependence of the payoff function on the full equilibrium allows for interesting feedback effects on the innovation decision by firms. These effects have to be taken into account throughout. They are of particular interest in our small open economy analysis. In contemporaneous work, Bornstein and Peter (2023) also introduce non-linear monopoly pricing into a general equilibrium framework, but with a focus very different from ours: They analyze what non-linear pricing implies for misallocation.

<sup>3</sup> This is in line with the well-known fact that richer countries export (and import) higher qualities (Schott, 2004; Hallak, 2006; Khandelwal, 2010; Bastos and Silva, 2010; Feenstra and Romalis, 2014; Flach, 2016; Schetter, 2020).

<sup>4</sup> This appendix presents (1) standard growth regressions using growth in GDP per capita as the dependent variable and (2) industry-level growth regressions using growth in export quality taken from Feenstra and Romalis (2014) as the dependent variable. These regressions control for an interaction of inequality and openness.

Our paper is related to two main strands of literature. First, there is a large and growing literature that seeks to understand how inequality impacts the growth prospects of a country via the demand for product innovation. Matsuyama (2002) shows in a model of learning by doing that the effect of inequality on growth may be non-monotonic and that conventional measures of inequality such as the Gini-coefficient are not a sufficient statistic for these effects. Foellmi and Zweimüller (2006) consider a model of expanding varieties where new varieties address consumers' needs following their preference hierarchy. Inequality stimulates growth via an associated higher demand for luxury goods. Foellmi et al. (2014) consider product and process innovation, where process innovation prepares 'luxury goods' for mass production, in line with product cycles observed from the data. Latzer (2018) presents a Schumpeterian growth model featuring agents with non-homothetic preferences over quality. She shows how the desire to better discriminate between consumers of different incomes ('surplus appropriation effect') induces incumbents to invest in R&D and can give rise to multi-quality firms in equilibrium.<sup>5</sup> All of these models share in common that they are considering closed economies. And while a change in the income distribution may have non-trivial overall effects on growth, it is the case that a ceteris paribus increase in the income of the rich is beneficial for growth because it increases their willingness to pay for innovated goods. We show that this channel may be very different and may, in fact, be reversed in a small open economy.<sup>6</sup>

Second, our paper is thus also related to the large literature analyzing the growth-effects of international trade (e.g. Grossman and Helpman (1991a), Acemoglu (2003), Galor and Mountford (2008), Nunn and Trefler (2010), Acemoglu et al. (2015), Gersbach and Schneider (2015), Schaefer and Schneider (2015), Arkolakis et al. (2018), Buera and Oberfield (2020), Diodato et al. (2022), Gersbach et al. (2023), Sampson (2023)). Openness to trade leads to higher competition as foreign firms enter the market. This might reduce R&D incentives for domestic firms and therefore lead to lower growth (Aghion and Howitt, 1992, 1996). At the same time, however, technology spillovers might arise as externalities from international trade (Grossman and Helpman, 1991b; Eaton and Kortum, 1999; Buera and Oberfield, 2020). In line with these opposing effects, empirical evidence on the impact of (foreign) competition on innovation and growth is mixed (Nickell, 1996; Blundell et al., 1999; Schmitz, 2005; Aghion et al., 2005, 2009; Hashmi, 2013; Bloom et al., 2015).<sup>7</sup> The key novelty of our work is that we analyze how the effects of international trade openness interact with inequality in shaping a country's growth prospects.<sup>8,9</sup>

More broadly our paper is also related to a larger literature on how business stealing effects can have negative consequences for growth but for different reasons than in our model. See e.g. Glode and Ordonez (2024) where technological progress leads to increased incentives to invest in wasteful rent seeking activities.

The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 considers the closed economy and section 4 the small open economy. Section 5 provides extensions and discussions. Finally, section 6 concludes. All proofs and further details are provided in the appendix and the accompanying supplementary material.

## 2. Model

We develop a model with non-homothetic preferences for quality and Schumpeterian growth through quality upgrading. We begin by developing the closed-economy model, which will serve as the benchmark for our open-economy analysis.

### 2.1. Households

The economy is populated by a continuum of infinitely-lived households of measure 1,  $j \in [0, 1]$ . Households derive utility from consumption of a continuum of differentiated goods,  $i \in [0, 1]$ , and a homogeneous good,  $z$ . Each differentiated good  $i$  represents a distinct consumption need of households that can be satisfied by consumption of one unit of exactly one of the quality versions of the good available at time  $t$  (time is discrete)  $q_i \in Q_i(t)$ . This means, the choice of quality is a discrete choice and there is an infinite

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Across a range of specifications, using either a large number of country controls or country fixed effects, we find that for developing countries this interaction term is typically significantly negative in the industry-level regressions, and still negative—albeit not in all cases significant—in the country-level regressions, in line with our theoretical predictions.

<sup>5</sup> This is in contrast to canonical Schumpeterian models (Aghion and Howitt, 1992) where the replacement effect (Arrow, 1962) outweighs the efficiency effect (Gilbert and Newbery, 1982; Reinganum, 1983).

<sup>6</sup> Hence, at a general level, our work is also related to Matsuyama (2019), who provides a thorough account of Engel's law in a global economy, and its implications for endogenous comparative advantage, structural change, and product cycles, among others. In his case, however, preferences are non-homothetic across sectors, while we consider non-homotheticities within sectors to study Schumpeterian growth. Moreover, he does not consider the effect of inequality within countries on growth. In that latter regard our paper also complements a large body of work that analyzes the reverse causal relationship, i.e. how innovation and growth impact inequality (see e.g. Chu et al. (2019, 2021); Acemoglu and Restrepo (2022); Moll et al. (2022); Aghion et al. (2023); Chu and Peretto (2023); Bloom et al. (2024) for recent contributions).

<sup>7</sup> Amiti and Khandelwal (2013) show that lower import tariffs (i.e. more competition) lead to quality upgrading for products close to the frontier, but discourages upgrading for products further away from the frontier, in line with our theoretical findings.

<sup>8</sup> Our work is thus also, but less closely, related to literature that incorporates non-homothetic preferences into models of international trade on the one hand (e.g. Flam and Helpman (1987), Stokey (1991), Matsuyama (2000), Fajgelbaum et al. (2011), Fieler (2011), Jaimovich and Merella (2012), or Foellmi et al. (2018)), and that analyzes long-lasting consequences of international trade on the other hand (e.g. Atkin et al. (2021); Egger et al. (2023)).

<sup>9</sup> Our work also adds a novel perspective to the discussions on infant industry protection. The theoretical literature on infant industry protection emphasizes the importance of learning-externalities either within or across industries (Krugman, 1987; Lucas Jr., 1988; Young, 1991; Matsuyama, 1992; Krugman and Elizondo, 1996; Puga and Venables, 1999; Hausmann and Rodrik, 2003; Rodrik, 2004; Melitz, 2005; Greenwald and Stiglitz, 2006). Our model also features an externality from innovation on aggregate productivity, and a potentially detrimental effect of trade on growth, which is the basis for infant industry protection. We argue, however, that this effect critically depends on the income distribution in developing countries.

degree of substitution between different quality versions of the same good. We use  $q_i^j(t)$  to denote the quality  $q_i \in \mathcal{Q}_i(t)$  consumed by household  $j$  and  $x_i^j(q_i; t)$  the associated quantity, i.e.  $x_i^j(q_i^j(t); t) = 1$  and  $x_i^j(q_i; t) = 0$  for all other qualities. If at time  $t$  household  $j \in [0, 1]$  consumes a bundle  $\{q_i^j(t)\}_{i \in [0,1]}$  of differentiated goods and  $z^j(t)$  units of the homogeneous good, then their instantaneous utility is given by

$$u\left(\{q_i^j(t)\}_{i \in [0,1]}, z^j(t)\right) = \int_0^1 (q_i^j(t))^{1-\beta} di (z^j(t))^\beta . \tag{1}$$

Equation (1) implies that there is a complementarity between the quantity of the homogeneous good and the qualities of the differentiated goods, i.e. richer households have a higher willingness to pay for quality. This will play a central role in our analysis.<sup>10</sup>

Households differ in their endowment with effective labor,  $\omega$ , which they inelastically supply to the labor market. We consider two types of households, a ‘high type’ with high labor endowment,  $\omega^H$ , and a ‘low type’ with low labor endowment,  $\omega^L$ , and use superscript  $h \in \{H, L\}$  to refer to the two different types of households. The share of high types is  $\lambda$  implying a corresponding share  $1 - \lambda$  of low types. Households earn wage rate  $w$  per unit of effective labor, which we choose to be the numéraire, i.e. we have  $w(t) = 1$  at all times.

We assume that households are hand-to-mouth and consume their income in each period. Thus, the household decision problem is a static problem and so is the firm problem considered below.<sup>11</sup> As a consequence, we simplify notation by ignoring the dependence of all variables on time  $t$  unless explicitly stated otherwise. Further, households of the same type  $h$  face the same utility maximization problem, and we will henceforth identify households by their types. In every period, a household of type  $h$  chooses  $q_i^h$  and  $z^h$  to maximize

$$\begin{aligned} \max_{\{q_i^h\}_{i \in [0,1]}, z^h} & \left[ \int_0^1 q_i^{h1-\beta} di \right] z^{h\beta} \\ \text{s.t.} & \int_0^1 p_i(q_i^h) di + p_z z^h \leq I^h, \end{aligned} \tag{2}$$

where  $I^h$  denotes per-period income of household type  $h$ ,  $p_i(q_i)$  the price of quality  $q_i$  of variety  $i$ , and  $p_z$  the price of the homogeneous good. The separability of the instantaneous utility function in combination with the fact that each differentiated good has measure zero imply that the household chooses  $q_i^h$  to maximize

$$\max_{q_i^h} q_i^{h1-\beta} z^{h\beta} - \mu^h p_i(q_i^h), \tag{3}$$

where  $\mu^h$  is the shadow value of income which, by the envelope theorem, is equal to

$$\mu^h := \frac{du^h(\cdot)}{dI^h} = \frac{\frac{\partial u^h}{\partial z^h}}{p_z} = \frac{\beta Q^h z^{h\beta-1}}{p_z} . \tag{4}$$

Here and below we use  $Q^h := \int_0^1 q_i^{h1-\beta} di$  to denote the quality aggregator of household  $h$ . Substituting equation (4) for  $\mu^h$  in decision problem (3), we get that the household chooses  $q_i^h$  to maximize

$$\max_{q_i^h} q_i^{h1-\beta} \underbrace{\frac{z^h p_z}{\beta Q^h}}_{:=\theta^h} - p_i(q_i^h). \tag{5}$$

From the household’s maximization problem we obtain the demand for the homogeneous good  $z^h$  as well as the demands for the differentiated goods  $\{x_i^h(\cdot)\}_{i \in [0,1]}$  with  $x_i^h(q_i^h) = 1$  and zero for all other qualities. Importantly, from the perspective of innovating firm  $i$ ,  $\theta^h := \frac{z^h p_z}{\beta Q^h}$  is a sufficient statistic for household characteristics, determining their willingness to pay for quality.  $\theta^h$  is exogenous to the firm and observed only by the household. It depends on the full general equilibrium in the economy.

<sup>10</sup> A unit requirement for consumption has previously been used by e.g. Jaskold Gabszewicz and Thisse (1980); Shaked and Sutton (1982); Latzer (2018) to model non-homothetic preferences for quality.

<sup>11</sup> In supplementary material S1, we specify the dynamic household problem for the case where households are allowed to save and borrow and argue that with symmetric initial asset endowment of zero and without any aggregate investment opportunities, households will not save and consume their income in each period along the balanced growth path.

### 2.2. Homogeneous good production

The production technology for the homogeneous good is given by

$$z = a_z A L_z,$$

where  $A$  denotes the aggregate state of technology as detailed below,  $L_z$  denotes effective labor input, and  $a_z$  is a time-invariant productivity parameter. There is perfect competition in the market for the homogeneous good, implying that its equilibrium price equals marginal costs

$$p_z = \frac{1}{a_z A}. \tag{6}$$

### 2.3. Differentiated good production and innovation

The quantity  $x_i(q)$  of variety  $i$  at quality  $q$  can be produced according to

$$x_i(q) = \frac{1}{q} a_q A L_i(q),$$

where  $L_i(q)$  denotes effective labor input and  $a_q$  is a time-invariant productivity parameter.

Blueprints for quality versions of each differentiated good  $i$  are inherited from the previous period up to the threshold quality level  $\bar{q}_{i,-1}$ . These blueprints are publicly available and there is a competitive fringe of firms that might enter the market.

Blueprints for new, higher-quality versions of each differentiated good can be developed through innovation. Firms engage in a patent race as described below and for the successful firm innovation entails additional development costs

$$h\left(\frac{\bar{q}_i}{\bar{q}_{i,-1}}\right) \tag{7}$$

to push the technological frontier for product  $i$  from  $\bar{q}_{i,-1}$  to  $\bar{q}_i$ , where  $h(\cdot)$  is  $C^2$  and satisfies:  $h(1) = 0$ ,  $h'(1) = 0$ , and  $h''(\cdot) > 0$ . These costs are denoted in terms of the numéraire, but paid using the homogeneous good.<sup>12</sup> Successful innovation results in a one-period patent for all qualities  $q_i \in (\bar{q}_{i,-1}, \bar{q}_i]$ . To enter the patent race, firms pay a fixed cost  $f$  that is also denoted in terms of the numéraire, but paid using the homogeneous good. The probability of winning is  $f/(M_i f)$ , where  $M_i$  is the mass of firms that entered the patent race for  $i$ . In equilibrium, firms then enter until profits net of fixed cost are zero in expectation, analogous to zero-profit conditions in the literature, e.g. Krugman (1980); Hopenhayn (1992); Melitz (2003). We assume that all households hold a perfectly diversified portfolio of entering firms, implying that profit income is zero for all households.

When patents expire, production knowledge accumulated in the R&D process and in the production of new, high-quality varieties spills over to the entire economy. Such spillovers give rise to the following aggregate technology  $A(t + 1)$

$$A(t + 1) = \int_0^1 \bar{q}_i(t) di.$$

We focus on the case of a common inherited quality level  $\bar{q}_i(t - 1) = \bar{q}(t - 1) \forall i \in [0, 1]$  as an initial condition at  $t = 0$ , i.e.  $\bar{q}_i(-1) = \bar{q}(-1) \forall i \in [0, 1]$ . As all differentiated good sectors are identical in terms of the firms' decision problems, in the closed economy the unique equilibrium is symmetric across the differentiated goods sectors—see section 3.3. This implies that  $A(t) = \bar{q}(t - 1)$  for all  $t \geq 0$  resulting in the following law of motion for aggregate technology<sup>13</sup>

$$A(t + 1) = \frac{\bar{q}(t)}{\bar{q}(t - 1)} A(t).$$

Whenever possible, we simplify notation by using  $A = \bar{q}_{-1}$  and correspondingly  $A_{+1} = \bar{q}$ .

### 3. Closed economy

We are now ready to analyze the closed economy. We begin by characterizing the firms' decision problem which will be central for our results.

<sup>12</sup> Hence, we take a lab equipment approach to R&D. Denoting the cost in terms of the numéraire implies that they increase over time with the economy's technological level  $A$  if measured in units of the homogeneous good  $z$ . This can be inferred from (6) and reflects the standard assumption of lab equipment specifications that it is harder and hence more costly to achieve the same relative increase in quality when the technological level  $A$  is higher (see e.g. Aghion and Howitt (2005)). This assumption allows for a balanced growth path where R&D investments are a constant share of GDP over time. In the context of our open economy model it implies that—given the same R&D cost function  $h(\bar{q}/\bar{q}_{-1})$  and identical parameters  $a_z$  and  $a_q$  across countries (which we assume)—no country has a comparative advantage in production vs innovation in the sense that a given proportional increase of  $\bar{q}$  involves the same R&D costs relative to GDP everywhere. See the beginning of section 4 and footnote 21 for further details on innovation in the SOE.

<sup>13</sup> The law of motion for aggregate technology implies that process innovation is a byproduct of product innovation. In supplementary material S2.2, we discuss an extension with deliberate process innovation.

### 3.1. Firms' decision problem

In analyzing the firms' decision problem we first note that the competitive fringe for all pre-existing qualities  $q_i \leq \bar{q}_{i,-1}$  pushes down their price to marginal cost,

$$p_i(q) = \frac{1}{a_q A} q, \quad \forall q \leq \bar{q}_{i,-1},$$

implying zero profits on these qualities. By contrast, an innovating firm can freely set the price  $p_i(q)$  for all qualities  $q \in (\bar{q}_{i,-1}, \bar{q}_i]$  and chooses  $\bar{q}_i$  and  $\{p_i(q)\}_{\bar{q}_{i,-1}}^{\bar{q}_i}$  to maximize its total profits. With two types of households, the decision problem of the innovating firm for good  $i$  can be written as follows:

**Lemma 1** (Firms' decision problem). *The decision problem of innovating firm  $i$  is equivalent to:*

$$\max_{q_i^H, p_i^H, q_i^L, p_i^L, \bar{q}_i} \lambda \left( p_i^H - \frac{1}{a_q A} q_i^H \right) + (1 - \lambda) \left( p_i^L - \frac{1}{a_q A} q_i^L \right) - h \left( \frac{\bar{q}_i}{\bar{q}_{i,-1}} \right) \quad (8)$$

$$\text{s.t. } \theta^h v(q_i^h) - p_i^h \geq \max_{q \in [0, \bar{q}_{i,-1}]} \left\{ \theta^h v(q) - \frac{1}{a_q A} q \right\}, \quad h \in \{L, H\}, \quad (\text{IR}^h)$$

$$\theta^H v(q_i^H) - p_i^H \geq \theta^L v(q_i^L) - p_i^L, \quad (\text{IC}^H)$$

$$\theta^L v(q_i^L) - p_i^L \geq \theta^H v(q_i^H) - p_i^H, \quad (\text{IC}^L)$$

$$q_i^h \leq \bar{q}_i, \quad h \in \{L, H\},$$

where  $v(q) := q^{1-\beta}$  and where the firm considers type  $\theta^h := \frac{p_z z^h}{\beta O^h}$  of household  $h \in \{L, H\}$  as exogenously given. The value  $\theta^h$  is private knowledge to the household.

The proof of Lemma 1 is given in appendix A.1. The firms' decision problem is one of optimal non-linear monopoly pricing over qualities, but with an endogenous choice of the upper bound on qualities and where the distribution of household types is given by the endogenous distribution of  $\theta$ .<sup>14</sup> The set of constraints in the firms' optimization problem is a reflection of the revelation principle, according to which the optimal set of contracts is one contract for each type of households such that each household has an incentive to truthfully reveal their type. Accordingly, the first set of constraints requires that contracts are *individually rational* (IR), that is, each household must prefer their contract over their best outside option, which is to consume the best option from the set of qualities  $q_i \leq \bar{q}_{i,-1}$  available at marginal cost. The second set of constraints requires that contracts are *incentive compatible* (IC), stipulating that every household must prefer their contract over the contract designed for the other household type.<sup>15</sup> Finally, the last constraint dictates that qualities cannot exceed the current technological frontier  $\bar{q}_i$  for the respective variety.

Lemma 2 shows that the household's willingness to pay for quality  $\theta$  as defined in section 2.1 is an increasing function of household income. This implies that households with higher income demand products of higher quality levels.

**Lemma 2** (Monotonicity of household types).  *$\theta^h$  is strictly increasing in  $I^h$ .*

The proof of Lemma 2 is given in appendix A.2.

### 3.2. Solution to firms' maximization problem

Lemma 2 implies that there exists a threshold income  $\hat{I}$  such that households with income  $\hat{I}$  would choose  $\bar{q}_{-1}$  if all qualities were priced at marginal cost. As it is never optimal for firms to charge a price below marginal costs, this in turn implies that households with income  $I \leq \hat{I}$  always demand quality  $q_i \leq \bar{q}_{-1}$  in equilibrium and only households with income  $I > \hat{I}$  may consume innovative goods. Lemma 3 derives this threshold income  $\hat{I}$ .

**Lemma 3** (Threshold income for demand for innovative qualities). *Household with income  $I \leq \hat{I} := \frac{1}{a_q(1-\beta)}$  always consumes quality  $q_i \leq \bar{q}_{-1}$  in equilibrium.*

<sup>14</sup> Hence, in the closed economy the decision problem differs in two important ways from the textbook case of non-linear monopoly pricing over qualities (e.g. Bolton and Dewatripont (2005)). Moreover, in the open economy, foreign competition introduces a second set of individual rationality constraints—see footnote 2 for further discussions on these differences, and section 4 for the firm problem in the open economy.

<sup>15</sup> This constraint arises because  $\theta^h$  is private knowledge. If  $\theta^h$  was known to the firm, it could extract each household's full willingness to pay for quality, taking into consideration only the individual rationality constraints.

The proof of Lemma 3 is given in appendix A.3. Lemma 3 immediately implies that if the incomes of both household types are below the threshold, i.e. if  $I^h \leq \hat{I}$ ,  $h = L, H$ , firms will not innovate and offer the households' demanded qualities  $q_i^h \leq \bar{q}_{-1}$  at marginal costs.

