University of Pavia and University of Milano

Department of Scienze Economiche e Aziendali

PhD program in Economics

Essays on supply-side fiscal policies for Covid-19

Supervisor:
Prof. Patrizio Tirelli

Candidate:
Emanuele Colombo Azimonti
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Introduction

This dissertation is an essay on supply-side fiscal policies against the Covid-19 pandemic. We investigate the effects of the pandemic on the economy and we discuss the role of different fiscal policies, whose focus is both the containment of the economic recession and of the spread of the disease. This thesis is divided in two chapters; they share a novel characterization of the Covid-19 shock, whose endogenous persistence depends on the share of contact-intensive activities in the economy. Another common element is the possibility for firms to reallocate part of the production towards an online retail technology, which, in contrast with the contact-intensive one, does not contribute to the persistence of the pandemic. Additionally, we are able to expand this core structure with the introduction of firm dynamics, that is a framework where firms can have access to the online retail only if they pay an entry cost and they are still profitable enough to compete in the market.
Overall, this structure allows us to focus on the gains deriving from a larger exploitation of the online commerce and to derive policy implications about the effectiveness of various supply-side fiscal tools.
Our methodology takes into account both the peculiarities of the DSGE pandemic literature (mainly, in terms of policy analysis) and those of the macro-epidemiological one, especially the individuation of a trade off between health and economic policies and in terms of individual incentives.

In the first chapter, we develop a model that allows for online retail trade and for endogenous Covid-related health expenditures. Firms can partially shift their production from the contact-intensive (more contagious) retail to online trade. In this framework, we imagine the pandemic as a labor supply shock, whose persistence is increasing in the level of contact-intensive activities; this allow us to proxy the impact of Covid-19, which spreads mainly due to interpersonal contact and physi-

cal interaction. This core structure is expanded with the introduction of public expenditure for health services, as an additional tool against the virus. Our model incorporates the health-vs-economy dilemma between stabilising consumption and mitigating the shock through a contraction of economic activity. The existence of such a trade-off is crucial when designing any policy intervention against the pandemic.
We adopt a Ramsey-optimal approach in order to investigate the public intervention in the economy. The planner can use a mix of fiscal policies, namely a combination of an online subsidy and labor income taxation (a proxy for lockdowns).
As a result, the market equilibrium at best imperfectly internalises the infection risk from contact-intensive retail trade, and the anticipation of health costs has large contractionary effects. The Ramsey planner exploits the subsidy to online trade to limit lockdown policies. Relative to the market equilibrium, the optimal policy stimulates consumption and contains the surge in health expenditures, mitigating both the recession and the persistence of the Covid-19 shock.

The second chapter expands the first with the introduction of firm dynamics. More precisely, we assume that all firms can exploit the contact-intensive retail channel, but only a subset can reallocate part of the production by entering the online retail channel. The possibility to participate to the online markets depends on the idiosyncratic level of online productivity, on the existence of a fixed entry cost and, finally, on the impact of the pandemic. As a matter of facts, the outbreak of Covid-19 affects asymmetrically the production costs faced by the two retail channels, as the contact-intensive becomes relatively more costly and inefficient than the online one. We use this framework to investigate how supply-side fiscal policies can create a further incentive for the online transition.
According to our results, the pandemic stimulates the expansion of the online sector, mainly thanks to the aforementioned effect on sectoral costs. Moreover, the fiscal intervention (with an online subsidy and a tax on contact-intensive production) encourages entry and reallocation in the online sector, by lowering the efficiency requirement needed to join the online market. This mechanism has sizable effects both on the mitigation of the disease and the containment of the economic downturn.

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Chapter 1

Covid-19 supply-side fiscal policies to escape the health-vs-economy dilemma

Emanuele Colombo Azimonti, Luca Portoghese, Patrizio Tirelli

Abstract We develop a model that allows for online retail trade and for endogenous Covid-related health expenditures. The market equilibrium at best imperfectly internalises the infection risk from contact-intensive retail trade, and the anticipation of health costs has large contractionary effects. The Ramsey planner exploits a subsidy to online trade to limit lockdown policies. Relative to the market equilibrium, the optimal policy stimulates consumption and contains the surge in health expenditures, mitigating both the recession and the persistence of the Covid-19 shock.

1.1 Introduction

We propose a simple macroeconomic model where an unconventional supply shock allows to mimick the adjustment to a pandemic shock. In our model firms have the option of switching between contact-intensive and online retail trade, where the former contributes to the persistence of the shock and the latter does not. All this paves the way for the analysis of optimal supply-side fiscal policies that must strike a balance between shock mitigation and the conventional macroeconomic stabilisation objective.