If  $I^H > \hat{I} \geq I^L$ , only the rich households may demand a quality level  $q_i^H > \bar{q}_{i,-1}$ , while  $q_i^L \leq \bar{q}_{-1}$  will be supplied at marginal costs. In this case, a firm's supply of  $q_i^H$  at price  $p_i^H := p_i(q_i^H)$  is determined by solving the firm's problem with  $(IR^H)$  binding and  $q_i^H = \bar{q}_i$ . That is,  $(q_i^H, p_i^H)$  is the unique solution to

$$(IR^H) \quad \theta^H v(q_i^H) - p_i^H = \theta^H v(\bar{q}_{i,-1}) - \frac{\bar{q}_{i,-1}}{a_q A}, \tag{9a}$$

$$\lambda \left[ \theta^H v'(q_i^H) - \frac{1}{a_q A} \right] = h' \left( \frac{q_i^H}{\bar{q}_{i,-1}} \right) / \bar{q}_{i,-1}. \tag{9b}$$

Equation (9a) dictates that the high types must be indifferent between consuming  $q_i^H$  and their best outside option of consuming  $\bar{q}_{-1}$  at marginal costs. Equation (9b) requires that in optimum, the willingness to pay of the high-types for a marginally higher quality—net of the extra cost of producing that higher quality—must just offset the extra cost of innovation.

Lastly, if  $I^H > I^L > \hat{I}$ , both household types may demand quality levels above the current technological frontier  $\bar{q}_{i,-1}$ , and firms need to decide whether to offer separate quality-price packages to the rich and the poor households or whether instead to pool demand with one quality-price offer to both household types. With separating offers for qualities  $q_i^H > q_i^L > \bar{q}_{i,-1}$ , conditions  $(IR^L)$  and  $(IC^H)$  are binding while  $(IR^H)$  and  $(IC^L)$  are slack. The profit maximizing offers  $(q_i^h, p_i^h)$  are the unique solution to

$$(IR^L) \quad \theta^L v(q_i^L) - p_i^L = \theta^L v(\bar{q}_{i,-1}) - \frac{\bar{q}_{i,-1}}{a_q A}, \tag{10a}$$

$$(IC^H) \quad \theta^H v(q_i^H) - p_i^H = \theta^H v(q_i^L) - p_i^L, \tag{10b}$$

$$\theta^L v'(q_i^L) - \frac{1}{a_q A} = \lambda \left( \theta^H v'(q_i^L) - \frac{1}{a_q A} \right), \tag{10c}$$

$$\lambda \left[ \theta^H v'(q_i^H) - \frac{1}{a_q A} \right] = h' \left( \frac{q_i^H}{\bar{q}_{i,-1}} \right) / \bar{q}_{i,-1}. \tag{10d}$$

Intuitively, condition (10c) weighs the gains from marginally increasing the quality for the low types—and, hence, their willingness-to-pay—against the marginal cost of producing that quality and of marginally tightening (IC) for the high types. (10d) weighs the gains from marginally increasing the quality for the high types against the marginal cost of production and of innovation.

Let  $(\bar{q}_i^h, \bar{p}_i^h)$  be the solution to equation system (10a) - (10d). We can then distinguish three cases: First, if  $\bar{q}_i^L \leq \bar{q}_{i,-1}$  the initial premise that  $q_i^L > \bar{q}_{i,-1}$  is violated and the firm will offer  $\bar{q}_{i,-1}$  to the low income households at marginal costs  $\bar{q}_{i,-1}/(a_q A)$ , while  $q_i^H$  and  $p_i^H$  are determined by solving (9a) - (9b).

Second, if  $\bar{q}_i^H > \bar{q}_i^L \geq \bar{q}_{i,-1}$ , the firm offers  $(q_i^H, p_i^H) = (\bar{q}_i^H, \bar{p}_i^H)$  to the high income households and  $(q_i^L, p_i^L) = (\bar{q}_i^L, \bar{p}_i^L)$  to the low income households.

Third, if  $\bar{q}_i^L \geq \bar{q}_i^H$ , the premise that  $q_i^H > q_i^L$  is violated and the firm will pool the demand of the high and low income households by offering only one quality-price combination. The profit maximizing combination is determined by solving the firms' problem with  $q_i^L = q_i^H = q_i$  and  $p_i^L = p_i^H = p_i$  and condition  $(IR^L)$  binding for  $(q_i, p_i)$ . This yields the necessary conditions

$$(IR^L) \quad \theta^L v(q_i) - p_i = \theta^L v(\bar{q}_{i,-1}) - \frac{\bar{q}_{i,-1}}{a_q A}, \tag{11a}$$

$$\theta^L v'(q_i) - \frac{1}{a_q A} = h' \left( \frac{q_i}{\bar{q}_{i,-1}} \right) / \bar{q}_{i,-1}. \tag{11b}$$

In sum, innovating firms offer quality-price bundles  $(q_i^h, p_i^h)$ ,  $h = L, H$ , according to the different cases specified above. Pre-existing quality levels  $q_i \leq \bar{q}_{i,-1}$  are priced at marginal costs  $p_i(q_i) = q_i/(a_q A)$ , while all other quality levels  $\hat{q}_i > \bar{q}_{i,-1}$ ,  $\hat{q}_i \neq q_i^h$ ,  $h = L, H$ , are not offered by the firm or, alternatively, at a price  $p_i(\hat{q}_i) = \hat{p}_i$  sufficiently high but finite such that households do not demand qualities  $\hat{q}_i$ . The supply of variety  $i$  at quality level  $q_i^H$  is  $\lambda$  and that of quality level  $q_i^L$  is  $(1 - \lambda)$ , with zero supply of any other quality levels.

According to the production function for differentiated goods, the firms' labor demand thus amounts to  $L_i(q_i^H) + L_i(q_i^L) = \frac{\lambda q_i^H + (1-\lambda)q_i^L}{a_q A}$ .

Lastly, the solution to the firms' maximization problem involves research investments  $f$  and development costs  $h(\bar{q}_i/\bar{q}_{-1})$  to increase the highest quality of variety  $i$  to  $\bar{q}_i$ .

### 3.3. Equilibrium

We define an equilibrium in the closed economy as a static equilibrium in each period while noting that our model has an infinite time horizon. As noted in sections 2.1 and 3.1, the maximization problems of the households and firms are static. Therefore, the equilibrium of our model is an infinite sequence of static one-period equilibria.

**Definition 1** (Equilibrium in the closed economy). An equilibrium is a set of prices  $\{ \{ p_i^e(q) \}_{(i,q) \in [0,1] \times \mathcal{Q}_i}, p_z^e, w^e = 1 \}$ , quantities  $\{ \{ x_i^{H,e}(q), x_i^{L,e}(q) \}_{(i,q) \in [0,1] \times \mathcal{Q}_i}, z^{H,e}, z^{L,e} \}$ , labor demand  $\{ \{ L_i^e(q) \}_{(i,q) \in [0,1] \times \mathcal{Q}_i}, L_z^e \}$ , upper bounds for quality  $\{ \bar{q}_i \}_{i \in [0,1]}$ , and a mass of firms entering the patent race  $M_i^e$  and R&D-investments  $h(\bar{q}_i^e / \bar{q}_{i-1})$  of the winning firm for all  $i$ , such that the zero-profit condition in the patent race holds, profits of the firms producing the differentiated goods and the homogeneous good are maximized, the households' utilities are maximized and all markets clear, i.e.

$$\lambda x_i^{H,e}(q) + (1 - \lambda) x_i^{L,e}(q) = \frac{1}{q} a_q A L_i^e(q), \quad \forall (i, q) \in [0, 1] \times \mathcal{Q}_i \quad (\text{differentiated good markets}), \quad (12a)$$

$$\lambda z^{H,e} + (1 - \lambda) z^{L,e} = a_z A L_z^e - a_z A \int_0^1 f M_i^e + h(\bar{q}_i^e / \bar{q}_{i-1}) di \quad (\text{homogeneous good market}), \quad (12b)$$

$$\int_0^1 L_i^e(q_i^{H,e}) + L_i^e(q_i^{L,e}) di + L_z^e = \lambda \omega^H + (1 - \lambda) \omega^L \quad (\text{labor market}), \quad (12c)$$

where  $q_i^{H,e}$  and  $q_i^{L,e}$  in (12c) are the quality levels for which demand  $x_i^{H,e}(q) > 0$  and  $x_i^{L,e}(q) > 0$ , respectively.

The different cases in the solution to the firms' decision problem play the central role for characterizing the equilibrium in our economy. Using again  $(\hat{q}^h, \hat{p}^h)$  to refer to the solution to the system of equations (10a) - (10d), we can formulate:

**Proposition 1** (Equilibrium in the closed economy). The equilibrium in the closed economy is unique and satisfies for  $h = \{ H, L \}$ :  $I^h = \omega^h$  and  $(q_i^{h,e}, p_i^{h,e}) = (q^{h,e}, p^{h,e})$ . The equilibrium in the differentiated good markets can be characterized as follows:

- (A) If  $\hat{I} \geq I^H \geq I^L$ , the firms do not innovate and offer qualities  $q^{h,e} = (1 - \beta) I^h a_q A \leq \bar{q}_{-1}$  at prices equal to marginal costs  $p^{h,e} = q^{h,e} / (a_q A)$ .
- (B) If  $I^H > \hat{I} \geq I^L$ , or  $I^H > I^L > \hat{I}$  and  $\hat{q}^L \leq \bar{q}_{-1}$ , there is a separating equilibrium where the firms innovate to provide quality  $q^{H,e} > \bar{q}_{-1}$  for the high types, which is offered at price  $p^{H,e}$  above marginal cost. The bundle  $(q^{H,e}, p^{H,e})$  is the unique solution to (9a) and (9b) where  $\theta^{H,e} = (I^H - p^{H,e}) / (\beta(q^{H,e})^{1-\beta})$ . The low types consume quality  $q^{L,e} \leq \bar{q}_{-1}$  priced at marginal costs  $p^{L,e} = q^{L,e} / (a_q A)$ .
- (C) If  $I^H > I^L > \hat{I}$  and  $\hat{q}^H > \hat{q}^L > \bar{q}_{-1}$ , there is a separating equilibrium with  $(q^{h,e}, p^{h,e}) = (\hat{q}^h, \hat{p}^h)$ . That is, the firms innovate to quality  $q^{H,e} > \bar{q}_{-1}$  and offer qualities  $\bar{q}_{-1} < q^{L,e} < q^{H,e}$  with mark-ups of different sizes at  $p^{L,e} < p^{H,e}$ .
- (D) If  $I^H \geq I^L > \hat{I}$  and  $\hat{q}^L \geq \hat{q}^H$ , there is a pooling equilibrium. Firms innovate to offer  $q^{H,e} = q^{L,e} = q^e$  at a price  $p^e$  that includes a mark-up.  $(q^e, p^e)$  are the unique solution to equations (11a) and (11b) where  $\theta^{L,e} = (I^L - p^e) / (\beta(q^e)^{1-\beta})$ .

The proof of Proposition 1 along with the other equilibrium values is provided in appendix A.4.<sup>16</sup> The intuition for the equilibrium is as follows. With  $I^H > \hat{I}$ , firms have an incentive to innovate to satisfy the quality demands of the high income households. The firm has to decide how much to invest in innovation and whether to offer two different quality levels for the different household types. In case (B), the firm innovates to cater to the quality demand of the rich households, while the poor households consume an already existing quality at marginal costs. If  $I^L \leq \hat{I}$  this is because the poor households' optimal quality level is lower than  $\bar{q}_{-1}$ . If  $I^L > \hat{I}$  serving the poor pre-existing qualities is nevertheless optimal if the gains in profits from providing quality  $q > \bar{q}_{-1}$  to the poorer households do not compensate for the implied losses in profits from the rich customer base. These lower profits arise because the firm would have to reduce the price for  $q_i^H$  to keep the rich households consuming the higher quality. In (C),  $I^L > \hat{I}$  and the share of the poor,  $1 - \lambda$ , is sufficiently large while income differences between the poor and the rich are sufficiently high such that it is beneficial to push out the technological level to offer high qualities for the rich and lower qualities, but still higher than  $\bar{q}_{-1}$ , for the poor. This is a separating equilibrium where both types of households are served by innovating firms, that is, we observe multi-quality firms in equilibrium. Finally, if  $I^L > \hat{I}$  and the share of the rich,  $\lambda$ , and income differences between rich and poor households are small enough, it is optimal for firms to pool households to economize on costs of innovation (case (D)).

We note that  $q^{L,e}$ ,  $q^{H,e}$ ,  $p^{L,e}$ , and  $p^{H,e}$  are jointly pinned down by systems of equations that depend only on these variables and time-invariant parameters. Moreover,  $q^{L,e}$  and  $q^{H,e}$  enter these equations only via  $\frac{q^{L,e}}{\bar{q}_{-1}}$  and  $\frac{q^{H,e}}{\bar{q}_{-1}}$ —see the detailed version of Proposition 1 in appendix A.4. Consequently, the aggregate growth rate  $g^e = \frac{q^{H,e}}{\bar{q}_{-1}}$  is constant over time, which constitutes the following corollary.<sup>17</sup>

<sup>16</sup> Note that case (A) of Proposition 1 summarizes the cases (ii) and (iii)(A) of the technical version of the proposition in appendix A.4, while the cases (B) and (C) of Proposition 1 correspond to the cases (iii)(B) and (iii)(C) in the technical formulation. As we show in the appendix, the equilibrium allocation of labor, the price  $p^e$ , and the price quality bundles  $(q^{h,e}, p^{h,e})$  are unique. However, we note that there is an infinite number of price vectors satisfying the equilibrium conditions, differing in the prices of qualities that are not supplied in equilibrium.

<sup>17</sup> Note that the above also implies that a household's position vis-à-vis  $\hat{I}$  remains constant over time.

**Corollary 1** (Balanced growth path). *There is a unique balanced growth path (BGP) which is reached instantaneously. Along the BGP, the growth rate is  $g^e = \frac{q^{H,e}}{\bar{q}_{-1}}$ .*

Next, we examine how a change in the income distribution impacts equilibrium outcomes and in particular  $q^{H,e}$ , which governs growth. To simplify notation, we will throughout dispose of the superscript  $e$  to indicate equilibrium outcomes.

### 3.4. Inequality and growth: closed economy

Without loss of generality, we normalize endowments with effective labor such that

$$E[\omega] = \lambda\omega^H + (1 - \lambda)\omega^L = 1 \equiv \bar{\omega}.$$

We further choose

$$\omega^H = 1 + \sigma\sqrt{(1 - \lambda)/\lambda}, \quad \omega^L = 1 - \sigma\sqrt{\lambda/(1 - \lambda)},$$

where  $\sigma \geq 0$ . This specification allows to separate changes in the variance of the income distribution,  $\sigma^2$ , from changes in its skewness.<sup>18</sup> To analyze the impact of inequality on growth in our economy, we then focus on changes in  $\sigma$ . This corresponds to a Lorenz-dominated shift of the income distribution and allows isolating our main mechanism of interest—an inequality-induced higher willingness to pay for innovation—and its implications in a closed vs open economy. Moreover, we concentrate on the most comprehensive case where  $\hat{I} < \bar{\omega}$  and, hence, innovation incentives can be driven by both low and high income households. The following proposition characterizes the growth effects of a change in inequality for this case.

**Proposition 2** (Inequality and growth in the closed economy). *If  $\hat{I} < \bar{\omega}$ , there is a U-shaped relationship between the variance of the income distribution and economic growth. The lowest growth rate is at the level of  $\sigma$  where the equilibrium type changes from a pooling to a separating equilibrium.*

The proof of Proposition 2 is given in appendix A.5.<sup>19</sup> The intuition of Proposition 2 can be summarized as follows. At an equal distribution of incomes,  $\sigma = 0$ , we trivially observe a pooling equilibrium. As  $\hat{I} < \bar{\omega}$ , all households like to consume higher quality levels than  $\bar{q}_{-1}$ , implying positive gains from innovation. Hence, economic growth is positive in such case. As we increase  $\sigma$  starting from an equal income distribution, we now have to consider the different types of equilibria described in Proposition 1. Initially, a pooling equilibrium will persist, but with the low income households showing a lower willingness to pay for quality and thereby leading to lower equilibrium quality and prices. As we increase inequality further, the willingness to pay for quality of high income households is eventually large enough to justify additional investments in quality upgrading on the side of innovating firms despite the fact that only a fraction  $\lambda < 1$  of households are rich. This means that there will be a separating equilibrium where innovation incentives are driven by the willingness-to-pay of the rich. In this equilibrium, innovation incentives and growth are increasing in  $\sigma$ . Taken together, there is a U-shaped relationship between inequality  $\sigma$  and growth, where the minimum is reached at the point where the economy switches from a pooling to a separating equilibrium, as illustrated in Fig. 1.

These growth effects of inequality are similar to what has previously been found in the literature and, in particular, the positive ‘willingness-to-pay’ effect in a separating equilibrium. In what follows, we show how this result hinges on the closed-economy assumption and how international trade can have profound consequences for the inequality-growth nexus in the developing world.

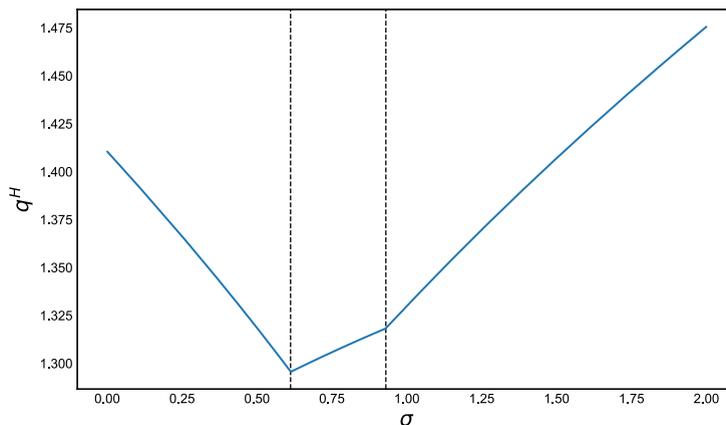
## 4. Small open economy

We now consider a small open economy (SOE) variant of our model where households can satisfy their demand for any of the goods by importing them from a rest of the world (ROW) that is technologically more advanced, but perfectly symmetric to the SOE otherwise. Specifically, we assume that  $a_z^{ROW} = a_z$ ,  $a_q^{ROW} = a_q$ , and  $\bar{q}_{-1} < \bar{q}_{-1}^{ROW}$ . As before, we consider the case of a common inherited quality in the first period across varieties, i.e.  $\bar{q}_{i,-1} = \bar{q}_{-1} \forall i \in [0, 1]$ , and analogously for the ROW.<sup>20</sup> Trade between the SOE and the ROW is subject to an iceberg trade cost  $\tau \geq 1$  that is the same across all sectors. Domestically or foreign produced versions of any given quality of a good are perfect substitutes. We are interested in how the ability to import high-quality goods impacts

<sup>18</sup> The skewness is  $SK(\omega) = (1 - 2\lambda)/(\sqrt{\lambda(1 - \lambda)})$ . Our specification of endowments with effective labor relates to Foellmi et al. (2014) and Latzer (2018) as follows. As in Foellmi et al. (2014) and Latzer (2018), an increase in  $\sigma$  increases the income gap and leaves the share of poor households unchanged. Therefore, an increase in  $\sigma$  always increases inequality and a policy reducing  $\sigma$  leads to a Lorenz-dominating shift. On the other hand, an increase in income concentration (i.e. a decrease in  $\lambda$ ) in our setting also increases the income gap. Hence, a change in  $\lambda$  leads to a Lorenz-crossing shift, as we cannot disentangle changes in income concentration and the income gap when varying  $\lambda$ . Unlike the specification in Foellmi et al. (2014) and Latzer (2018),  $\lambda$  is therefore not monotonously related to measures of inequality.

<sup>19</sup> The statements for a separating equilibrium where both types are still served are partially based on numerical solutions for a broad range of parameter specifications, see appendix A.5.

<sup>20</sup> In our set-up, comparative advantage thus only arises from the extensive quality margin, i.e. from the fact that the ROW can produce a broader range of qualities when compared to the SOE. We note that we could allow for the SOE to have a comparative advantage in homogeneous good production. This would imply that foreign competition becomes binding at lower levels of inequality, but our subsequent discussions of the inequality-growth nexus in the SOE would otherwise remain the same.



Note: The figure shows the equilibrium values of quality  $q^H$  for different values of inequality  $\sigma$  and where the average income exceeds the income threshold for consuming innovative goods,  $\hat{I} < \bar{\omega}$ . The dashed lines indicate changes in the type of equilibrium, first from a pooling to a separating equilibrium and then to a separating equilibrium where only rich households are served by innovating firms. The remaining parameter values are homogeneous good share  $\beta = 0.5$ , share of rich households  $\lambda = 0.2$ , inherited quality level  $\bar{q}_{-1} = 1$ , and productivity parameter  $a_q = 12$ . Furthermore, marginal development cost are  $h'(x) = x - 1$ .

Fig. 1. Equilibrium quality levels  $q^H$  for different values of inequality  $\sigma$ .

innovation and growth in developing countries. To that end, we consider an SOE that is sufficiently far from the frontier such that (i) it is never optimal for firms in the SOE to innovate in order to serve the rich in the ROW<sup>21</sup>; and (ii) households can import their preferred quality from the ROW at the prevailing equilibrium prices, without being bound by the highest available quality level in the ROW. In section 5.1, we discuss how our main insights change if the preferred import option of the rich in the SOE is bound by the maximum quality in the ROW.

To analyze the equilibrium in the SOE, we note that the set-up immediately implies that there cannot be two-way trade of any given quality of a good. Hence, balanced trade is possible only if the SOE imports some high qualities  $q_i > \bar{q}_i$  from abroad and exports the homogeneous good  $z$  and / or qualities  $q_i \leq \bar{q}_i$  of the differentiated goods. In turn, this requires that the SOE can price the homogeneous good competitively in the world market. Thus, with positive trade it holds that

$$p_z^{ROW} = \tau \frac{w}{a_z A} = \frac{w^{ROW}}{a_z A^{ROW}}. \tag{13}$$

To compete with firms in the SOE, foreign firms are willing to serve consumers in the SOE at their marginal costs scaled by the iceberg trade cost  $\tau$ .<sup>22</sup> The marginal costs for firms from the ROW of providing quality  $q$  to consumers in the SOE are:  $\tau \frac{q w^{ROW}}{a_q A^{ROW}}$ . Using the domestic wage as the numéraire again,  $w = 1$ , and equation (13), thus yields for the price of foreign qualities in the SOE

$$p^f(q) = \tau^2 \frac{1}{a_q A} q, \tag{14}$$

where we use superscript  $f$  to denote an offer from foreign firms to consumers in the SOE. We summarize these insights in the following lemma, and relegate a formal proof to appendix A.6.

**Lemma 4 (Import prices in the SOE).** *To compete against innovating firms in the SOE, foreign firms are willing to serve consumers in the SOE at a price  $p^f(q)$  given by (14).*

#### 4.1. The domestic firms' decision problem in the SOE

The previous discussions imply that firms in the SOE cannot make profits from selling differentiated goods to the ROW—see also the proof of Lemma 4. Yet, the availability of imported qualities impacts innovation incentives in the SOE, because imported qualities introduce a second set of individual rationality constraints for households: In the SOE, a contract offered by a domestic monopolist must not only be preferable to a household's best choice among the domestic competitive fringe, but also to its best import option. This leads to the following augmented decision problem for innovating firms in the SOE:

<sup>21</sup> This is the case if the SOE is sufficiently far from the frontier and large innovation steps are sufficiently expensive, e.g. in the limiting case where  $h(x)$  becomes vertical at some level  $\hat{x} > 1$ .

<sup>22</sup> If  $q_i \leq \bar{q}_{i-1}^{ROW}$ , imported goods are offered at marginal costs because of the competitive fringe in the ROW. Regarding  $\bar{q}_i^{ROW} > q_i > \bar{q}_{i-1}^{ROW}$ , the foreign firms are willing to offer these qualities at marginal costs when under competitive pressure from the domestic firms because for every price above marginal costs they make strictly positive profits from serving households in the SOE without impacting their profit potential in the ROW. Foreign qualities priced at marginal costs is thus the relevant competitive benchmark that innovating firms in the SOE must reach to sell their innovative goods domestically—see decision problem (15).

$$\begin{aligned}
 & \max_{q_i^H, p_i^H, q_i^L, p_i^L, \bar{q}_i} \lambda \left( p_i^H - \frac{1}{a_q A} q_i^H \right) + (1 - \lambda) \left( p_i^L - \frac{1}{a_q A} q_i^L \right) - h \left( \frac{\bar{q}_i}{\bar{q}_{i,-1}} \right) & (15) \\
 & \text{s.t. } \theta^h v(q_i^h) - p_i^h \geq \max_{q \in [0, \bar{q}_{i,-1}]} \left\{ \theta^h v(q) - \frac{1}{a_q A} q \right\}, & (\text{IR}^h) \\
 & \theta^h v(q_i^h) - p_i^h \geq \max_{q \in [0, \bar{q}_{i,ROW}]} \left\{ \theta^h v(q) - \tau^2 \frac{1}{a_q A} q \right\}, & (\text{IR}^{th}) \\
 & \theta^H v(q_i^H) - p_i^H \geq \theta^H v(q_i^L) - p_i^L, & (\text{IC}^H) \\
 & \theta^L v(q_i^L) - p_i^L \geq \theta^L v(q_i^H) - p_i^H, & (\text{IC}^L) \\
 & q_i^h \leq \bar{q}_i, \quad h \in \{L, H\}.
 \end{aligned}$$

Decision problem (15) is the same as decision problem (8) in the closed economy, but with the additional (IR<sup>th</sup>) constraints. It is useful to simplify this decision problem by solving for the value of the best import option of a household of type  $\theta^h$ . In particular, household  $h$ 's best import option satisfies  $\theta^h v'(q^{h,f}) = \tau^2 / (a_q A)$  and hence

$$q^{h,f} = \left[ \frac{(1 - \beta)\theta^h a_q A}{\tau^2} \right]^{\frac{1}{\beta}}. \tag{16}$$

This implies for the value of the best import option

$$\theta^h v(q^{h,f}) - \frac{q^{h,f} \tau^2}{a_q A} = [\theta^h]^{\frac{1}{\beta}} [\bar{q}_{-1}]^{\frac{1-\beta}{\beta}} \underbrace{\left[ \frac{a_q (1 - \beta)}{\tau^2} \right]^{\frac{1-\beta}{\beta}}}_{:= \chi(\tau)} \beta. \tag{17}$$

Observe from equation (17), that the value of the best import option is convex in  $\theta^h$ , reflecting the fact that higher types not only value quality more, but that they also consume higher quality. As we will see, this convexity implies that for sufficiently high levels of inequality, (IRf) is binding for the high types. In what follows, we assume that iceberg trade-costs exceed a minimum threshold  $\underline{\tau}$ .

**Assumption 1.**

$$\tau \geq \underline{\tau} := \left[ \frac{\beta^{\frac{2\beta-1}{\beta}} \left[ 1 - \frac{1}{a_q} \right]^{1/\beta} [a_q (1 - \beta)]^{\frac{1-\beta}{\beta}}}{1 - \frac{1}{a_q} (1 + \beta)} \right]^{\frac{\beta}{2(1-\beta)}}$$

As the following Lemma 5 shows, Assumption 1 precludes that low types find it optimal to import their differentiated products. This is attractive for our purposes for two reasons: First, it allows centering the discussions on how the possibility of rich households to import high quality from abroad impacts the growth effects of inequality, which is our main focus of interest. Second, it implies that without inequality at  $\sigma = 0$ , (IRf) is strictly non-binding for both types, which will allow to identify the different effects of inequality on growth in the SOE as discussed in section 4.4.

**Lemma 5 (Constraint (IR<sup>L</sup>) non-binding).** *Let Assumption 1 be satisfied. Then constraint (IR<sup>th</sup>) is either redundant or binding for the high types.*

The proof of Lemma 5 is given in appendix A.7. We note that Assumption 1 is sufficient but not necessary for our analysis to apply.

**4.2. Solution to firms' profit maximization problem in the SOE**

As discussed above, the only difference between the firms' profit maximization problem in the closed and the small open economy is the inclusion of the foreign competition constraints (IR<sup>th</sup>), where according to Lemma 5 only (IR<sup>th</sup>) may be binding. Therefore, if for any given levels of willingness to pay for quality  $\theta^h$  the solution to the firms' problem in the closed economy as derived in subsection 3.1 does not violate (IR<sup>th</sup>), then it is also the solution to the firms' maximization problem in the SOE. Consequently, we focus on the case where (IR<sup>th</sup>) is binding. We organize our discussion as in subsection 3.1 by first considering the case  $I^H > \hat{I} \geq I^L$ , where there is no demand for innovation from the low income households, and then turn to the case  $I^H > I^L > \hat{I}$ , where the low income households may consume innovated qualities as well. We refer to these cases by (B-SOE) and (C-SOE) to indicate that they relate to the cases (B) and (C) in Proposition 1.

**(B-SOE)** For  $I^H > \hat{I} \geq I^L$ , the domestic firm supplies quality level  $q_i^L \leq \bar{q}_{i,-1}$  at marginal costs to the low types as in the closed economy. For the high income households, however, with  $(IRf^H)$  binding, the best import option is more attractive than consuming  $\bar{q}_{i,-1}$ . Consequently,  $(IRf^H)$  replaces  $(IR^H)$  in optimality condition (9a), and the offer  $(q_i^H, p_i^H)$  solves

$$(IRf^H) \quad \theta^H v(q_i^H) - p_i^H = [\theta^H]^{\frac{1}{\beta}} [\bar{q}_{i,-1}]^{\frac{1-\beta}{\beta}} \chi(\tau), \tag{18a}$$

$$\lambda \left[ \theta^H v'(q_i^H) - \frac{1}{a_q A} \right] = h' \left( \frac{q_i^H}{\bar{q}_{i,-1}} \right) / \bar{q}_{i,-1}, \tag{18b}$$

where condition (18b) is the same as (9b) in the closed economy. As opposed to the closed economy, the candidate solution  $(q_i^{H,B}, p_i^{H,B})$  to optimality conditions (18a) and (18b) may imply negative profits if the value of the best import option is too high. Therefore, the domestic firm offers  $(q_i^H, p_i^H) = (q_i^{H,B}, p_i^{H,B})$  to the high income households if

$$\pi^B(q_i^{H,B}, p_i^{H,B}) = \lambda \left( p_i^{H,B} - \frac{1}{a_q A} q_i^{H,B} \right) - h \left( \frac{q_i^{H,B}}{\bar{q}_{i,-1}} \right) \geq 0, \tag{19}$$

and abstains from making an offer to the rich households otherwise.<sup>23</sup>

**(C-SOE)** Considering the case  $I^H > I^L > \hat{I}$  next, we first recall that in the closed economy in the separating equilibrium, conditions  $(IR^L)$  and  $(IC^H)$  are binding, and the profit maximizing offers  $(q_i^h, p_i^h)$  are the solution to (10a) to (10d). With the foreign competition constraint  $(IRf^H)$  binding, this solution is no longer feasible and the innovating firms' optimal contracts instead solve

$$(IR^L) \quad \theta^L (v(q_i^L) - v(\bar{q}_{i,-1})) + \frac{1}{a_q} = p_i^L, \tag{20a}$$

$$(IRf^H) \quad \theta^H v(q_i^H) - p_i^H = [\theta^H]^{\frac{1}{\beta}} [\bar{q}_{i,-1}]^{\frac{1-\beta}{\beta}} \chi(\tau), \tag{20b}$$

$$(IC^H) \quad \theta^H v(q_i^H) - p_i^H \geq \theta^H v(q_i^L) - p_i^L, \tag{20c}$$

$$\lambda \theta^H v'(q_i^H) - \lambda \frac{1}{a_q A} - h' \left( \frac{q_i^H}{\bar{q}_{i,-1}} \right) \frac{1}{\bar{q}_{i,-1}} = 0. \tag{20d}$$

System of equations (20a) to (20d) differs in two key ways from its counterpart in the closed economy. First, to compete with foreign high-quality providers, innovating firms in the SOE need to improve the value of their offer to the rich households due to constraint  $(IRf^H)$ . Second, this in turn relaxes incentive constraint  $(IC^H)$ , which allows innovating firms to earn higher profits from the poor households by increasing  $q^L$  and  $p^L$  while holding constant  $\theta^L v(q^L) - p^L$  (i.e. guaranteeing that  $(IR^L)$  remains binding), up to the point where  $(IC^H)$  is again binding, or up to  $q_i^L = (\theta^L (1 - \beta) a_q A)^{(1/\beta)}$  if  $(IC^H)$  remains slack. The latter yields the highest possible profits from serving the poor. In either case, (10c) no longer applies and the optimal solution must satisfy (20c) instead.<sup>24</sup>

Consider the candidate solution  $(q_i^{h,C}, p_i^{h,C})$  solving (20a)-(20d) as described. Again, we need to check whether it is actually profit-maximizing to serve the high income households under pressure from foreign competition. Alternatively, firms could choose to innovate to serve the poor households only. In this case, their optimal contract is determined by optimality conditions (9a) and (9b), but adapted to the low types:

$$(IR^L) \quad \theta^L v(q_i^L) - p_i^L = \theta^L v(\bar{q}_{i,-1}) - \frac{\bar{q}_{i,-1}}{a_q A}, \tag{21a}$$

$$(1 - \lambda) \left[ \theta^L v'(q_i^L) - \frac{1}{a_q A} \right] = h' \left( \frac{q_i^L}{\bar{q}_{i,-1}} \right) / \bar{q}_{i,-1}. \tag{21b}$$

We refer to this solution by  $(q_i^{L,Cf}, p_i^{L,Cf})$ . Hence, the domestic firm serves the rich if

$$\begin{aligned} \pi^C(\{q_i^{h,C}, p_i^{h,C}\}_h) &:= \lambda \left( p_i^{h,C} - \frac{1}{a_q A} q_i^{h,C} \right) + (1 - \lambda) \left( p_i^{L,C} - \frac{1}{a_q A} q_i^{L,C} \right) - h \left( \frac{q_i^{h,C}}{\bar{q}_{i,-1}} \right) \\ &\geq (1 - \lambda) \left( p_i^{L,Cf} - \frac{1}{a_q A} q_i^{L,Cf} \right) - h \left( \frac{q_i^{L,Cf}}{\bar{q}_{i,-1}} \right) =: \pi^{Cf}(q_i^{L,Cf}, p_i^{L,Cf}), \end{aligned} \tag{22}$$

<sup>23</sup> In the closed economy it is always optimal to serve the rich households in case B as  $I^H > \hat{I}$  and  $h'(1) = 0$ . The former implies that at  $q_i = \bar{q}_{i,-1}$ , the rich households' willingness to pay for a marginally higher  $q_i$  exceeds the marginal production cost of  $1/a_q$ . The latter implies that marginally increasing  $\bar{q}$  is costless. By contrast, in case (B-SOE) with  $(IRf^H)$  binding, it may be optimal not to serve the rich as their best import option is strictly better than consuming  $\bar{q}_{i,-1}$  at marginal costs.

<sup>24</sup> In the closed economy, it may be optimal for innovating firms not to serve the poor even if  $I^L > \hat{I}$ . This is because serving the poor  $q^L > \bar{q}_{i,-1}$  tightens constraint  $(IC^H)$ . In the SOE with  $(IRf^H)$  strictly binding, this cannot happen because in such case rich households strictly prefer their best import option over consuming  $\bar{q}_{i,-1}$  at marginal cost domestically.

and does not serve the rich otherwise. When condition (22)—or condition (19), for that matter—holds with equality, domestic firms are indifferent between innovating or not to serve the rich. As the following lemma shows, in such case they would offer a strictly lower quality to rich households than their best import option.

**Lemma 6** (Import quality higher than domestic quality). *When a firm in the SOE is just indifferent between innovating or not to serve rich households, the quality that it would offer to the rich households is strictly lower than the quality of the rich households’ best import option.*

The proof of Lemma 6 is given in appendix A.8. As we will see, Lemma 6 implies that in equilibrium firms in the SOE may innovate to serve rich households in some but not all differentiated goods sectors.

To formally characterize the equilibrium, it will be useful to define  $\Delta\pi^C(\{q_i^{h,C}, p_i^{h,C}\}_h, q_i^{L,Cf}, p_i^{L,Cf}) := \pi^C(\{q_i^{h,C}, p_i^{h,C}\}_h) - \pi^{Cf}(q_i^{L,Cf}, p_i^{L,Cf})$ . Moreover, to reduce notation, we note that the solutions  $(q_i^{h,C}, p_i^{h,C}), (q_i^{L,Cf}, p_i^{L,Cf})$  only depend on the model’s primitives,  $\bar{q}_{i-1}$  which is given in each period, and the levels of  $\theta^h$ , which are endogenously determined in equilibrium, but which the firms take as given. By inserting the firms’ optimal offers, we can then write  $\Delta\pi_i^C(\theta^H, \theta^L)$  and accordingly  $\pi_i^B(\theta^H)$  for the maximum profits in the previously discussed case (B-SOE).

### 4.3. Equilibrium in the small open economy

With some adjustments, we can define the equilibrium in the small open economy analogously to the one in the closed economy. Again, we define a static equilibrium, noting that the dynamic equilibrium is an infinite sequence of static one-period equilibria. As before, we refer to the set of qualities available for consumption as  $Q_i(t) = [0, \bar{q}_i^{ROW}(t)]$ , where we have made use of the fact that the SOE is technologically lagging. In contrast to the closed economy, the equilibrium in the SOE takes into account the foreign supply of differentiated goods, which we refer to by  $x^f(\cdot)$  and the SOE’s exports. As the detailed composition of these exports is a matter of indifference, we assume for expositional reasons that only the homogeneous good is exported and denote the exported quantity by  $z^{exp}$ .

**Definition 2** (Equilibrium in SOE). *An equilibrium in the SOE is a set of prices  $\{ \{p_i^e(q), p_i^{f,e}(q)\}_{(i,q) \in [0,1] \times Q_i}, p_z^e, p_z^{ROW,e}, w^e = 1 \}$ , quantities  $\{ \{x_i^{H,e}(q), x_i^{L,e}(q), x_i^{f,e}(q)\}_{(i,q) \in [0,1] \times Q_i}, z^{H,e}, z^{L,e}, z^{exp,e} \}$ , labor demand  $\{ \{L_i^e(q)\}_{i \in [0,1] \times Q_i}, L_z^e \}$ , upper bounds for quality  $\{ \bar{q}_i \}_{i \in [0,1]}$ , and a mass of firms entering the patent race  $M_i$  and R&D-investments  $h(\bar{q}_i/\bar{q}_{i-1})$  of the winning firm for all  $i$ , such that  $p_z^{ROW,e}$  and  $\{p_i^{f,e}(q)\}_{(i,q) \in [0,1] \times Q_i}$  are given by (13) and (14), respectively, and in the SOE the profits of the firms producing the differentiated goods and the homogeneous good are maximized, the households’ utilities are maximized, the zero-profit condition in the patent race holds, and all markets clear, i.e.*

$$\lambda x_i^{H,e}(q) + (1 - \lambda) x_i^{L,e}(q) = \frac{1}{q} a_q A L_i^e(q) + x_i^{f,e}(q), \forall (i, q) \in [0, 1] \times Q_i \quad (\text{differentiated good markets}), \tag{23a}$$

$$\lambda z^{H,e} + (1 - \lambda) z^{L,e} + z^{exp,e} = a_z A L_z^e - a_z A \int_0^1 f M_i^e + h(\bar{q}_i/\bar{q}_{i-1}) di \quad (\text{homogeneous good market}), \tag{23b}$$

$$\int_0^1 L_i^e(q_i^{H,e}) + L_i^e(q_i^{L,e}) di + L_z^e = \lambda \omega^H + (1 - \lambda) \omega^L \quad (\text{labor market}), \tag{23c}$$

$$\int_0^1 p_i^{f,e}(q_i^{H,f,e}) x_i^{f,e}(q_i^{H,f,e}) di = p_z^{ROW,e} z^{exp,e} \quad (\text{trade balance}), \tag{23d}$$

where  $q_i^{H,e}, q_i^{L,e}$  in (23c) are the quality levels for which domestic production is positive, i.e.  $L_i^e(q_i^{H,e}) > 0$  and  $L_i^e(q_i^{L,e}) > 0$ , respectively. Analogously,  $q_i^{H,f,e}$  in (23d) refers to the quality level where foreign supply is positive, i.e.  $x_i^{f,e}(q_i^{H,f,e}) > 0$  in an equilibrium with trade.