The outbreak of the Covid-19 pandemic shed light on the presence of a trade-off between the need of saving lives through health policies and the avoidance of the economic collapse, as a consequence of these policies. In this sense, Atkeson (2020) and Loayza (2020) warn how, when designing any possible public intervention, governments should always take into account the existence and the evolution of such a trade-off.

A number of contributions investigate the effects of the pandemic shock within a business cycle framework. Corrado et al. (2021) interpret pandemic-induced recession as the consequence of standard demand and supply shocks, exacerbated by adverse sector-specific disturbances in contact-intensive industries. Eichenbaum et al. (2022) highlight how a key feature of the Covid shock is that it acts like a negative shock to the demand for consumption and the supply of labor.
Guerrieri et al. (2022) show that a supply-side shock (i.e. a sectoral shutdown shock) can trigger a negative demand effect in other sectors, due to the complementarity that potentially arises in a multi-sector environment. In a similar vein, Baqaee and Farhi (2022) show that complementarities in production amplify Keynesian spillovers from supply shocks but mitigate them for demand shocks, and argue that for this reason demand stabilization policies in response to Covid-19 were relatively less effective. Buera et al. (2021) model the lockdown as an exogenous shutdown to a subset of entrepreneurs that operate in the economy, and impose an exogenous productivity increase for online trading firms to match the observed sectoral reallocation away from contact-intensive activities. Bayer et al. (2020), Elenev et al. (2020), and Faria-e Castro (2021) investigate the effectiveness of several fiscal tools, for example a decrease in income taxation, or an expansion of unemployment insur-

ance. All these studies maintain the focus on demand-side policies and a concern for the stabilisation objectives that characterise macroeconomic these policies.
By contrast, Loayza and Pennings (2020) and Dupor (2020) point out that a macroeconomic stimulus aimed at propelling aggregate demand may not necessarily be the best choice in the middle a pandemic, i.e. when the policy maker's preeminent goal is avoiding the spread of the disease.

We design a model that incorporates the trade-off between stabilising consumption and mitigating the shock through a contraction of economic activity. First, we follow Corrado et al. (2021) in modelling the pandemic shock as a systemic labor supply shock, but we also allow for the endogenous mitigation of the shock conditionally to a reduction of contact-intensive activities in total economic activity. Second, we allow for the possibility that profit-maximizing firms reallocate retail trading from contact-intensive to online activities. Third, our model accounts for the sharp increase in health expenditures after the shock. To the best of our knowledge, this is a new amplification channel of the macroeconomic adjustment to the shock.
Fourth, we investigate the design of Ramsey-optimal fiscal policies, where the planner relies on two tools, a sectoral production subsidy and an income tax rate. Our focus here is on the identification of policies that should mitigate the effects of the shock and favor consumption stabilization through a reallocation of retail trade towards online activities. The scope for fiscal intervention arises because retail trade through contact-intensive technologies generates a negative externality on the persistence of our proxy for the pandemic shock. Note that the income tax tool might be interpreted as a proxy for lock-down policies.

Our results in a nutshell. In the private sector equilibrium the pandemic shock generates a persistent contraction, and the reallocation away from contact-intensive trade is almost nil because both trades contract in a similar way. The increase in health expenditure triggers a strong contraction of private consumption through the standard crowding-out effect. Even if there is no sectoral reallocation, the fall in contact-intensive retail trade mitigates the shock. Relative to the market equilibrium, the Ramsey-optimal policy substantially mitigates the shock without exacerbating output losses. This is obtained combining a persistent stimulus to online trading with a contractionary increase in taxation.

We contribute to a rapidly growing literature that investigates the normative implications of the Covid-19 pandemic. A strand of literature focuses on the role of age-specific socioeconomic interactions to examine the effect of different containment measures on the spread of the pandemic. For example, Favero et al. (2020) and Rampini (2020) propose models which take heterogeneity in the population (in terms of different risk levels related to age and sectors) into account. The results claim that prudent policies of gradual return to work may save many lives with limited economic costs, as long as they differentiate by age group and risk sector. In a similar vein, Giagheddu and Papetti (2020) and Acemoglu et al. (2020) highlight how uniform social distancing measures are less effective compared with age-targeted measures. These papers focus on the effectiveness of age related containment measures, while our work is mainly interested in reallocation policies which can mitigate the pandemic shock.

Dealing with supply-side policies, Hubbard and Strain (2020a), Hubbard and Strain (2020b) and Hanson et al. (2020) argue that, in order to avoid firms bankruptcies, governments should provide financial aid in the form of grants to small firms (more likely to face a permanent revenue loss due to Covid-19), while should prefer loans to big firms, whose liquidity problems can be attenuated by greater internal resources and access to financial markets. While this literature mainly focuses on temporary financial measures, our work offers an alternative scope for supply-side interventions and highlights the advantages, both in terms of shock mitigation and of economic recovery, of providing an incentive to online trade.