To characterize the equilibrium, we need to account for the fact that firms in the SOE may innovate to serve rich households in some but not all differentiated goods sectors and the rich households import a share of the differentiated goods from the ROW. Indeed, this is typically the case in an equilibrium with trade as we will discuss further in the next section. For now, it suffices to note that in such case firms must be indifferent between innovating or not to serve the rich, i.e. it must hold that  $\Delta\pi_i^C(\theta^H, \theta^L) = 0$  and  $\pi_i^B(\theta^H) = 0$ , respectively, depending on whether  $I^L > \hat{I}$  or  $I^L \leq \hat{I}$ . Let  $N_f$  be the share of sectors in which rich households import the differentiated goods. Without loss of generality, we assume that these are sectors  $i \in [0, N_f]$ , and can then write the rich households’ taste for quality  $\theta^H$  in the SOE as follows

$$\theta^H = \frac{I^H - \int_0^{N_f} p_i^{H,f} di - \int_{N_f}^1 p_i^H di}{\beta \left( \int_0^{N_f} (q_i^{H,f})^{1-\beta} di + \int_{N_f}^1 (q_i^H)^{1-\beta} di \right)}. \tag{24}$$

Our previous considerations on the households' best import option and the solution to the firms' maximization problem as specified in subsection 4.1 indicate how the optimal bundles  $(q_i^{H,f}, p_i^{H,f}), (q_i^H, p_i^H)$  depend on the level of  $\theta^H$ , which in turn depends on these prices and quantities as shown in Equation (24). The equilibrium level of  $\theta^H$  is therefore a fixed point of (24). For a given share of imports  $N_f$ , we denote a solution to this fixed point problem (24) by  $\theta^{H,e}(N_f)$  and analogously for  $\theta^{L,e}(N_f)$ .<sup>25</sup> We are now ready to characterize the equilibrium in the SOE as follows:

**Proposition 3 (Equilibrium in the SOE).** *If the equilibrium in the closed economy as characterized in Proposition 1 does not violate condition (IR<sup>fH</sup>), then this is also the equilibrium in the SOE.*

*If the equilibrium in the closed economy violates condition (IR<sup>fH</sup>), then there exists an equilibrium in the SOE where the differentiated goods markets satisfy:*

(B-SOE) Let  $I^H > \hat{I} \geq I^L$ . Then the poor households consume  $q^{L,e} = (1 - \beta)a_q AI^L \leq \bar{q}_{-1}$  at marginal costs  $p^{L,e} = q^{L,e}/(a_q A)$  and

- (i) domestic firms serve the rich households in all sectors  $i$ , i.e.  $N_f^e = 0$ , if  $\pi^B(\theta^{H,e}(0)) \geq 0$ . The firms' offer  $(q^{H,e}, p^{H,e})$  is the solution to (18a) and (18b) where  $\theta^{H,e}(0) = (I^H - p^{H,e})/(\beta(q^{H,e})^{1-\beta})$ ;
- (ii) there is a dual economy with  $N_f^e \in (0, 1)$  if  $\pi^B(\theta^{H,e}(N_f^e)) = 0$ . In sectors  $i \leq N_f^e$ , the rich households import quality  $q^{H,f,e}$  at price  $p^{H,f,e}$  as given by (14) and (16). In sectors  $i > N_f^e$ , the domestic firms provide the bundle  $(q^{H,e}, p^{H,e})$  which is the solution to (18a) and (18b);
- (iii) the rich households import all differentiated goods, i.e.  $N_f^e = 1$ , if  $\pi^B(\theta^{H,e}(1)) < 0$ .<sup>26</sup>

(C-SOE) Let  $I^H > I^L > \hat{I}$ . Then

- (i) domestic firms serve the rich households in all sectors  $i$ , i.e.  $N_f^e = 0$ , if  $\Delta\pi^C(\{\theta^{h,e}(0)\}_h) \geq 0$ . The firms' offer  $(q^{h,e}, p^{h,e})$  is the solution to (20a) - (20d) where  $\theta^{h,e}(0) = (I^h - p^{h,e})/(\beta(q^{h,e})^{1-\beta})$ ;
- (ii) there is a dual economy with  $N_f^e \in (0, 1)$  if  $\Delta\pi^C(\{\theta^{h,e}(N_f^e)\}_h) = 0$ . In sectors  $i \leq N_f^e$ , the rich households import quality  $q^{H,f,e}$  at price  $p^{H,f,e}$  as given by (14) and (16) with  $\theta^H = \theta^{H,e}(N_f^e)$  and the domestic firms innovate for the poor households offering  $(q^{L,e}, p^{L,e}) = (q^{L,Cf,e}, p^{L,Cf,e})$  which is the solution to (21a) and (21b) with  $\theta^L = \theta^{L,e}(N_f^e)$ . In sectors  $i > N_f^e$ , the domestic firms offer the bundles  $(q^{h,e}, p^{h,e}) = (q^{h,Ce}, p^{h,Ce})$  which is the solution to (20a) - (20d) with  $\theta^h = \theta^{h,e}(N_f^e)$ ;
- (iii) the rich households import all differentiated goods, i.e.  $N_f^e = 1$  if  $\Delta\pi^C(\{\theta^{h,e}(1)\}_h) < 0$ .<sup>27</sup> The domestic firms innovate to offer to the low income households  $(q^{L,e}, p^{L,e}) = (q^{L,Cf,e}, p^{L,Cf,e})$  which is the solution to (21a) and (21b) with  $\theta^L = \theta^{L,e}(1)$ .

The proof of Proposition 3 is provided in appendix A.9. As in the equilibrium in the closed economy, given prices and quantities for the differentiated goods, the equilibrium values for the homogeneous good, exports and the distribution of labor across sectors follow. They are also provided in appendix A.9. Which of the different cases in Proposition 3 materializes depends on primitive parameters of our model. We provide further economic intuition in the next section, where we consider how inequality impacts growth in the SOE.

#### 4.4. Inequality and growth: small open economy

We now analyze equilibrium innovation and, hence, growth for sequentially increasing variances of the income distribution,  $\sigma$ . Analogous to the closed economy, we again consider the most comprehensive case where  $\bar{\omega} > \hat{I}$ .

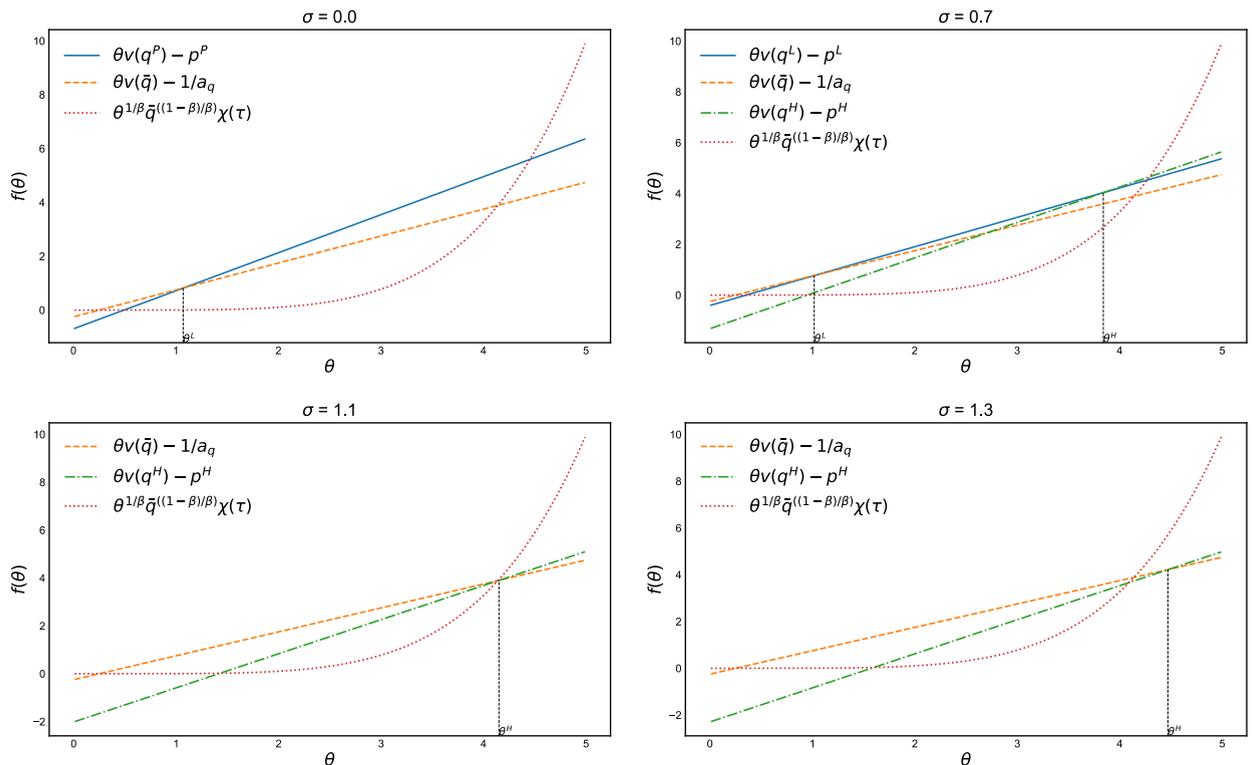
*Low levels of inequality – Foreign competition constraint (IR<sup>fH</sup>) non-binding, equilibrium in SOE identical to the one in the closed economy*

Starting from perfect equality  $\sigma = 0$ , condition (IR<sup>fH</sup>) is non-binding for both household types, and the equilibrium is identical to the equilibrium in the closed economy (see Proposition 3). This is illustrated in the top-left graph of Fig. 2. This graph shows a household's payoff from three different consumption choices for the differentiated good as a function of its type  $\theta$ : The payoff when consuming  $\bar{q}_{-1}$  from the domestic competitive fringe,  $\theta v(\bar{q}_{-1}) - \frac{1}{a_q}$  (orange dashed line); the payoff when consuming the optimal pooling contract offered by innovating domestic firms,  $\theta v(q^P) - p^P$  (blue solid line); and the payoff from the respective best import option,  $[\theta]^\frac{1}{\beta} [\bar{q}_{-1}]^\frac{1-\beta}{\beta} \chi(\tau)$  (red dotted line). Individual rationality for the low types (IR<sup>L</sup>) implies that the orange dashed and the blue solid lines intersect at  $\theta^L$  which is equal to  $\theta^H$  in this case. Clearly, this intersection lies above the red dotted line, i.e. both types strictly prefer the domestic firms' offer  $(q^P, p^P)$  over their best import option.

<sup>25</sup> We show existence of these fixed points in appendix A.9.

<sup>26</sup> If  $q^{H,f,e} \leq \bar{q}_{ROW,-1}$  according to (16), the rich households import quality  $q^{H,f,e}$  at price  $p^{H,f,e}$  as given by (14) and (16) with  $\theta^H = \theta^{H,e}(1)$ . Otherwise, with  $N_f^e = 1$  innovating firms from the ROW will charge a mark-up when serving rich households in the SOE. This is because foreign high-quality providers are subjected to less competitive pressure when selling their products to the SOE. The optimal contract by firms from the ROW in such case is not material for the main messages of the paper, and the characterization has therefore been omitted.

<sup>27</sup> See footnote 26.



Note: The figures illustrate the optimal contracts for different values of inequality  $\sigma$ . The remaining parameter values are homogeneous good share  $\beta = 0.2$ , share of rich households  $\lambda = 0.2$ , inherited quality level  $\bar{q}_{-1} = 1$ , trade costs  $\tau = 3$ , and productivity parameter  $a_q = 4$ . Furthermore, marginal development cost are  $h'(x) = x - 1$ .

Fig. 2. Illustration of the effect of inequality on innovation in the SOE.

As we increase inequality  $\sigma$ , this does not affect the value of consuming the highest quality from the competitive fringe as represented by the orange dashed line nor the value of the best import option as shown by the red dotted line in the top-left graph of Fig. 2. It does, however, decrease the poor households' willingness to pay for quality  $\theta^L$ , and consequently the prices in the pooling equilibrium  $q^P$ , and  $p^P$  (see Proposition 2). That is, the blue solid line shifts upwards and becomes flatter such that its intersection with the orange dotted line moves to the left. Most importantly, however, the increase in  $\sigma$  also increases the rich households' taste for quality  $\theta^H$ . As long as the foreign competition constraint ( $IR^{FH}$ ) is non-binding, a change in inequality  $\sigma$  trivially has the same effect on growth as in the closed economy: Growth initially declines while still in a pooling equilibrium and eventually increases when  $\sigma$  and, hence, income differences are large enough such that innovating firms find it optimal to separate high types from low types. This separating equilibrium is illustrated in the top-right graph of Fig. 2. The green dash-dotted line shows a household's payoff from consuming quality  $q^H$  as a function of its type  $\theta$ ,  $\theta v(q^H) - p^H$ . As before, individual rationality ( $IR^L$ ) implies that the orange dashed and the blue solid lines intersect at  $\theta^L$ . In addition, incentive compatibility for the high types ( $IC^H$ ) implies that the blue solid and the green dash-dotted lines intersect at  $\theta^H$ . As we can observe, both types still prefer their respective contract over their best import option.

*Intermediate levels of inequality – Foreign competition constraint ( $IR^{FH}$ ) binding. Foreign competition leads to higher growth in SOE than in the closed economy*

Continuing to increase inequality  $\sigma$ , the rich households' willingness to pay for quality  $\theta^H$  increases further and eventually is high enough such that high types are indifferent between consuming the domestic quality  $q^H$ —as determined by the optimal solution in the closed economy—and their best import option.<sup>28</sup> This is illustrated in the bottom-left graph of Fig. 2, where for clarity we show a scenario where this indifference occurs only after innovating domestic firms stopped serving the poor, i.e. it shows case (B-SOE). At this point, if we increase  $\sigma$  further, the foreign competition constraint ( $IR^{FH}$ ) becomes strictly binding for the high types.

How will innovating firms—and, hence, the economy—respond if their optimal contract for the rich is no longer feasible due to import competition? Then domestic firms may, in principle, find it optimal to stop serving the rich. In fact, this will eventually be the case for  $\sigma$  high enough, as we will see below. Initially, however, when ( $IR^{FH}$ ) is marginally binding, it is profitable for innovating firms to marginally improve the value of the contract for the rich such that they are again indifferent between consuming domestic quality

<sup>28</sup> ( $IR^{FH}$ ) must eventually be binding because the value of the best import option is convex in  $\theta$ , while in the closed economy the value of contract  $(q^H, p^H)$  for the rich is bounded from above by a straight line.

$q^H$  or their best import option. This means, there is an equilibrium according to Proposition 3 (B-SOE)(i) or (C-SOE)(i), depending on whether or not the domestic firms still serve the poor households with innovative qualities  $q^L > \bar{q}_{-1}$ .

In this case, the marginal increase in inequality  $\sigma$  has no direct effect on  $q^H$  and, hence, growth. This is because for a given  $\theta^H$ , the quality level  $q^H$  is pinned down by (18b) and (20d), respectively (which are identical).<sup>29</sup> Instead, to counter foreign competition, firms improve the value of their offer to the rich by lowering the price  $p^H$ . Importantly, however, this triggers a positive general equilibrium demand effect on growth, as the lower prices  $p^H$  for all differentiated goods allow the rich to economize on their spending on the differentiated goods. The associated income effect induces them to consume more of the homogeneous good and  $\theta^H$  increases, which in turn increases their demand for high quality. To satisfy this demand, domestic firms increase their offered quality  $q^H$ —see condition (18b) and (20d), respectively.

*High levels of inequality – Foreign competition constraint (IR<sup>H</sup>) binding, households import some or all of the differentiated goods*

As we keep increasing inequality  $\sigma$  and, hence,  $\theta^H$ , (IR<sup>H</sup>) tightens further, and this has the same qualitative effect on  $q^H$  and  $p^H$  as described previously. Eventually, however, the rich households' taste for quality  $\theta^H$  is so high and foreign competition fierce enough such that it is no longer profitable for firms in all domestic good sectors to serve the rich.<sup>30</sup> This leads to equilibrium (B-SOE)(ii), respectively (C-SOE)(ii) of Proposition 3, where the rich start importing some varieties from abroad, and they import strictly higher quality than what alternatively they would have been able to buy from domestic innovators as Lemma 6 shows. In turn, this implies that rich households do not immediately switch to importing all differentiated goods, but instead they gradually increase the share of differentiated goods they import,  $N_f$ , as  $\sigma$  increases further.<sup>31</sup>

To shed further light on this effect recall from Proposition 3 that with  $N_f \in (0, 1)$  firms must be indifferent between serving or not the rich. Let  $\bar{\sigma}$  be the level of inequality for which this indifference condition is just satisfied if all firms are still serving the rich (i.e. if  $N_f = 0$ ). Now, let us consider case (B-SOE), for simplicity, such that for  $\bar{\sigma}$  it holds that  $I^L \leq \hat{I}$  and innovative domestic firms only serve the rich households. The firms' indifference then implies that—with  $\bar{\sigma}$ —they make exactly zero profits, and these profits depend only on  $\theta^H$ .<sup>32</sup> Hence, as we increase  $\sigma$ , firms remain indifferent between serving or not the rich if  $\theta^H$  remains constant at  $\bar{\theta}^H$ , where

$$\bar{\theta}^H := \frac{\bar{I}^H - \int_0^1 \bar{p}^H di}{\beta \int_0^1 (\bar{q}^H)^{1-\beta} di} \tag{25}$$

is the rich households' type given  $\bar{\sigma}$  and where here and below we use  $\bar{x}$  to indicate the value of variable  $x$  for the case of  $\bar{\sigma}$ . It follows that for  $\sigma > \bar{\sigma}$  implying income level  $I^H > \bar{I}^H$ , the share of imports  $N_f$  is implicitly defined by

$$\bar{\theta}^H = \frac{I^H - \int_0^{N_f} \bar{p}^{H,f} di - \int_{N_f}^1 \bar{p}^H di}{\beta(\int_0^{N_f} (\bar{q}^{H,f})^{1-\beta} di + \int_{N_f}^1 (\bar{q}^H)^{1-\beta} di)} \tag{26}$$

In other words, when the high income households start consuming imported qualities,  $\theta^H$  no longer increases with  $\sigma$ —and, hence, neither do domestic quality  $q_i^H$  and the best import option  $q_i^{H,f}$ . Instead, an increase in income translates into a higher share of imported qualities and, as a consequence, less growth compared to the closed economy.

It is now also clear, that if  $I^H$  increases beyond a critical level  $\bar{I}^H$  implicitly defined by

$$\bar{\theta}^H = \frac{\bar{I}^H - \int_0^1 \bar{p}^{H,f} di}{\beta(\int_0^1 (\bar{q}^{H,f})^{1-\beta} di)} \tag{27}$$

then the rich households will consume imported qualities of all varieties  $i$ , and  $\theta^H$ , and the corresponding best imported quality level  $q^{H,f}$  increase again with  $\sigma$ . This case corresponds to equilibrium (B-SOE)(iii), respectively (C-SOE)(iii) in Proposition 3.

More generally, as long as inequality  $\sigma$  is not extremely high, firms in some differentiated good sectors are highly innovative and still serve the rich, while firms in other differentiated good sectors either innovate less to serve the poor [in case (C-SOE)] or stop

<sup>29</sup> In particular, condition (18b) and (20d), respectively, defines  $q^H$  as a function of  $\theta^H$  and equates the total marginal utility from increasing  $q^H$ ,  $\lambda \theta^H v'(q^H)$ , to the total marginal cost,  $\lambda \frac{1}{\alpha_A} + h' \left( \frac{q^H}{\bar{q}_{-1}} \right) \frac{1}{\bar{q}_{-1}}$ . Hence, ceteris paribus a change in  $q^H$  cannot be optimal: It would increase (decrease) the willingness-to-pay of the rich by less (more) than it would increase (decrease) the marginal cost of delivering quality to the rich.

<sup>30</sup> (IR<sup>H</sup>) must eventually be strictly binding because (i) the value of the import option scales with  $(\theta^H)^{1/\beta}$ —see equation (17); (ii) derivations analogous to the ones for equation (17) imply that the total domestic surplus (consumer + producer surplus) from serving the rich would also scale with  $(\theta^H)^{1/\beta}$  if there were no cost of innovation and the rich would consume their preferred quality at marginal cost; (iii) the cost of innovation and, consequently, the smaller increase in  $q^H$  imply that the total domestic surplus from serving the rich scales less than with  $(\theta^H)^{1/\beta}$ .

<sup>31</sup> This can be shown by contradiction. Given  $\bar{\sigma}$  as defined in the main text before (25), firms' profits are just equal whether or not they innovate to serve the rich. Hence, if  $\sigma$  marginally increases further and (IR<sup>H</sup>) tightens more, firms in the SOE would, ceteris paribus, be better off not to serve the rich. Now, suppose by way of contradiction, that all domestic firms—or, for that matter, any set of positive measure of firms—would stop serving the rich in response to the marginal increase in  $\sigma$ . From Lemma 6 we know that in such case households import strictly higher quality than what they would otherwise buy from innovating domestic firms and, hence, by (IR<sup>H</sup>), at a strictly higher price. In turn, this would trigger a drop in  $\theta^H$  implying that  $\theta^H < \bar{\theta}^H$ . But with  $\theta^H < \bar{\theta}^H$ , domestic firms would again be strictly better off innovating and serving the rich—due to less fierce international competition at lower levels of  $\theta^H$ —, a contradiction.

<sup>32</sup> When innovative firms still serve the poor, their profits when serving or not the rich also depend on  $\theta^L$  which also changes with  $\sigma$ . This complicates the subsequent discussions, but it does not impact the essence of the argument.

innovating altogether [in case (B-SOE)]. In either case, the quality  $\bar{q}_i$  in the importing sectors is lower when compared to the closed economy and  $A(t+1)$  is decreasing vis-à-vis the closed economy as we increase inequality  $\sigma$  further.

We can now formulate Proposition 4, which follows from the previous discussions.

**Proposition 4** (*Inequality and growth in the SOE*). *In the small open economy:*

- (i) *For small values of the variance in incomes  $\sigma$ , the only equilibrium is a no-trade equilibrium, that is, equilibrium outcomes are the same as in the closed economy.*
- (ii) *For intermediate values of the variance in incomes  $\sigma$ , constraint  $(IR^H)$  is binding and innovating firms block entry from foreign competitors by lowering  $p^H$ . Quality  $q^H$  and thus  $A(t+1)$  is higher than in the closed economy.*
- (iii) *For values of  $\sigma$  sufficiently high, domestic firms can no longer profitably compete with foreign firms in serving the rich households in all differentiated good sectors. In some sectors, high qualities are then imported. As  $\sigma$  increases further, eventually the share of sectors in which high qualities are imported increases and the domestic technological level  $A(t+1)$  decreases.*

The proposition carries the central message of this paper that the growth effects of inequality are very different in the SOE when compared to the closed economy. In the closed economy, an increase in inequality  $\sigma$  has a positive effect on growth whenever firms find it optimal to offer separate qualities for the rich and the poor households. By contrast, in an SOE with inequality high enough such that  $(IR^H)$  is binding, firms initially block entry of foreign competitors by lowering price  $p^H$ , leading to a positive general equilibrium effect on quality  $q^H$  and higher growth. This positive *demand effect* is rooted in the fact that—due to international competition—firms charge smaller mark-ups for high qualities.

As inequality increases further, it is eventually high enough such that some domestic firms no longer find it optimal to serve rich households, implying that foreign competition has a negative *business stealing effect* on innovation and, hence, economic growth. This business stealing effect gets bigger as we increase inequality further. This is for two reasons: On the one hand, the higher  $\sigma$ , the larger the share of differentiated goods that the rich import. On the other hand, if rich households import high qualities from abroad, firms in the SOE no longer benefit from their higher willingness to pay for quality as  $\sigma$  increases further. Corollary 2 summarizes these two central novel effects of inequality on growth in the SOE that derive from our analysis.

**Corollary 2** (*Growth effects of inequality in the SOE*). *Compared to the closed economy, there are two novel effects of inequality on growth in the small open economy:*

- (i) *a positive general equilibrium demand effect (+) that is triggered by increased competition from foreign firms;*
- (ii) *a negative business stealing effect (-), when inequality is so high that domestic firms are no longer able to compete with foreign entrants in all differentiated good sectors, and rich households start satisfying their demand for some varieties via importing high quality.*

Interestingly, the previous discussions also show how non-homothetic demand for quality along with the threat of import competition from abroad can give rise to an endogenous emergence of a dual economy in developing countries, even with an ex-ante perfectly symmetric set-up. The basic intuition is that the domestic population is not rich enough to satisfy all of its demand for quality by importing pricey high qualities from abroad.<sup>33</sup> We summarize this insight in the following corollary.

**Corollary 3** (*Dual economy in the SOE*). *In contrast to the closed economy, in an equilibrium with trade there is a ‘dual economy’ in the ex-ante perfectly symmetric SOE, i.e. some differentiated good sectors in the SOE are highly innovative, while others are lagging behind. Only for very high levels of inequality, the high income households import all types of differentiated goods and there will be no ‘dual economy’ as a consequence.*

## 5. Discussion and further results

This section provides further discussion, first of our theoretical set-up and results, and then of empirical patterns in the data.

### 5.1. Theoretical discussion

#### 5.1.1. A novel ‘Dutch disease’ effect

In developing our arguments, we have assumed that there is a one-to-one mapping between the level of technology and aggregate income. In line with that, we considered a ROW that is perfectly symmetric to the SOE but for its technological level. Yet, this is not necessarily the case in oil-rich countries, for example. It is therefore interesting to know how a country’s growth prospects change if we increase incomes, holding constant the domestic level of technology. Interestingly, in the closed economy frameworks previously

<sup>33</sup> Our paper thus also speaks to the literature on dual economies that goes back at least to Lewis (1954)—see LaPorta and Shleifer (2014) for an overview. In our set-up a dual economy emerges endogenously and at a general level our work is thus related to Porzio (2017), but the underlying mechanisms are very different. Porzio (2017) considers a model with sorting and matching of heterogeneous agents into becoming managers and workers. He shows how a dual economy can arise if firms in developing countries have the opportunity to adopt state-of-the-art technologies from abroad.

considered in the literature, this will typically boost growth. In our case, this is evident from considering equation (10d), which implies that firms respond to higher incomes—and therefore a higher  $\theta^H$ —by increasing  $q^H$ , reflecting the higher willingness to pay for quality on the side of the rich. In the SOE, however, the increase in income also implies that the value of the best import option for the household increases, and this can have an effect on innovation and growth similar to an increase in inequality. In particular, higher windfalls—for a given level of inequality—can imply that the SOE ends up being in scenario (iii) of Proposition 4 where the economy suffers from the negative business stealing effect. In such case, the economy may suffer from a novel negative ‘Dutch Disease’ type effect of windfall gains on growth. As opposed to the textbook case, the effect here is not centered on intersectoral reallocations,<sup>34</sup> but on the fact that windfall gains through e.g. oil revenues imply that households get richer vis-à-vis the domestic level of technology which can imply that domestic firms find it harder to compete with foreign high-quality providers.

### 5.1.2. Distance to frontier, inequality, and growth in the SOE

Our previous results critically depend on the ability of rich households to satisfy their demand for high quality via importing. In turn, this ability depends on a country’s distance to frontier, and we therefore discuss next the importance of a country’s distance to frontier for the inequality-growth nexus.

When increasing the technological level in the SOE, keeping constant the income distribution and the technological level in the ROW, the SOE’s GDP increases, benefiting both low and high income types. In turn, this increases the households’ demand for quality. As long as the SOE is sufficiently far from the frontier such an increase in aggregate technology has no effect on innovation and growth in the SOE. This is because equilibrium qualities are a constant multiple of the technological level inherited from the previous period as shown in Corollary 1.

This, however, is no longer true if the SOE is sufficiently close to the world’s technological frontier  $\bar{q}^{ROW}$ . In such case, and for high enough inequality, the rich households’ optimal imported quality  $q^{H,f}$  as defined in equation (16) may no longer be available because it is beyond the technological frontier in the ROW. Then the best import option for rich households is to demand the highest quality in the ROW  $\bar{q}^{ROW}$  at marginal cost. This implies that innovating domestic firms can compete with foreign firms for higher levels of inequality. More specifically, the foreign competition constraint ( $IR^H$ ) is binding for higher levels of inequality only, and whenever it is binding, the outside option  $\bar{q}^{ROW}$  has a lower value to the household. We provide further technical details in supplementary material S2.1 and summarize the main insights in the following proposition.

**Proposition 5** (Distance to frontier, inequality, and growth in the SOE). *Let  $\bar{\sigma}(\tau, A)$  be the highest inequality level  $\sigma$  such that domestic firms still find it optimal to serve the rich if all other domestic firms do. Let  $\bar{q}^{H,f}$  denote the unconstrained optimal import quality of rich households in this case. An increase in the level of technology in the SOE  $\bar{q}_{-1}$  increases  $\bar{\sigma}(\tau, A)$  and thus allows firms in the SOE to successfully compete against foreign high-quality providers for higher levels of inequality if the SOE is sufficiently close to the technological frontier such that  $\bar{q}^{H,f} > \bar{q}^{ROW}$ .*

### 5.1.3. Further discussion and extensions

The model could be extended in various ways as we now briefly explain. Further details are provided in supplementary material S2.2.

In our model, there are no knowledge spillovers from the rest of the world (ROW) to the domestic country but only knowledge spillovers within the domestic economy. This reflects that firms in most developing countries are still far from the world’s technological frontier (see e.g. Cirera et al. (2022); Schetter (2022)). Reasons can be a lack of absorptive capacity (Cohen and Levinthal, 1989), and that knowledge spillovers often have local public good character (see e.g. Zucker et al. (1998)). Of course, no international spillovers is a stark simplification. Supplementary material S2.2 illustrates how our model can accommodate international knowledge spillovers. As long as such spillovers are not too strong, they do not impact our main mechanism. Interestingly, knowledge spillovers might get reinforced if they bring the economy closer to the technological frontier such that it can now better compete with foreign technology leaders as discussed in section 5.1.2.

Our model set-up focuses on innovations in the quality levels of differentiated goods and assumes that efficiency improvements in production are a by-product of quality-upgrading. Of course, process innovations are often a purposeful activity by firms that can play an important role for the provision of higher quality products to the poorer part of society (Foellmi et al., 2014). Supplementary material S2.2 presents a simplified set-up with process innovation for qualities that are inherited from the past. Such innovation intensifies competition with the domestic competitive fringe. In turn, this implies that constraint ( $IR^H$ ) is binding at higher levels of inequality only, but the impact of international competition on the inequality-growth nexus is otherwise the same.

To carve out the novel causal effects of inequality on growth in the SOE, we have considered an economy with two types of households. With more than two types, the analysis would be more involved, but the effects identified in Proposition 4 would still be at play. Supplementary material S2.2 briefly discusses this with reference to the limiting case with a continuum of types.

Lastly, the previous analyses held constant a country’s openness to trade. Our set-up implies that the growth effects of trade liberalizations depend on a country’s distance to frontier, echoing previous findings in the literature (Aghion et al., 2005, 2009;

<sup>34</sup> The basic argument is that an oil boom causes a real appreciation of the domestic currency and therefore decreases an economy’s competitiveness in other tradable sectors. If the primary sector has a lower growth potential, this undermines an economy’s long-run growth prospects. See e.g. Cordon and Neary (1982); Sachs and Warner (1995).

Amiti and Khandelwal, 2013). It adds to this literature by highlighting that such growth effects also critically depend on the level of inequality in a country. See supplementary material S2.2 for further discussion.

## 5.2. Empirical patterns

Our analysis of the growth effects of inequality in developing countries isolates a negative business stealing effect that is particularly prevalent in unequal and open countries. This implies that inequality has a smaller or even negative effect on growth in an open economy when compared to a closed economy as previously considered in the literature. While a full econometric analysis is beyond the scope of the paper, we were nevertheless interested as to whether this theoretical prediction can speak to basic correlations in the data. In supplementary material S3, we provide an empirical illustration comprising (1) standard growth regressions using growth in GDP per capita as the dependent variable and (2) industry-level growth regressions using growth in export quality taken from Feenstra and Romalis (2014) as the dependent variable. In these regressions, we then control for an interaction of inequality and openness. Across a range of specifications, using either a large number of country controls or country fixed effects, we illustrate that for developing countries this interaction term is typically significantly negative in the industry-level regressions, and still negative—albeit not in all cases significant—in our country-level regressions, in line with our theoretical predictions.<sup>35</sup> While our theory can thus speak to basic correlations in the data, we emphasize that these regressions only show associations and they involve variables that are difficult to measure. These regressions must therefore be interpreted with caution. See the supplementary material for further details.

## 6. Conclusion

In this paper, we analyzed how inequality impacts growth in developing countries in the context of a Schumpeterian model with growth through quality upgrading and non-homothetic demand for quality. Our key insights show that the growth effects of inequality are very different in an open when compared to a closed economy: Higher inequality boosts the willingness to pay for high quality of rich households, which stimulates innovation and growth in the closed economy.

In the open economy, however, this increased taste for quality also makes importing high qualities from abroad more attractive. For low levels of inequality this triggers a positive demand effect on innovation as domestic firms deter entry from foreign competitors by lowering their price on high qualities. For sufficiently high levels of inequality, however, this is no longer profitable and rich households start satisfying their demand for quality via importing, giving rise to a dual economy and a negative business stealing effect of inequality on growth. The size of this effect critically depends on a country's stage of development and its openness to trade. Overall, our theory suggests that in the developing world inequality is more harmful for growth in open as opposed to closed economies.

While these observations have so far largely gone unnoticed in the literature, we believe that they are of first order importance for our understanding of the growth prospects of developing countries, and they are of immediate relevance for redistributive and trade policies: In essence, our findings show how a strong (upper-) middle class can be key for sustained growth in the developing world, and how for low levels of development tariffs can have a beneficial effect on growth. The latter point is related to previous findings in the literature (Aghion et al., 2005, 2009; Amiti and Khandelwal, 2013). Our work shows how the growth effects of such policies critically depend on inequality.

Our model makes several simplifying assumptions. In developing countries, we may find stronger entry barriers for firms, more macroeconomic volatility, and slower imitation of technological advances by a competitive fringe of firms, for example. Moreover, inequality and openness to trade impact growth through additional channels, including knowledge spillovers, investments in human capital, or political institutions. Incorporating such factors will be an interesting avenue in future work to study the robustness of our findings. It would also be interesting to analyze how inequality and openness impact growth in countries at the world's technological frontier.

## CRedit authorship contribution statement

**Ulrich Schetter:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing, Project administration, Funding acquisition, Supervision, Visualization. **Maik T. Schneider:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing, Project administration. **Adrian Jäggi:** Formal analysis, Writing – original draft, Writing – review & editing, Data curation, Visualization, Software.

## Declaration of competing interest

The authors wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

<sup>35</sup> In that regard, our paper also relates to the empirical literature analyzing the linkages from income inequality to economic growth. This literature tends to find a negative effect, but the evidence is far from being conclusive—see e.g. Bénabou (1997); Barro (2000); Voitchovsky (2005); Easterly (2007); Ostry et al. (2014); Halter et al. (2014); Brueckner and Lederman (2018).

**Data availability**

Data will be made available on request.

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**Appendix A. Proofs**

*A.1. Proof of Lemma 1*

We show a variant of Lemma 1 with a continuous closed set of types  $\Theta$ , before explaining that the case with two types then follows immediately as a special case.

**Lemma 1'.**

The decision problem of innovating firm  $i$  is equivalent to:

$$\begin{aligned} \max_{\{q_i(\theta), p_i(\theta)\}_{\theta \in \Theta, \bar{q}_i}} & \int_{\theta \in \Theta} \left[ p_i(\theta) - \frac{1}{a_q A} q_i(\theta) \right] f_\theta(\theta) d\theta - h \left( \frac{\bar{q}_i}{\bar{q}_{i,-1}} \right) \\ \text{s.t.} & \theta v(q_i(\theta)) - p_i(\theta) \geq \max_{q \in [0, \bar{q}_{i,-1}]} \left\{ \theta v(q) - \frac{1}{a_q A} q \right\}, \quad \forall \theta \in \Theta, \tag{IR} \\ & \theta v(q_i(\theta)) - p_i(\theta) = \max_{\hat{\theta} \in \Theta} \{ \theta v(q_i(\hat{\theta})) - p_i(\hat{\theta}) \}, \quad \forall \theta \in \Theta, \tag{IC} \\ & q_i(\theta) \leq \bar{q}_i, \quad \forall \theta \in \Theta, \end{aligned}$$

where  $v(q) := q^{1-\beta}$  and where the firm considers type  $\theta^h := \frac{p_z z^h}{\beta Q^h}$ ,  $Q^h := \int_0^1 q_i^{h^{1-\beta}} di$ , of household  $h$  as exogenously given.  $\theta^h$  is private knowledge to the households and is distributed according to  $f_\theta(\theta)$  with support  $\Theta$ , with this probability density function (pdf) being common knowledge.

**Proof.** To show the desired result, we proceed in two steps.

**Step 1.** From the discussions in the main text—decision problem (5)—we know that households choose quality to maximize

$$\max_{q_i^h} q_i^{h^{1-\beta}} \underbrace{\frac{z^h p_z}{\beta Q^h}}_{:= \theta^h} - p_i(q_i^h).$$

From the perspective of innovating firm  $i$ ,  $\theta^h := \frac{z^h p_z}{\beta Q^h}$  is a sufficient statistic for household characteristics, which is exogenous to the firm and observed only by the household.  $\theta$  is distributed according to  $f_\theta(\theta)$ , which depends on the full general equilibrium in the economy.

Let  $\Theta$  denote the set of pairwise distinct elements in  $\{\theta^j\}_{j \in [0,1]}$ . Then, by the revelation principle (cf. e.g. Mas-Colell et al. (1995, Proposition 23.C.1)), the innovating firm can limit attention to truthful revelation mechanisms, i.e. for each  $\theta \in \Theta$  a quality-price bundle  $(q_i(\theta), p_i(\theta))$  such that households find it optimal to truthfully reveal their type, that is

$$\theta v(q_i(\theta)) - p_i(\theta) = \max_{\hat{\theta} \in \Theta} \{ \theta v(q_i(\hat{\theta})) - p_i(\hat{\theta}) \}, \quad \forall \theta \in \Theta, \tag{IC}$$

where  $v(q_i) = q_i^{1-\beta}$  and  $p_i(\theta) := p_i(q_i(\theta))$ .

**Step 2.** The competitive fringe implies that all qualities  $q_i \leq \bar{q}_{i,-1}$  are offered at marginal cost, which, in turn, implies that every household must weakly prefer its offered contract  $(q_i(\theta^h), p_i(\theta^h))$  to its best choice among all qualities  $q \leq \bar{q}_{i,-1}$

$$\theta v(q_i(\theta)) - p_i(\theta) \geq \max_{q \in [0, \bar{q}_{i,-1}]} \left\{ \theta v(q) - \frac{1}{a_q A} q \right\}, \quad \forall \theta \in \Theta. \tag{IR}$$

Lemma 1' then follows from accounting for the endogenous choice of  $\bar{q}_i$  and the feasibility constraint  $q(\theta) \leq \bar{q}_i, \forall \theta \in \Theta$ . Finally, Lemma 1 follows from noting that with two types  $\theta^h, h = L, H$  only,  $f_\theta(\cdot)$  reduces to a discrete distribution with  $f_\theta(\theta^H) = \lambda, f_\theta(\theta^L) = 1 - \lambda$  and  $f_\theta(\theta) = 0$  otherwise.  $\square$

A.2. Proof of Lemma 2

Consider any two types  $\theta^H, \theta^L \in \Theta$ . We show that

$$\theta^H > \theta^L \Rightarrow I^H > I^L, \tag{i}$$

$$\theta^H = \theta^L \Rightarrow I^H = I^L. \tag{ii}$$

The result then follows.

(i) The following conditions are necessary for incentive compatibility for both types:

$$\theta^H v(q_i(\theta^H)) - p_i(\theta^H) \geq \theta^H v(q_i(\theta^L)) - p_i(\theta^L) \tag{IC^H}$$

$$\theta^L v(q_i(\theta^L)) - p_i(\theta^L) \geq \theta^L v(q_i(\theta^H)) - p_i(\theta^H). \tag{IC^L}$$

Rearranging terms and combining the two conditions, we get

$$\theta^H [v(q_i(\theta^H)) - v(q_i(\theta^L))] \geq \theta^L [v(q_i(\theta^H)) - v(q_i(\theta^L))].$$

Using  $\theta^H > \theta^L$  along with the fact that  $v'(\cdot) > 0$ , we get  $q_i(\theta^H) \geq q_i(\theta^L) \forall i \in [0, 1]$ , and, hence

$$Q^H = \int_0^1 (q_i(\theta^H))^{1-\beta} di \geq \int_0^1 (q_i(\theta^L))^{1-\beta} di = Q^L. \tag{A.1}$$

Moreover, incentive compatibility requires that  $p_i(\theta^H) \geq p_i(\theta^L) \forall i \in [0, 1]$ , implying that

$$\int_0^1 p_i(\theta^H) di \geq \int_0^1 p_i(\theta^L) di. \tag{A.2}$$

Finally, by the monotonicity of households' preferences, the budget constraint always holds with equality, i.e. we have

$$p_z z^h = I^h - \int_0^1 p_i(\theta^h) di \quad \forall h \in [0, 1]. \tag{A.3}$$

Combining (A.1), (A.2), and (A.3) with the definition of  $\theta$ , we conclude

$$\theta^H > \theta^L \Rightarrow I^H > I^L.$$

(ii) It remains to show that

$$\theta^H = \theta^L \Rightarrow I^H = I^L.$$

We proceed by contradiction. Suppose there exist two types of households with  $I^H > I^L$  satisfying  $\theta^H = \theta^L$ . Then it must be that  $Q^H > Q^L$  and that  $\int_0^1 p_i(\theta^H) di > \int_0^1 p_i(\theta^L) di$ . Hence, for some measurable subset  $\hat{I} \subseteq [0, 1]$  we must have that

$$q_i(\theta^H) > q_i(\theta^L) \quad \forall i \in \hat{I},$$

where incentive compatibility for both  $H$  and  $L$  requires

$$\theta^h v(q_i(\theta^H)) - p_i(\theta^H) = \theta^h v(q_i(\theta^L)) - p_i(\theta^L) \quad \forall i \in [0, 1], h \in \{L, H\}. \tag{A.4}$$

This, however, contradicts profit maximization by innovating firms  $i \in \hat{I}$ . To see this, note that for firm  $i$  to offer two distinct contracts to one type of households, both contracts must yield the same profit to the firm. Consider, for concreteness, the case of  $q_i(\theta^H) < \bar{q}_i$ .<sup>36</sup> Then, we must have

$$p_i(\theta^H) - p_i(\theta^L) = \frac{1}{a_q A} (q_i(\theta^H) - q_i(\theta^L)). \tag{A.5}$$

(A.4), (A.5), and the concavity of  $v(\cdot)$  imply that for every  $\bar{q}_i \in (q_i(\theta^L), q_i(\theta^H))$  there exists a  $\bar{p}_i \in (p_i(\theta^L), p_i(\theta^H))$  such that

$$\theta^h v(q_i(\theta^L)) - p_i(\theta^L) = \theta^h v(\bar{q}_i) - \bar{p}_i, \quad h \in \{L, H\},$$

and

<sup>36</sup> It is straightforward to extend the argument to the case of  $q_i(\theta^H) = \bar{q}_i$ .

$$\bar{p}_i - p_i(\theta^L) > \frac{1}{a_q A} (\bar{q}_i - q_i(\theta^L)) .$$

The contract  $(\bar{q}_i, \bar{p}_i)$  yields higher profits for the firm than both  $(q_i(\theta^H), p_i(\theta^H))$  and  $(q_i(\theta^L), p_i(\theta^L))$ . It satisfies (IC) and (IR) for households  $L, H$ . Moreover, it weakly relaxes (IC) to all other households because it is less preferred than  $(q_i(\theta^L), p_i(\theta^L))$  by all types  $\theta < \theta^L$  and less preferred than  $(q_i(\theta^H), p_i(\theta^H))$  by all types  $\theta > \theta^H$ . Hence, offering  $(q_i(\theta^H), p_i(\theta^H))$  and  $(q_i(\theta^L), p_i(\theta^L))$  cannot be profit maximizing.  $\square$

### A.3. Proof of Lemma 3

We begin with a preliminary observation and then show the desired result.

**Lemma A.1.** *The equilibrium price of quality  $q_i^h$ ,  $h \in \{L, H\}$ , of any differentiated good  $i \in [0, 1]$  is never below its marginal cost of production, i.e.  $p_i^h \geq \frac{q_i^h}{a_q A}$ ,  $h \in \{L, H\}$ .*

**Proof.** We proceed by contradiction. Suppose innovating firm  $i$  offers contracts  $(q_i^h, p_i^h)$  and  $(\hat{q}_i^h, \hat{p}_i^h)$ ,  $h \neq \hat{h} \in \{L, H\}$ , and where  $p_i^h < \frac{q_i^h}{a_q A}$  and  $\hat{p}_i^h \geq \frac{\hat{q}_i^h}{a_q A}$ .<sup>37</sup> Contract  $(q_i^h, p_i^h)$  is loss making for firm  $i$ . Consider the following variant to these contracts:  $\tilde{q}_i^h = \hat{q}_i^h$ , and  $\tilde{p}_i^h = \hat{p}_i^h$  and

$$\tilde{q}_i^h = \underset{q \in \{[0, \bar{q}_{-1}], q_i^h, \hat{q}_i^h\}}{\operatorname{argmax}} \{ \theta^h v(q) - \tilde{p}_i^h \}$$

$$\text{s.t. } \tilde{p}_i^h = \begin{cases} \frac{\tilde{q}_i^h}{a_q A}, & \text{if } \tilde{q}_i^h \in \{[0, \bar{q}_{-1}], q_i^h\} \\ \hat{p}_i^h, & \text{if } \tilde{q}_i^h = \hat{q}_i^h \end{cases} .$$

By construction, contract  $(\tilde{q}_i^h, \tilde{p}_i^h)$  satisfies (IR) and (IC) for households  $h$ . Moreover, as either  $\tilde{q}_i^h = q_i^h$  and  $\tilde{p}_i^h > p_i^h$ , or  $(\tilde{q}_i^h, \tilde{p}_i^h)$  is a contract that has already been available previously, contract  $(\tilde{q}_i^h, \tilde{p}_i^h)$  satisfies (IR) and (IC) for household  $\hat{h}$ . Yet, contracts  $(\tilde{q}_i^h, \tilde{p}_i^h)$ ,  $(\hat{q}_i^h, \hat{p}_i^h)$  yield strictly larger profits to firm  $i$  when compared to contracts  $(q_i^h, p_i^h)$  and  $(\hat{q}_i^h, \hat{p}_i^h)$ , a contradiction to the latter being profit maximizing.  $\square$

Now, suppose all qualities were offered at marginal cost. Then household  $h \in \{H, L\}$  would maximize its instantaneous utility (1) subject to

$$\int_0^1 q_i^h \frac{1}{a_q A} di + z^h \frac{1}{a_z A} = I^h .$$

Standard derivations then imply that  $q_i^h = q^h \forall i \in [0, 1]$  and that

$$q^h \frac{1}{a_q A} = (1 - \beta) I^h . \tag{A.6}$$

Now, the solution to (A.6) is household  $h$ 's consumed quality unless this quality level is not available or some other quality is sold at a price below marginal cost. By Lemma A.1, the latter will never occur in equilibrium. Moreover, the competitive fringe for pre-existing qualities implies that qualities  $q_i \leq \bar{q}_{-1}$  are offered at marginal cost in equilibrium. Recalling that  $A = \bar{q}_{-1}$ , it follows that a household with income  $\hat{I} := \frac{1}{a_q(1-\beta)}$  just finds it optimal to consume quality  $\bar{q}_{-1}$ . Moreover, the solution according to (A.6) is increasing in  $I^h$ . Hence, households with income  $I < \hat{I}$  find it optimal to consume quality  $q_i < \bar{q}_{-1}$ .  $\square$

### A.4. Proof of Proposition 1

We first state the formal version of Proposition 1, which we prove in the following.

#### Proposition 1'

There is a unique equilibrium satisfying for  $h = \{H, L\}$ :  $q_i^{h,e} = q^{h,e}$  and  $p_i^{h,e} = p^{h,e} \forall i \in [0, 1]$ . Depending on parameter values, this equilibrium can be characterized according to one of the following cases:

<sup>37</sup> Note that the firm will never price both contracts below marginal cost because this would imply that it is making losses and staying out of business and making zero profits is always an option for the firm.

(i)  $I^L \leq I^H \leq \hat{I}$  :

$$q^{L,e} = (1 - \beta)a_q A I^L, p^{L,e} = \frac{1}{a_q A} q^{L,e},$$

$$q^{H,e} = (1 - \beta)a_q A I^H, p^{H,e} = \frac{1}{a_q A} q^{H,e}.$$

(ii)  $I^H > \hat{I} \geq I^L$  :

$$q^{L,e} = (1 - \beta)a_q A I^L, p^{L,e} = \frac{1}{a_q A} q^{L,e},$$

$q^{H,e} > \bar{q}_{-1}$  and  $p^{H,e}$  are the unique solution to:

$$\frac{I^H - p^{H,e}}{\beta} \left[ 1 - \left( \frac{\bar{q}_{-1}}{q^{H,e}} \right)^{1-\beta} \right] + \frac{1}{a_q} = p^{H,e},$$

$$\lambda \frac{1-\beta}{\beta} (I^H - p^{H,e}) - \lambda \frac{1}{a_q} \frac{q^{H,e}}{\bar{q}_{-1}} - \frac{q^{H,e}}{\bar{q}_{-1}} h' \left( \frac{q^{H,e}}{\bar{q}_{-1}} \right) = 0.$$

(iii)  $I^H \geq I^L > \hat{I}$  :

(A) If the solution to the system of equations in part (B) involves  $q^L \leq \bar{q}_{-1}$ , there is a separating equilibrium with  $q^{L,e} = \bar{q}_{-1}, p^{L,e} = \frac{1}{a_q}$  and where  $q^{H,e}, p^{H,e}$  are the solutions to the equations as shown in (ii).

(B) There is a separating equilibrium where  $q^{L,e}, p^{L,e}, q^{H,e}$ , and  $p^{H,e}$  are the unique solution to:

$$\frac{I^L - p^{L,e}}{\beta} \left[ 1 - \left( \frac{\bar{q}_{-1}}{q^{L,e}} \right)^{1-\beta} \right] + \frac{1}{a_q} = p^{L,e},$$

$$\frac{I^H - p^{H,e}}{\beta} \left[ 1 - \left( \frac{q^{L,e}}{q^{H,e}} \right)^{1-\beta} \right] + p^{L,e} = p^{H,e},$$

$$I^L - \lambda (I^H - p^{H,e}) \left( \frac{q^{L,e}}{q^{H,e}} \right)^{1-\beta} - (1 - \lambda) \frac{\beta}{(1 - \beta)a_q} \frac{q^{L,e}}{\bar{q}_{-1}} = p^{L,e},$$

$$\lambda \frac{1-\beta}{\beta} (I^H - p^{H,e}) - \lambda \frac{1}{a_q} \frac{q^{H,e}}{\bar{q}_{-1}} - \frac{q^{H,e}}{\bar{q}_{-1}} h' \left( \frac{q^{H,e}}{\bar{q}_{-1}} \right) = 0.$$

(C) If the solution to the system of equations in part (B) involves  $q^L \geq q^H$ , there is a pooling equilibrium, i.e.  $q^{L,e} = q^{H,e} = q^{P,e}$  and  $p^{L,e} = p^{H,e} = p^{P,e}$  which are the unique solution to:

$$\frac{I^L - p^{P,e}}{\beta} \left[ 1 - \left( \frac{\bar{q}_{-1}}{q^{P,e}} \right)^{1-\beta} \right] + \frac{1}{a_q} = p^{P,e},$$

$$\frac{1-\beta}{\beta} (I^L - p^{P,e}) - \frac{1}{a_q} \frac{q^{P,e}}{\bar{q}_{-1}} - \frac{q^{P,e}}{\bar{q}_{-1}} h' \left( \frac{q^{P,e}}{\bar{q}_{-1}} \right) = 0.$$

We prove each part of Proposition 1' in turn and then derive the remaining equilibrium values (iv).

(i) Follows from the proof of Lemma 3 and the fact that qualities  $q \leq \bar{q}_{-1}$  are offered at marginal cost.

(ii) From the proof of Lemma 3 we know that for all differentiated goods we have:  $q^{L,e} = (1 - \beta)a_q A I^L$  and  $p^{L,e} = \frac{1}{a_q A} q^{L,e}$  and that household  $H$ 's preferred option among freely available qualities is  $\bar{q}_{-1}$ . Moreover, it is never optimal for the firm to upgrade quality more than what is needed to serve the high types, i.e. we have  $\bar{q}_i = \max\{\bar{q}_{-1}, q_i^H\}$ . Hence, firm  $i$ 's decision problem simplifies to

$$\max_{p_i^H, q_i^H} \lambda \left[ p_i^H - \frac{1}{a_q A} q_i^H \right] - h \left( \frac{q_i^H}{\bar{q}_{-1}} \right)$$

s.t.  $\theta^H v(q_i^H) - p_i^H \geq \theta^H v(\bar{q}_{-1}) - \frac{1}{a_q A} \bar{q}_{-1}.$  (IR<sup>H</sup>)

As the firm's profits are strictly increasing in  $p_i^H$ , (IR<sup>H</sup>) always holds with equality in equilibrium. Rearranging (IR<sup>H</sup>), substituting in for  $p_i^H$  in the objective, and differentiating with respect to  $q_i^H$ , we get the following necessary conditions for profit maximization:

$$\theta^H v(q_i^H) - \theta^H v(\bar{q}_{-1}) + \frac{1}{a_q A} \bar{q}_{-1} = p_i^H, \tag{IR<sup>H</sup>}$$

$$\lambda \theta^H v'(q_i^H) - \lambda \frac{1}{a_q A} - \frac{1}{\bar{q}_{-1}} h' \left( \frac{q_i^H}{\bar{q}_{-1}} \right) = 0. \tag{A.7}$$

Note that for every  $\theta^H > 0$ , the first order conditions (IR<sup>H</sup>) and (A.7) have at most one solution, implying that any equilibrium has to be symmetric across differentiated goods. Using the symmetry,  $A = \bar{q}_{-1}$ , the fact that  $I^H - p^H = p_z z^H$ , the definitions of  $\theta$  and  $v(\cdot)$ , and rearranging terms, we get

$$\frac{I^H - p^H}{\beta} \left[ 1 - \left( \frac{\bar{q}_{-1}}{q^H} \right)^{1-\beta} \right] + \frac{1}{a_q} = p^H, \tag{A.8}$$

$$\lambda \frac{1-\beta}{\beta} (I^H - p^H) - \lambda \frac{1}{a_q} \frac{q^H}{\bar{q}_{-1}} - \frac{q^H}{\bar{q}_{-1}} h' \left( \frac{q^H}{\bar{q}_{-1}} \right) = 0, \tag{A.9}$$

which are the expressions shown in Proposition 1'. Finally, to see that these equations have a unique solution and that this solution involves  $q^{H,e} > \bar{q}_{-1}$ , observe that (A.8) describes an increasing relationship between  $p^H$  and  $q^H$  starting from  $p^H = \frac{1}{a_q}$  and  $q^H = \bar{q}_{-1}$  and converging to  $p^H = \frac{I^H}{1+\beta} + \frac{\beta}{(1+\beta)a_q}$  as  $q^H \rightarrow \infty$ , while (A.9) describes a decreasing relationship between  $p^H$  and  $q^H$  starting from  $p^H = I^H - \frac{\beta}{(1-\beta)a_q}$  and  $q^H = \bar{q}_{-1}$ , and reaching  $p^H = 0$  at the solution of

$$\frac{1-\beta}{\beta} \lambda I^H = \frac{\lambda}{a_q} \frac{\hat{q}^H}{\bar{q}_{-1}} + \frac{\hat{q}^H}{\bar{q}_{-1}} h' \left( \frac{\hat{q}^H}{\bar{q}_{-1}} \right).$$

The result then follows from  $I^H > \frac{1}{a_q(1-\beta)}$ .

(iii) We show existence and uniqueness of the equilibrium by construction. In particular, we follow the standard procedure for addressing this optimization problem, i.e. we eliminate (IR<sup>H</sup>) as it is redundant and consider the firm's maximization problem ignoring (IC<sup>L</sup>). Noting further that (IR<sup>L</sup>) and (IC<sup>H</sup>) are always binding,<sup>38</sup> this yields the following first-order conditions for profit maximization:

$$\theta^L (v(q_i^L) - v(\bar{q}_{-1})) + \frac{1}{a_q} = p_i^L, \tag{A.10}$$

$$\theta^H (v(q_i^H) - v(q_i^L)) + p_i^L = p_i^H, \tag{A.11}$$

$$\theta^L v'(q_i^L) - \lambda \theta^H v'(q_i^L) - (1-\lambda) \frac{1}{a_q A} \leq 0, \tag{A.12}$$

$$\lambda \theta^H v'(q_i^H) - \lambda \frac{1}{a_q A} - h' \left( \frac{q_i^H}{\bar{q}_{-1}} \right) \frac{1}{\bar{q}_{-1}} = 0, \tag{A.13}$$

with the complementary slackness condition for (A.12) being

$$\left[ \theta^L v'(q_i^L) - \lambda \theta^H v'(q_i^L) - (1-\lambda) \frac{1}{a_q A} \right] [q_i^L - \bar{q}_{-1}] = 0. \tag{A.14}$$

For  $\theta^L$  and  $\theta^H$  given, these equations have exactly one solution. If this solution implies  $q_i^H \geq q_i^L$ , it characterizes the uniquely optimal choice of firm  $i$ . If it involves  $q_i^H < q_i^L$ , then the uniquely optimal choice is instead to pool consumers.<sup>39</sup> We will get back to this point later and characterize the separating equilibrium first, if it exists.

Note first that the fact that for  $\theta^L$  and  $\theta^H$  given, equations (A.10) to (A.14) have a unique solution implies that there can only exist a symmetric separating equilibrium. This equilibrium can be derived by the following algorithm that takes into account the endogeneity of  $\theta^h$ ,  $h \in \{L, H\}$ , with respect to the equilibrium outcomes:

(1) For every  $\hat{q}^L$ , there is a unique  $\hat{p}^L$  satisfying (A.10). For  $\hat{q}^L$  and  $\hat{p}^L$  given, (A.11) describes a monotonously increasing relation between  $p^H$  and  $q^H$ , starting at  $\hat{q}^H = \hat{q}^L$  and  $\hat{p}^H = \hat{p}^L$  and converging to  $\hat{p}^H = \frac{I^H + \hat{p}^L \beta}{1+\beta}$  as  $\hat{q}^H \rightarrow \infty$ . (A.13), on the other hand, describes a monotonously decreasing relation between  $p^H$  and  $q^H$ , starting at  $\hat{q}^H = \bar{q}_{-1}$  and  $\hat{p}^H = I^H - \frac{\beta}{(1-\beta)a_q}$  and reaching  $\hat{p}^H = 0$  at the solution of

$$\frac{1-\beta}{\beta} \lambda I^H = \frac{\lambda}{a_q} \frac{\hat{q}^H}{\bar{q}_{-1}} + \frac{\hat{q}^H}{\bar{q}_{-1}} h' \left( \frac{\hat{q}^H}{\bar{q}_{-1}} \right).$$

Hence, for every  $\hat{q}^L$ , (A.10), (A.11), and (A.13) have at most one solution for  $\hat{p}^L$ ,  $\hat{p}^H$ ,  $\hat{q}^H$ .

(2) Start with  $\hat{q}^L = \bar{q}_{-1}$  and follow the steps as described above. Plug the derived  $\hat{q}^L$ ,  $\hat{q}^H$ ,  $\hat{p}^L$ ,  $\hat{p}^H$  into (A.12).<sup>40</sup> If inequality (A.12) is satisfied,  $\hat{q}^L$ ,  $\hat{q}^H$ ,  $\hat{p}^L$ ,  $\hat{p}^H$  are the unique equilibrium values (case A).

<sup>38</sup> If not, the firm could increase profits by raising  $p^L$  and / or  $p^H$ .

<sup>39</sup> This solution may involve  $q_i^H < q_i^L$  because the cost of innovation is made dependent on  $q_i^H$  in the above first-order-conditions, i.e. these conditions apply only if  $q_i^H \geq q_i^L$ . If  $q_i^H < q_i^L$  they ignore the fact that the cost of innovation would be governed by  $q_i^L$  in such case.

<sup>40</sup> Note that by  $I^H > \frac{1}{(1-\beta)a_q}$  there is indeed a solution for (A.10), (A.11), and (A.13) with  $\hat{q}^L = \bar{q}_{-1}$ .

(3) If inequality (A.12) is violated, add some small  $\Delta > 0$  to  $\hat{q}^L$  and repeat the steps in (1). Continue adding  $\Delta > 0$  to  $\hat{q}^L$  until (A.12) is satisfied.<sup>41</sup> If the inequality is strict, apply a bisection algorithm for convergence to the equilibrium values (case B).<sup>42</sup>

(4) The unique symmetric solution to equations (A.10)-(A.14) may imply  $q^L \geq q^H$ . In such case there exists no separating equilibrium, and the unique equilibrium is a symmetric pooling equilibrium which is the solution to

$$\theta^L (v(q^P) - v(\bar{q}_{-1})) + \frac{1}{a_q} = p^P, \tag{A.15}$$

$$\theta^L v'(q^P) - \frac{1}{a_q A} - \frac{1}{\bar{q}_{-1}} h' \left( \frac{q^P}{\bar{q}_{-1}} \right) = 0. \tag{A.16}$$

Using the definitions of  $\theta$  and  $v(\cdot)$  along with the fact that  $A = \bar{q}_{-1}$  yields the expressions given in Proposition 1' (case C).

(5) It remains to be shown that an equilibrium according to case (A) and (B), respectively, is unique if it exists. To see this, assume that a symmetric separating equilibrium exists with  $\hat{q}^L < \hat{q}^H$  and note first that the arguments in steps (1) to (3) above imply that if an equilibrium according to case (A) and (B) exists, there can be no other separating equilibrium. To see that there can also be no pooling equilibrium in such case, suppose that there exists some  $\tilde{q}^L$  such that equations (A.10), (A.11), and (A.13) are simultaneously satisfied if  $\tilde{q}^H = \tilde{q}^L = \tilde{q}$  for all  $i$ . As by assumption there is a symmetric separating equilibrium with  $\hat{q}^H > \hat{q}^L$ , step (3) then implies that for these values the inequality in condition (A.12) must be strict. This, in combination with the fact that equation (A.13) holds implies that the left-hand-side of equation (A.16) would be negative for this value, i.e. in a potential pooling equilibrium it must be that  $q < \tilde{q}$ . But for  $q^L < \tilde{q}$  we know from the reasoning above that the unique symmetric solution to equations (A.10), (A.11), and (A.13) implies  $q^H > q^L$ , i.e. there can be no pooling equilibrium.

(iv) Lastly, given prices and quantities for the differentiated goods, the other equilibrium values follow: Homogeneous good consumption of household  $h$  is  $z^{h,e} = (I^h - p^{h,e})a_z A$ , which it buys at the equilibrium price  $p_z^e = 1/(a_z A)$ . Moreover, labor demand is given by

$$\int_0^1 L_i^e(q^{H,e}) + L_i^e(q^{L,e}) di + L_z^e = \lambda \frac{q^{H,e}}{a_q A} + (1 - \lambda) \frac{q^{L,e}}{a_q A} + \lambda(I^H - p^{H,e}) + (1 - \lambda)(I^L - p^{L,e}) + \int_0^1 f M_i^e + h(\tilde{q}_i^e / \bar{q}_{-1}) di,$$

which is remunerated at the normalized equilibrium wage rate  $w^e = 1$ .  $\square$

### A.5. Proof of Proposition 2

For  $\sigma = 0$ , the unique equilibrium is a pooling equilibrium with positive growth. As  $\sigma$  increases, and, hence,  $I^L$  decreases, the growth rate in the pooling equilibrium declines. To see this, observe that as  $I^L$  decreases, both equilibrium conditions for the pooling equilibrium shift downwards in the  $q^P, p^P$  diagram, but that the shift of (A.16) is larger, implying that both  $q^P$ , and  $p^P$  decline. This, in turn, implies that higher- $\sigma$  pooling equilibria are associated with a higher  $\theta^H$ . Hence, for some  $\sigma$  large enough, condition (A.13) holds with equality.<sup>43</sup>

As we increase  $\sigma$  further, we switch from a pooling equilibrium to a separating equilibrium. In the separating equilibrium, an increase in  $\sigma$  has two different effects: (i) The associated increase in  $I^H$  has a strictly positive effect on growth.<sup>44</sup> (ii) The associated decrease in  $I^L$  has an indirect effect on growth as its effect on  $q^L$  and  $p^L$  impacts  $p^H$  and, hence,  $\theta^H$  which, in turn, pins down  $q^H$  via (A.13). This effect may initially be negative but will eventually be positive for  $\sigma$  large enough such that  $\theta^L$  and  $q^L$  sufficiently small.<sup>45</sup> In particular, totally differentiating (A.10), we get

$$dp^L = d\theta^L [v(q^L) - v(\bar{q}_{-1})] + \theta^L v'(q^L) dq^L,$$

while totally differentiating (A.10) and using that  $q^H, p^H$ , and  $\theta^H$  are constant, we get

$$dp^L = \theta^H v'(q^L) dq^L.$$

Combining the previous two equations and rearranging terms, we get

<sup>41</sup> Note that by (A.10) increasing  $\hat{q}^L$  results in a higher  $\hat{p}^L$  and a lower  $\hat{\theta}^L$ . This does not affect (A.13), but shifts the solutions to (A.11) in the  $q^H, p^H$  diagram down and to the right, i.e. according to (A.11)  $\hat{q}^H$  is now associated with a lower  $\hat{p}^H$ . Together, this implies that the unique solution to (A.11) and (A.13) now yields a higher  $\hat{q}^H$  and a lower  $\hat{p}^H$ . Further, by (A.13), it is associated with a higher value  $\hat{\theta}^H$ . A higher  $\hat{q}^L$  in conjunction with a lower (higher)  $\hat{\theta}^L$  ( $\hat{\theta}^H$ ) implies that the left hand side of (A.12) is decreasing.

<sup>42</sup> Note that this is indeed an equilibrium and in particular that the above reasoning also implies that no firm has an incentive to deviate by pooling types in its sector. This follows from the fact that given  $\theta$ , i.e. given the equilibrium strategy of all other firms in the economy, the solution to equations (A.10) to (A.14) is uniquely optimal.

<sup>43</sup> Note that (A.13) and (A.16) together imply that (A.12) will also hold with equality and that for  $q^H = q^L$  and therefore  $p^H = p^L$  (A.11) trivially holds.

<sup>44</sup> To show (i), we proceed by contradiction. In particular, note that (A.13) defines an increasing relationship between  $\theta^H$  and  $q^H$ , i.e. for growth to decline it must be that  $\theta^H$  declines as well. (A.10) and (A.12) then imply that  $q^L$  must increase while  $\theta^L$  decreases. But then equations (A.10) and (A.11) imply that  $p^H$  must decrease as well, a contradiction to  $\theta^H$  being decreasing given that  $I^H$  increases and  $q^H$  decreases.

<sup>45</sup> Further details are available in the working paper version of this article.

**Table A.1**  
Parameter choices for numerical solutions.

$\lambda$ :	{0.05, 0.15, ..., 0.95}
$\beta$ :	{0.05, 0.15, ..., 0.95}
$a_q$ :	{2, 4, ..., 20}
$c$ :	{1, 2, 4, 8, 12}
$\alpha$ :	{0.05, 0.2, 1, 10, 20}

$$\frac{dq^L}{d\theta^L} = \frac{v(q^L) - v(\bar{q}_{-1})}{(\theta^H - \theta^L)v'(q^L)}. \tag{A.17}$$

Equation (A.17) characterizes how  $q^L$  has to change in response to a marginal change in  $\theta^L$  for  $(I^L)$  and  $(IC^H)$  still to be satisfied given  $q^H$ ,  $p^H$ , and  $\theta^H$ . On the other hand, noting that in a separating equilibrium equation (A.12) holds with equality and totally differentiating, using again that  $\theta^H$  stays constant by assumption, we get

$$\frac{dq^L}{d\theta^L} = \frac{v'(q^L)}{v''(q^L)} \frac{1}{\lambda\theta^H - \theta^L}. \tag{A.18}$$

Equation (A.18) characterizes the optimal change of  $q^L$  in response to a marginal change of  $\theta^L$  for a given  $\theta^H$ . Now, if the right-hand-side of (A.18) is larger than the right-hand-side of (A.17), then the optimal response of  $q^L$  to a marginal decrease of  $\theta^L$  is larger in absolute terms than the one needed to have  $(IC^H)$  satisfied at the old levels of  $q^H$  and  $p^H$ . As the high types value quality more, this decreases the attractiveness of contract  $(q^L, p^L)$  to high types which, in turn, allows firms to increase  $p^H$ . As a consequence, growth is lower via the negative general equilibrium effect of a higher  $p^H$  on  $\theta^H$ . In other words, a decrease of  $I^L$  lowers growth if

$$\frac{v'(q^L)}{v''(q^L)} \frac{1}{\lambda\theta^H - \theta^L} > \frac{v(q^L) - v(\bar{q}_{-1})}{(\theta^H - \theta^L)v'(q^L)}.$$

Using the definition of  $v(\cdot)$  and rearranging terms, this is equivalent to

$$\frac{\theta^H - \theta^L}{\theta^L - \lambda\theta^H} < \frac{\beta}{1 - \beta} \left[ 1 - \left( \frac{\bar{q}_{-1}}{q^L} \right)^{1-\beta} \right].$$

Now, the right-hand-side of the above condition approaches zero as  $q^L \rightarrow \bar{q}_{-1}$  while the left-hand-side is strictly positive, which shows that, indeed a decrease in  $I^L$  eventually has a positive effect on growth. We show numerically that for a broad range of parameter specifications the direct effect via an increase of  $I^H$  always dominates. In particular, we numerically solve for  $q^H$  as a function of  $\sigma$  for the parameter specifications as specified in Table A.1, assuming  $h'(x) = c(x - 1)^\alpha$ . For each possible combination of these parameter specifications,  $q^H$  is increasing as a function of  $\sigma$  in a separating equilibrium.<sup>46</sup>

#### A.6. Proof of Lemma 4

The expression in Lemma 4 has been derived in the main text under the condition that in equilibrium the SOE prices the homogeneous good competitively in the ROW (equation (13)). To show this, note first that firms from the SOE cannot serve consumers in the ROW at a price that is higher than the marginal cost of firms from the ROW in their home market. For the homogeneous good and for differentiated goods with quality  $q \leq \bar{q}_{-1}^{ROW}$ , this follows immediately from the competitive fringe in the ROW. For differentiated goods with quality  $q > \bar{q}_{-1}^{ROW}$ , this would violate profit maximization of firms in the ROW.<sup>47</sup>

The previous arguments imply that the SOE cannot sell a product in the ROW at a price above marginal costs of producing that same product in the ROW. The symmetry of the set-up in combination with the fact that it is never optimal to serve the ROW at a

<sup>46</sup> If  $\sigma$  is large enough such that the economy reaches the point where innovating firms find it optimal to no longer serve the low types—i.e. if the solution to the system of equations in Proposition 1' (iii)(B) involves  $q^L \leq \bar{q}_{-1}$ —this is necessarily the case and an increase in  $\sigma$  has a positive effect on growth. Formally, this follows from noting that as  $I^H$  increases, both (A.8) and (A.9) shift upwards with (A.9)—which is downward sloping—shifting more.

<sup>47</sup> To see that, suppose by way of contradiction that the SOE sells quality  $\hat{q} > \bar{q}_{-1}^{ROW}$  of some differentiated good  $i$  at a price  $\hat{p}(\hat{q}) > \frac{w^{ROW} \hat{q}}{a_q A^{ROW}}$  to a positive measure  $(1 - \lambda^{ROW}) > 0$  of consumers in the ROW. By assumption, the SOE does not serve rich households in the ROW, i.e.  $(1 - \lambda^{ROW}) < 1$  and  $\bar{q}_i^{ROW} > \hat{q}$ . It follows that in such case firm  $i$  in the ROW makes variable profits  $\hat{\pi}_i^{ROW} = \lambda^{ROW} \left( \hat{p}^{H:ROW} - \frac{w^{ROW}}{a_q A^{ROW}} \bar{q}_i^{ROW} \right)$ . Suppose instead that the firm offered quality  $\hat{q}$  at a price  $\hat{p} - \epsilon$  with  $\epsilon$  positive but small and quality  $\bar{q}_i^{ROW}$  at a price  $\hat{p}^{H:ROW} - \epsilon$ , with all other prices unaffected. Clearly, in such case rich households in the ROW would still opt to consume  $\bar{q}_i^{ROW}$  while poor households would now prefer to buy quality  $\hat{q}$  domestically rather than importing it from the SOE. It follows that with these contracts, firm  $i$ 's profits would be

$$\hat{\pi}_i^{ROW,2} = \lambda^{ROW} \left( \hat{p}^{H:ROW} - \epsilon - \frac{w^{ROW}}{a_q A^{ROW}} \bar{q}_i^{ROW} \right) + (1 - \lambda^{ROW}) \left( \hat{p} - \epsilon - \frac{w^{ROW} \hat{q}}{a_q A^{ROW}} \right) = \hat{\pi}_i^{ROW} + (1 - \lambda^{ROW}) \left( \hat{p} - \frac{w^{ROW} \hat{q}}{a_q A^{ROW}} \right) - \epsilon.$$

For  $0 < \epsilon < (1 - \lambda^{ROW}) \left( \hat{p} - \frac{w^{ROW} \hat{q}}{a_q A^{ROW}} \right)$ , this would strictly increase firm  $i$ 's variable profits in the ROW while not affecting its cost of innovation. This, however, contradicts that serving only the rich at a price  $\hat{p}^{H:ROW}$  is profit maximizing.

price below marginal costs in the SOE inclusive of trade costs then immediately implies that in an equilibrium with positive exports it must hold

$$\tau \frac{w}{a_z A} \leq \frac{w^{ROW}}{a_z A^{ROW}} .$$

But then, this condition must hold with equality—equation (13)—for if not, the homogeneous good firms in the SOE would make strictly positive profits from serving the ROW, which would contradict profit maximization of homogeneous good firms in the SOE as they are price takers in the ROW.  $\square$

A.7. Proof of Lemma 5

We show that constraint (IRf) cannot be binding for the low types. With  $I^L \leq \hat{I}$  this is trivially the case. We thus consider the case of  $I^L > \hat{I}$  and show that low types prefer quality  $\bar{q}_{-1}$  over any imported quality.

As argued in the main body of the text, it is never optimal to import quality  $q \leq \bar{q}_{-1}$ . Hence, constraint (IRf) can only be binding if the preferred importing quality satisfies  $q > \bar{q}_{-1}$ . Combined with the fact that the marginal utility of quality is increasing in  $\theta$ , this implies that low types will prefer quality  $\bar{q}_{-1}$  over their best import option if this is the case for some  $\hat{\theta} \geq \theta^L$ .

The income of low types is bounded from above by 1. Moreover,  $\theta^L$  is decreasing in both,  $q^L$  and  $p^L$ . We conclude that  $\theta^L$  is bounded from above by

$$\bar{\theta}^L := \frac{1 - \frac{1}{a_q}}{\beta \bar{q}_{-1}^{1-\beta}} .$$

A household of type  $\bar{\theta}^L$  prefers quality  $\bar{q}_{-1}$  over its best import option if

$$\bar{\theta}^L v(\bar{q}_{-1}) - \frac{1}{a_q} \geq [\bar{\theta}^L]^{\frac{1}{\beta}} [\bar{q}_{-1}]^{\frac{1-\beta}{\beta}} \chi(\tau) .$$

Using the definitions of  $\bar{\theta}^L$ ,  $v(\cdot)$ , and  $\chi(\tau)$ , this can be rewritten as

$$\frac{1 - \frac{1}{a_q}}{\beta \bar{q}_{-1}^{1-\beta} \bar{q}_{-1}^{1-\beta}} - \frac{1}{a_q} \geq \left[ \frac{1 - \frac{1}{a_q}}{\beta \bar{q}_{-1}^{1-\beta}} \right]^{\frac{1}{\beta}} [\bar{q}_{-1}]^{\frac{1-\beta}{\beta}} \left[ \frac{a_q(1-\beta)}{\tau^2} \right]^{\frac{1-\beta}{\beta}} \beta .$$

Solving for  $\tau$  and simplifying terms yields the expression given in Assumption 1.  $\square$

A.8. Proof of Lemma 6

To show the desired result, we consider the limiting case where domestic firms are just indifferent between innovating or not to serve the rich and then proceed by contradiction. In particular, we show that  $q^{H,f} \leq q^H$  contradicts that it is optimal for domestic firms not to serve the rich households, where  $q^{H,f}$  denotes the quality of the best import option and  $q^H$  denotes the optimal domestically-provided quality.

Suppose that  $q^{H,f} \leq q^H$ . The best importing quality satisfies the first-order condition for utility maximization of the rich  $\theta^H v'(q^{H,f}) = \frac{\tau^2}{a_q A}$ , implying that

$$p^{H,f} = \frac{\tau^2}{a_q A} q^{H,f} = \theta^H v'(q^{H,f}) q^{H,f} , \tag{A.19}$$

where  $p^{H,f}$  denotes the price of imported quality  $q^{H,f}$ . In the limiting case where domestic firms are just indifferent between serving or not the rich households, (IRf) is binding for the rich and, hence,  $\theta^H v(q^{H,f}) - p^{H,f} = \theta^H v(q^H) - p^H$ . Therefore,

$$p^H = \theta^H [v(q^H) - v(q^{H,f}) + p^{H,f}] = \theta^H \int_{q^{H,f}}^{q^H} v'(x) dx + \theta^H v'(q^{H,f}) q^{H,f} \geq \theta^H v'(q^H) q^H .$$

The second equality follows from using the fundamental theorem of calculus and equation (A.19). The inequality follows from the fact that  $v(\cdot)$  is concave and that  $q^{H,f} \leq q^H$ , by assumption, and from simplifying terms. The above inequality is strict whenever  $q^{H,f} < q^H$ .

Now, there are two possibilities for when domestic firms are indifferent between serving or not the rich households. (i) Either they make zero profits and are equally well off stopping to innovate altogether. (ii) Or they would be equally well off innovating at a lower rate to just serve the poor. We show that neither is possible.

(i) The first order condition for  $q^H$  implies

$$\lambda \theta^H v'(q^H) - \lambda \frac{1}{a_q A} - \frac{1}{\bar{q}_{-1}} h' \left( \frac{q^H}{\bar{q}_{-1}} \right) = 0 . \tag{A.20}$$

Clearly, the fact that  $p^H \geq \theta^H v'(q^H)q^H$  and the convexity of  $h(\cdot)$  imply that firms are making strictly positive profits from just serving the rich households, i.e. a solution with no innovation cannot be optimal.

(ii) The fact that firms cannot be indifferent between serving both types of households or just the poor follows from a revealed preference argument. In particular, in the separating equilibrium, it must be that  $(IR^L)$  is binding.<sup>48</sup> Moreover, firms could opt to offer poor households contract  $(\tilde{q}^L, \tilde{p}^L)$ , where we use this to denote the contract that firms would offer the low types in the hypothetical scenario where they just serve these types. This contract also satisfies  $(IR^L)$  with equality, i.e. low types are indifferent between contracts  $(\tilde{q}^L, \tilde{p}^L)$  and  $(q^L, p^L)$ . We now show that offering  $(\tilde{q}^L, \tilde{p}^L)$  and  $(\tilde{q}^H, \tilde{p}^H)$  would yield strictly higher profits than when just offering  $(\tilde{q}^L, \tilde{p}^L)$ , where  $\tilde{q}^H = q^H$  and  $\tilde{p}^H$  is as defined below. In turn, this implies that the optimal contracts in the separating equilibrium yield strictly higher profits than when just offering  $(\tilde{q}^L, \tilde{p}^L)$ .

If  $\tilde{q}^L \leq q^L$ , this follows immediately because the change in the contract of the poor would not affect the contract for the rich and because firms make positive profits from serving the rich.

If  $\tilde{q}^L > q^L$  and  $(IC^H)$  is not binding, the same reasoning from before applies. If  $(IC^H)$  is binding, then the price of  $q^H$  changes to  $\tilde{p}^H = \tilde{p}^L + \int_{\tilde{q}^L}^{q^H} \theta^H v'(x)dx$ , and we have

$$\begin{aligned} & (1 - \lambda) \left( \tilde{p}^L - \frac{1}{a_q A} \tilde{q}^L \right) - h \left( \frac{\tilde{q}^L}{\tilde{q}_{-1}} \right) \\ & < \left( \tilde{p}^L - \frac{1}{a_q A} \tilde{q}^L \right) - h \left( \frac{\tilde{q}^L}{\tilde{q}_{-1}} \right) \\ & < \left( \tilde{p}^L - \frac{1}{a_q A} \tilde{q}^L \right) - h \left( \frac{\tilde{q}^L}{\tilde{q}_{-1}} \right) + \int_{\tilde{q}^L}^{q^H} \lambda \theta^H v'(x) - \lambda \frac{1}{a_q A} - \frac{1}{\tilde{q}_{-1}} h' \left( \frac{x}{\tilde{q}_{-1}} \right) dx \\ & = \left( \tilde{p}^L - \frac{1}{a_q A} \tilde{q}^L \right) + \lambda (\tilde{p}^H - \tilde{p}^L) - \lambda \frac{1}{a_q A} (q^H - \tilde{q}^L) - h \left( \frac{q^H}{\tilde{q}_{-1}} \right). \end{aligned} \tag{A.21}$$

The first inequality follows from  $\tilde{p}^L - \frac{1}{a_q A} \tilde{q}^L > 0$  and  $\lambda > 0$ . The second inequality follows from using (A.20) and the fact that  $v(\cdot)$  is concave and  $h(\cdot)$  is convex. The equality follows from solving the integral. The result then follows from noting that the expression in the last row is equal to total profits with this alternative separating contract.  $\square$

### A.9. Proof of Proposition 3

The fact that the various cases as characterized in Proposition 3 represent equilibria follows from the discussions in the main text. In particular, the solution to the firms' problem implies that firms maximize profits given households' utility maximizing consumption choice, and the definition of  $\theta^{h,e}(N_f)$  implies that in turn these solutions are consistent with the ensuing  $\theta^{h,e}$ . It remains to show that an equilibrium does indeed exist when  $(IR^H)$  is binding (step 1) and to characterize the equilibrium values in the markets other than the differentiated good markets for this case (step 2). This appendix discusses each step in turn.

**Step 1.** The characterization of the equilibrium in Proposition 3 relies on  $\theta^{h,e}(N_f^e)$ , which is the fixed point of (24) for a given  $N_f^e$ . That is, it considers situations where the ensuing solution to the domestic firms' profit maximization problem  $(q_i^h, p_i^h)$  implies levels of  $\theta^h$  that are in turn consistent with the firms' solution given the respective equilibrium level of  $N_f^e$ . To show existence, we therefore need to show that such fixed points exist for some  $N_f^e \in [0, 1]$ . We begin with showing two lemmata that characterize existence of fixed points of (24) for arbitrary given  $N_f$ .

**Lemma A.2.** *There is an infimum  $\underline{N}_f \geq 0$  such that for all  $N_f > \underline{N}_f$ , the fixed point  $\theta^{H,e}(N_f)$  exists.*

**Proof.** To show that (24) possesses a fixed point  $\theta^{H,e}(N_f)$ , we can rewrite it as

$$I^H - \int_0^{N_f} \beta \theta^H \cdot (q_i^{H,f})^{1-\beta} + p_i^{H,f} di - \int_{N_f}^1 \beta \theta^H \cdot (q_i^H)^{1-\beta} + p_i^H di = 0. \tag{A.22}$$

Using equations (14) and (16), we obtain

<sup>48</sup>  $(IR^L)$  is never binding. Hence, the only possibility where  $(IR^L)$  is not binding is a hypothetical case where  $(IC^L)$  is binding, for otherwise firms could increase profits by increasing  $p^L$  which would only relax  $(IC^H)$ .  $(IC^L)$ , however, cannot be binding because, by assumption, the rich are indifferent between consuming the domestically produced quality  $q^H$  or importing a weakly lower quality. As richer households have a stronger taste for quality, poor households must then weakly prefer the best import choice of the rich households over  $(q^H, p^H)$  and, therefore, strictly prefer their own best import choice, i.e.  $(IR^L)$  would have to be strictly binding in such case, a contradiction.

$$\int_0^{N_f} \beta \theta^H \cdot (q_i^{H,f})^{1-\beta} + p_i^{H,f} di = N_f \cdot (\theta^H)^{\frac{1}{\beta}} \left[ \frac{(1-\beta)a_q A}{\tau^2} \right]^{\frac{1-\beta}{\beta}}.$$

The contract offered by the domestic firm is determined by (IR<sup>H</sup>) and (18b) (which is equivalent to (20d) as well as to the corresponding equation in the closed economy (10d)). We first note that (18b) defines a unique level  $q_i^H$  only depending on  $\theta^H$ , the previous period's maximum quality level  $\bar{q}_{i-1}$ , and the exogenous primitives of the model. Further, note that the implicit function theorem implies that the function  $q_i^H(\theta^H)$  defined by (18b) is an increasing function in  $\theta^H$ .<sup>49</sup> The condition (IR<sup>H</sup>) then defines the price  $p_i^H$  such that for the firms' optimal  $q_i^H(\theta^H)$ , (IR<sup>H</sup>) holds with equality. With  $\bar{q}_{i-1} = \bar{q}_{-1}$  for all  $i$ , we get<sup>50</sup>

$$\int_{N_f}^1 \beta \theta^H \cdot (q_i^H)^{1-\beta} + p_i^H di = (1 - N_f) \cdot (\theta^H)^{\frac{1}{\beta}} [(1-\beta)a_q A]^{\frac{1-\beta}{\beta}} \cdot \left[ (1+\beta) \left( 1 + \frac{a_q}{\lambda} h' \left( \frac{q^H(\theta^H)}{\bar{q}_{-1}} \right) \right)^{-\frac{1-\beta}{\beta}} - \beta \tau^{-2\frac{1-\beta}{\beta}} \right].$$

To show the desired result, we now proceed in two steps. We first show that for  $N_f = 1$  a fixed point must exist and then show the desired result.

(i)  $N_f = 1$ : If rich households import all differentiated goods, then (A.22) reads

$$I^H - (\theta^H)^{\frac{1}{\beta}} \left[ \frac{(1-\beta)a_q A}{\tau^2} \right]^{\frac{1-\beta}{\beta}} = 0.$$

As the left-hand side monotonously declines with  $\theta^H$ , there is a unique root

$$\theta^{H,e}(N_f = 1) = (I^H)^{\beta} \left[ \frac{\tau^2}{(1-\beta)a_q A} \right]^{1-\beta}.$$

Also note that  $\theta^{H,e}(N_f = 1)$  increases with  $I^H$ .

(ii)  $N_f \in (0, 1)$ : If only a share of differentiated goods are imported, (A.22) can be written as

$$I^H - N_f \cdot (\theta^H)^{\frac{1}{\beta}} \left[ \frac{(1-\beta)a_q A}{\tau^2} \right]^{\frac{1-\beta}{\beta}} - (1 - N_f) \cdot (\theta^H)^{\frac{1}{\beta}} [(1-\beta)a_q A]^{\frac{1-\beta}{\beta}} \cdot \left[ (1+\beta) \left( 1 + \frac{a_q}{\lambda} h' \left( \frac{q^H(\theta^H)}{\bar{q}_{-1}} \right) \right)^{-\frac{1-\beta}{\beta}} - \beta \tau^{-2\frac{1-\beta}{\beta}} \right] = 0. \tag{A.24}$$

We know that the left-hand-side (LHS) of (A.24) declines with  $\theta^H$  for  $N_f = 1$ . Since the LHS of (A.24) is continuous in  $N_f$ , there must exist a  $0 \leq \underline{N}_f < 1$  such that for all  $N_f > \underline{N}_f$  the root  $\theta^{H,e}(N_f)$  exists.  $\square$

Based on Lemma A.2, we define  $N_f^{min} = \underline{N}_f$  if  $\theta^{H,e}(\underline{N}_f)$  exists, and  $N_f^{min} = \underline{N}_f + \epsilon$  otherwise for some small but positive  $\epsilon$  such that  $\theta^{H,e}(N_f^{min})$  exists. Lemma A.2 implies that for all  $N_f \geq N_f^{min}$ , a fixed point for  $\theta^H$  exists. Lemma A.3 below shows that for any given  $N_f \geq N_f^{min}$  there are corresponding fixed points  $\theta^{L,e}(N_f)$  with respect to the firms' offers for the low income households.

**Lemma A.3.** For any  $N_f \in [N_f^{min}, 1]$  and corresponding fixed point  $\theta^{H,e}(N_f)$ , there exists a fixed point  $\theta^{L,e}(N_f)$  such that the optimal contract for the low income households  $(q_i^L, p_i^L)$  by the domestic firms given  $\theta^L = \theta^{L,e}(N_f)$  in turn implies that  $\theta^L = \theta^{L,e}(N_f)$ .

**Proof.** To prove Lemma A.3, we need to distinguish the cases (i) where the domestic firm offers qualities  $q_i^L \leq \bar{q}_{-1}$  and (ii) where it offers qualities  $q_i^L > \bar{q}_{-1}$ .

<sup>49</sup> Applying the implicit function theorem to (18b) yields

$$\frac{dq_i^H}{d\theta^H} = \frac{\lambda v'(q_i^H)}{-\lambda \theta^H v''(q_i^H) + h''(q_i^H/\bar{q}_{i-1})/\bar{q}_{i-1}^2} > 0. \tag{A.23}$$

The sign is positive as  $v(\cdot)$  is concave and  $h(\cdot)$  is convex.

<sup>50</sup> More concretely, we use  $p_i^H = \theta^H v(q_i^H) - (\theta^H)^{\frac{1}{\beta}} (\bar{q}_{-1})^{\frac{1-\beta}{\beta}} \chi(\tau)$  from (IR<sup>H</sup>) and

$$q_i^H = (\theta^H)^{\frac{1}{\beta}} [(1-\beta)a_q A]^{-\frac{1}{\beta}} \left[ 1 + \frac{a_q}{\lambda} h' \left( \frac{q_i^H}{\bar{q}_{-1}} \right) \right]^{-\frac{1}{\beta}},$$

from equation (18b). Some minor transformations considering that  $A = \bar{q}_{-1}$  then yield the expression of interest.

(i) This reflects case (B-SOE) in the domestic firms' problem in subsection 4.1 where  $I^L \leq \hat{I}$ . Then the low income households consume the same quality  $q_i^L = [\theta^L(1 - \beta)a_q A]^{\frac{1}{\beta}} \leq \bar{q}_{-1}$  at marginal costs independent of the share  $N_f$  of imported high quality goods for the rich households. We thus obtain

$$\theta^{L,e}(N_f) = (I^L)^\beta [(1 - \beta)a_q A]^{\beta-1}, \quad \forall N_f \in [0, 1].$$

(ii) This corresponds to the case (C-SOE) in the domestic firms' problem in subsection 4.1 where  $I^L > \hat{I}$ . Then we can write for  $\theta^L$  with given  $N_f$ :

$$\theta^L = \frac{I^L - \int_0^{N_f} p_i^{L,Cf} di - \int_{N_f}^1 p_i^{L,C} di}{\beta \left( \int_0^{N_f} (q_i^{L,Cf})^{1-\beta} di + \int_{N_f}^1 (q_i^{L,C})^{1-\beta} di \right)}. \tag{A.25}$$

This can be re-written as

$$I^L - \int_0^{N_f} \beta \theta^L \cdot (q_i^{L,Cf})^{1-\beta} + p_i^{L,Cf} di - \int_{N_f}^1 \beta \theta^L \cdot (q_i^{L,C})^{1-\beta} + p_i^{L,C} di = 0. \tag{A.26}$$

Replacing  $p_i^{L,C}$  and  $p_i^{L,Cf}$  via condition  $(IR^L)$ , i.e. via equation (20a) (or equivalently (21a)), and taking into account the symmetry of offers in sectors  $i \leq N_f$  and  $i > N_f$ , respectively, we obtain

$$I^L - N_f \cdot \left[ \theta^L ((1 + \beta) \cdot (q^{L,Cf})^{1-\beta} - (\bar{q}_{-1})^{1-\beta}) + \frac{1}{a_q} \right] - (1 - N_f) \cdot \left[ \theta^L ((1 + \beta) \cdot (q^{L,C})^{1-\beta} - (\bar{q}_{-1})^{1-\beta}) + \frac{1}{a_q} \right] = 0. \tag{A.27}$$

It then remains to be shown that  $q^{L,Cf}$  and  $q^{L,C}$  are increasing functions of  $\theta^L$  for (A.27) to possess a unique root  $\theta^{L,e}(N_f)$ . For  $q^{L,Cf}$  this follows from (21b) via the implicit function theorem.<sup>51</sup>

When the domestic firm serves the high income households, we have to distinguish the cases where  $(IC^H)$ , i.e. (20c), is slack or binding. Starting with the case where (20c) is slack, the domestic firm innovates up to  $q^H$ , and  $q^{L,C}$  can be freely chosen. In this case we obtain  $q^{L,C} = [\theta^L(1 - \beta)a_q A]^{\frac{1}{\beta}} > \bar{q}_{-1}$ , which increases with  $\theta^L$ . If  $(IC^H)$  is binding, it determines  $q^{L,C}$ . Using  $(IR^{H1})$  and  $(IR^L)$  (20a), it can be written as

$$F(\theta^L, q^{L,C}) = \theta^{H,e}(N_f) v(q^{L,C}) - \theta^L v(q^{L,C}) + \theta^L v(\bar{q}_{-1}) - \frac{1}{a_q} - [\theta^{H,e}(N_f)]^{\frac{1}{\beta}} (\bar{q}_{-1})^{\frac{1-\beta}{\beta}} \chi(\tau) = 0.$$

The implicit function theorem yields

$$\frac{d q^{L,C}}{d \theta^L} = - \frac{\frac{\partial F}{\partial \theta^L}}{\frac{\partial F}{\partial q^{L,C}}} = \frac{v(q^{L,C}) - v(\bar{q}_{-1})}{(1 - \beta)(q^{L,C})^{-\beta} (\theta^{H,e}(N_f) - \theta^L)} > 0.$$

Consequently, the left-hand side of (A.27) is strictly decreasing in  $\theta^L$  for any given  $N_f$  and thus, there exists a unique root  $\theta^{L,e}(N_f)$ .

In addition, we note (i) that the implicit function theorem implies that  $\theta^{L,e}(N_f)$  increases with  $I^L$  for any given  $N_f$  and (ii) that  $\theta^{L,e}(N_f)$  is a continuous function of  $N_f$ .  $\square$

We now build on Lemmata A.2 and A.3 to show that an equilibrium exists. We proceed in four steps:

(i) We note that  $\pi^B(\theta^{H,e}(N_f))$  and  $\Delta\pi^C(\{\theta^h(N_f)\}_h)$  are continuous functions of  $N_f$  for  $N_f \in [N_f^{min}, 1]$ . This is the case as Lemma A.2 implies that equation (A.24) defines an implicit function  $\theta^{H,e}(N_f)$  that—according to the implicit function theorem—is continuous over  $N_f \in [N_f^{min}, 1]$ . Similarly, from Lemma A.3 we obtain the corresponding function  $\theta^{L,e}(N_f)$  that is also continuous on  $[N_f^{min}, 1]$ . Recalling the definitions of  $\pi^B(\theta^H)$  and  $\Delta\pi^C(\{\theta^h\}_h)$  in subsection 4.1,  $\pi^B(\theta^H)$  and  $\Delta\pi^C(\{\theta^h\}_h)$  are continuous functions of  $\theta^H$  and  $\{\theta^h\}_h$ , respectively. With  $\theta^{h,e}(N_f)$  continuous on  $[N_f^{min}, 1]$ ,  $\pi^B(\theta^{H,e}(N_f))$  and  $\Delta\pi^C(\{\theta^{h,e}(N_f)\}_h)$  are thus continuous on  $N_f \in [N_f^{min}, 1]$ .

(ii) From Lemma A.2 and A.3 we know that  $\theta^{H,e}(1)$  and  $\theta^{L,e}(1)$  exist. Hence, if  $\pi^B(\theta^{H,e}(1)) \leq 0$  or  $\Delta\pi^C(\{\theta^{h,e}(1)\}_h) \leq 0$ , respectively, there exists an equilibrium with  $N_f^e = 1$ , i.e. where rich households import all differentiated goods from the ROW (B-SOE(iii))

<sup>51</sup> That is, we obtain from the implicit function theorem:

$$\frac{d q_i^L}{d \theta^L} = \frac{(1 - \lambda)v'(q_i^L)}{-(1 - \lambda)\theta^L v''(q_i^L) + h''(q_i^L/\bar{q}_{i-1})/\bar{q}_{i-1}^2} > 0. \tag{A.28}$$

The sign is positive as  $v(\cdot)$  is concave and  $h(\cdot)$  is convex.

and C-SOE(iii), respectively). Similarly, if  $N_f^{min} = 0$  and  $\pi^B(\theta^{H,e}(0)) \geq 0$  or  $\Delta\pi^C(\{\theta^{h,e}(0)\}_h) \geq 0$ , respectively, there exists an equilibrium with  $N_f^e = 0$ , i.e. where rich households purchase all differentiated goods from innovative local firms (B-SOE(i) and C-SOE(i), respectively). It thus remains to be shown that an equilibrium exists if (a)  $\pi^B(\theta^{H,e}(1)) > 0$  and either  $N_f^{min} > 0$  or  $N_f^{min} = 0$  and  $\pi^B(\theta^{H,e}(0)) < 0$ , and (b)  $\Delta\pi^C(\{\theta^{h,e}(1)\}_h) > 0$  and either  $N_f^{min} > 0$  or  $N_f^{min} = 0$  and  $\Delta\pi^C(\{\theta^{h,e}(0)\}_h) < 0$ , respectively.

(iii) If  $N_f^{min} = 0$ ,  $\pi^B(\theta^{H,e}(0)) < 0$  and  $\pi^B(\theta^{H,e}(1)) > 0$ , then by step (i), there exists an  $N_f^e \in (0, 1)$  for which  $\pi^B(\theta^{H,e}(N_f^e)) = 0$ , which then constitutes an equilibrium as firms are indeed indifferent between serving or not the rich (B-SOE(ii)). Similarly, if  $N_f^{min} = 0$ ,  $\Delta\pi^C(\{\theta^{h,e}(0)\}_h) < 0$  and  $\Delta\pi^C(\{\theta^{h,e}(1)\}_h) > 0$ , there exists an  $N_f^e \in (0, 1)$  for which  $\Delta\pi^C(\{\theta^{h,e}(N_f^e)\}_h) = 0$ , which then constitutes an equilibrium (C-SOE(ii)).

(iv) Lastly, Lemma A.4 below shows that if  $N_f^{min} > 0$ , there must be an  $\tilde{N}_f \geq N_f^{min}$  for which  $\pi^B(\theta^{H,e}(\tilde{N}_f)) < 0$  (in case B-SOE), respectively  $\Delta\pi^C(\{\theta^{h,e}(\tilde{N}_f)\}_h) < 0$  (in case C-SOE). Step (i) then again implies that there exists an  $N_f^e \in (0, 1)$  for which  $\pi^B(\theta^{H,e}(N_f^e)) = 0$  and  $\Delta\pi^C(\{\theta^{h,e}(N_f^e)\}_h) = 0$ , respectively, which thus constitutes an equilibrium (B-SOE(ii) and C-SOE(ii), respectively).

**Lemma A.4.** *If  $N_f^{min} > 0$  and  $\pi^B(\theta^{H,e}(1)) > 0$  in case (B-SOE), respectively  $\Delta\pi^C(\{\theta^{h,e}(1)\}_h) > 0$  in case (C-SOE), then for each of the cases (B-SOE) and (C-SOE) there exists a  $\tilde{N}_f \geq N_f^{min}$  such that  $\pi^B(\theta^{H,e}(\tilde{N}_f)) < 0$  and  $\Delta\pi^C(\{\theta^{h,e}(\tilde{N}_f)\}_h) < 0$ , respectively.*

**Proof.** The proof of Lemma A.4 proceeds in three steps:

(i) First, for a given share of imported goods  $N_f$ , we define the right-hand side of (24) as function  $G(\theta^H; N_f)$ . That is,

$$G(\theta^H; N_f) := \frac{I^H - N_f p^{H,f} - (1 - N_f) p^H}{\beta(N_f v(q^{H,f}) + (1 - N_f) v(q^H))}. \tag{A.29}$$

For (24) to not possess a fixed point,  $G(\theta^H; N_f)$  must eventually be increasing in  $\theta^H$ . Recall that  $q^H$  is an implicit function of  $\theta^H$  via (18b) (resp. (20d)) and with this,  $p^H$  is an implicitly defined function of  $\theta^H$  via (IR<sup>H</sup>), i.e. (20b). Since  $q^{H,f}$ ,  $q^H$ , and  $p^{H,f}$  increase with  $\theta^H$ , we obtain that  $\frac{\partial G(\theta^H; N_f)}{\partial \theta^H} > 0$  only if  $N_f < 1$  and  $\frac{\partial p^H}{\partial \theta^H} < 0$ .

(ii) Consider condition (IR<sup>H</sup>) which characterizes price  $p^H$ :

$$p^H(\theta^H) = \theta^H \cdot (q^H(\theta^H))^{1-\beta} - (\theta^H)^{\frac{1}{\beta}} (\bar{q}_{-1})^{\frac{1-\beta}{\beta}} \chi(\tau).$$

This condition is independent of  $N_f$ . Moreover, if  $N_f^{min} > 0$ , it must be decreasing in  $\theta^H$  for  $\theta^H$  sufficiently large by step (i). Now, for  $\theta = \theta^{H,e}(1)$ , the firms make profits from serving the rich by assumption. Hence,  $p^H(\theta^{H,e}(1))$  must be greater than marginal production costs, which are increasing in  $q^H$  and, hence,  $\theta^H$ . It follows that if  $N_f^{min} > 0$ , there must exist  $\theta^{mc}$ , where the price  $p^H$  equals the marginal production costs of (optimal) quality  $q^H$  as defined by (20d), i.e.  $p^H(\theta^{mc}) = \frac{q^H(\theta^{mc})}{a_q A}$ . Further, it must be that  $\theta^{mc} > \theta^{H,e}(1)$ .

(iii) From (i) we know that  $G(\theta^H; N_f = 1)$  is a strictly decreasing function, which possesses a fixed point at  $\theta^{H,e}(1)$ . It then follows from our discussions that  $G(\theta^{mc}; N_f = 1) < \theta^{H,e}(1) < \theta^{mc}$ . This is illustrated in Fig. A.1.

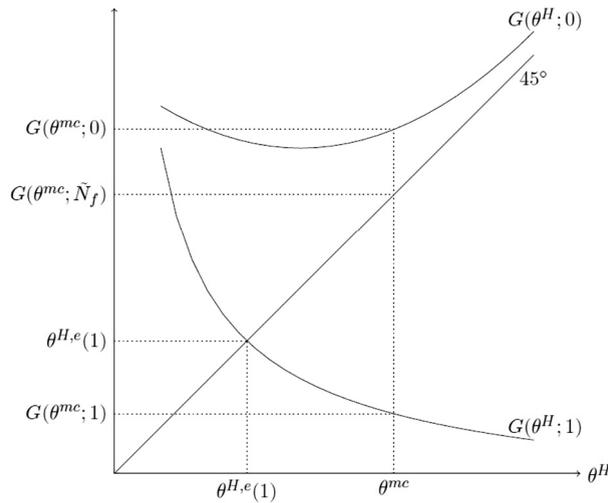
On the other hand, when  $N_f^{min} > 0$ , then  $G(\theta^H; N_f = 0)$  does not intersect with the bisectrix (see Fig. A.1). Consequently,  $G(\theta^{mc}; N_f = 0) > \theta^{mc}$ . Thus, by continuity of  $G(\theta^{mc}; N_f)$  in  $N_f$ , there must exist a  $\tilde{N}_f$  such that  $G(\theta^{mc}; \tilde{N}_f) = \theta^{mc}$ , i.e. for  $\tilde{N}_f$  it holds  $\theta^{H,e}(\tilde{N}_f) = \theta^{mc}$ . Moreover, as (24) does not have a fixed point for  $N_f < N_f^{min}$ , it must be that  $\tilde{N}_f \geq N_f^{min}$ . But with the price only covering marginal costs, the additional profits from serving the rich must be negative as innovation costs cannot be covered.  $\square$

**Step 2.** It remains to provide the other equilibrium values.

In case (B-SOE), homogeneous good consumption of the rich households is  $z^{H,e} = (I^H - N_f^e p^{H,f,e} - (1 - N_f^e) p^{H,B,e}) a_z A$  and for the poor households  $z^{L,e} = (I^L - p^{L,B,e}) a_z A$ . The amount of exported homogeneous goods is  $z^{exp,e} = N_f \lambda \tau \frac{a_z}{a_q} q^{H,f,e}$ . Labor demand is given by

$$\int_{N_f^e}^1 L_i^e(q^{H,B,e}) di + \int_0^1 L_i^e(q^{L,B,e}) di + L_z^e = (1 - N_f^e) \lambda \frac{q^{H,B,e}}{a_q A} + (1 - \lambda)(1 - \beta) I^L + \lambda (I^H - N_f^e p^{H,f,e} - (1 - N_f^e) p^{H,B,e})$$

$$+ (1 - \lambda) \beta I^L + N_f \lambda \frac{\tau^2}{a_q A} q^{H,f,e} + \left[ \int_{N_f^e}^1 f M_i^e + h(q^{H,B,e} / \bar{q}_{-1}) di \right].$$



Note: The figure illustrates  $G(\theta^H; 0)$  and  $G(\theta^H; 1)$  as functions of  $\theta^H$  and indicates the fixed point  $G(\theta^{mc}; \bar{N}_f) = \theta^{mc}$ . Note that the important property of  $G(\theta^H; 0)$  is that it does not intersect with the bisectrix and regarding  $G(\theta^H; 1)$  that it is strictly declining with  $\theta^H$ .

Fig. A.1. Illustration of  $G(\theta^H; 0)$  and  $G(\theta^H; 1)$ .

In case (C-SOE), homogeneous good consumption of the rich households is  $z^{H,e} = (I^H - N_f^e p^{H,f,e} - (1 - N_f^e) p^{H,C,e}) a_z A$  and for the poor households  $z^{L,e} = (I^L - N_f^e p^{L,C,f,e} - (1 - N_f^e) p^{L,C,e}) a_z A$ . The amount of exported homogeneous goods is  $z^{exp,e} = N_f^e \lambda \tau \frac{a_z}{a_q} q^{H,f,e}$ . Labor demand is given by

$$\begin{aligned} & \int_{N_f^e}^1 L_i^e(q^{H,C,e}) di + \int_0^{N_f^e} L_i^e(q^{L,C,f,e}) di + \int_{N_f^e}^1 L_i^e(q^{L,C,e}) di + L_z^e \\ &= \lambda(1 - N_f^e) \frac{q^{H,C,e}}{a_q A} + (1 - \lambda) \left[ N_f^e \frac{q^{L,C,f,e}}{a_q A} + (1 - N_f^e) \frac{q^{L,C,e}}{a_q A} \right] \\ & \quad + \lambda(I^H - N_f^e p^{H,f,e} - (1 - N_f^e) p^{H,C,e}) + (1 - \lambda)(I^L - N_f^e p^{L,C,f,e} - (1 - N_f^e) p^{L,C,e}) \\ & \quad + \lambda N_f^e \frac{\tau^2}{a_q A} q^{H,f,e} + \left[ \int_0^{N_f^e} f M_i^e + h(q^{L,C,f,e} / \bar{q}_{-1}) di + \int_{N_f^e}^1 f M_i^e + h(q^{H,C,e} / \bar{q}_{-1}) di \right]. \end{aligned}$$

Lastly, labor is remunerated at the normalized equilibrium wage rate  $w^e = 1$  and the homogeneous good priced at  $p_z^e = 1/(a_z A)$  reflecting marginal costs, while the SOE exports the homogeneous goods at the equilibrium price  $p_z^{ROW} = \tau/(a_z A)$  reflecting marginal costs times the iceberg trade costs. Also note the measure of firms participating in the patent races  $M_i^e$  does not differ for sectors which import and only innovate to serve the poor,  $i \in [0, N_f^e]$ , and for those where domestic firms compete to serve the rich households,  $i \in [N_f^e, 1]$ . This is because for  $N_f^e \in (0, 1)$ , expected profits must be the same across sectors.  $\square$

**Appendix B. Supplementary material**

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jet.2024.105887>.

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