Another strand of literature integrates macroeconomic and epidemiological models. Farboodi et al. (2021) and Eichenbaum et al. (2021) investigate how the pandemic shock affects households' economic incentives. Krueger et al. (2020) argue that the composition of the households' consumption bundle endogenously tilts towards less contact intensive sectors. In a similar vein, Alvarez et al. (2021) study the optimal lockdown policy to control the fatalities of a pandemic while minimizing the output costs of the lockdown; while their findings prescribe a tight initial lockdown, our model shows that the exploitation of online trade translates into less severe lockdowns policies.

We share with these works the importance of considering the existence of the health-economy trade-off when implementing lockdowns or other public health policies. Moreover, another commonality is that individual responses to the shock do not fully internalise their effects on the persistence of the pandemic shock.

The remainder of the chapter is organized as follows: section 1.2 describes the model, section 1.3 provides information on calibration and presents the results; section 1.4 concludes.

1.2 The model

The main actors in our model economy are: households, intermediate firms, final firms and the public sector. The model embeds frictions, both nominal and real: there are price stickiness and some level of rigidity in the reallocation of labor.

Households consume C_t, save through government bonds and supply differentiated labor services $N_{rf,t}$, $N_{ro,t}$ and $N_{I,t}$. Perfectly competitive intermediate firms sell their goods $S_{Irf,t}$ and $S_{Iro,t}$ to final firms, who are monopolistically competitive and face price rigidities; to produce their output, final firms can exploit two different technologies, a physical $S_{rf,t}$ (contact-intensive) and an online one $S_{ro,t}$.

The public sector provides two different types of subsidies: the first is aimed at offsetting the distortion involved by the presence of imperfect competition in the final market, while the second, ν_t is destined to boost the online production. Following the occurrence of the pandemic shock, the online subsidy is deployed as a tool for the containment of Covid-19, as it creates an incentive for a stronger utilisation of the less contagious channel (online). Moreover, the government levies labor income taxation t_t and issues public debt.

Aside the online subsidy, the model also allows for public health services expenditure g_t (financed through debt and labor income taxation), whose demand is an increasing function of the severity of the pandemic shock. Intermediate firms, in this context, produce health services $S_{Ig,t}$ in order to satisfy the public demand.

Finally, the economy is hit by a pandemic shock, whose persistence is endogenous to the share of contact-intensive productive activities.

The model economy is summarised in the flow diagram of Figure 1.1.

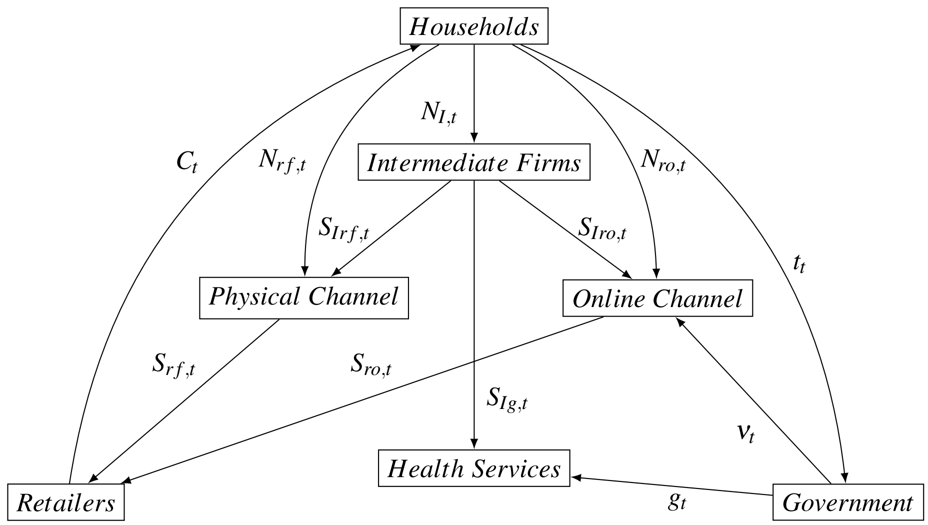

Figure 1.1: Flow diagram of the stylised economy

1.2.1 Intermediate Firms

The intermediate sector is characterised by the presence of fully competitive firms producing intermediate goods which will be used both as production inputs by the final sector and to produce public health services.
Firms have access to the following production function:

$$S_{I,t} = AN_{I,t}^{\alpha} \tag{1.1}$$

where $S_{I,t}$ is the intermediate output , $N_{I,t}$ is the labor used as productive factor in the intermediate production and A defines the level of productivity. Intermediate firms are subject to decreasing returns to scale, as $\alpha < 1$.
In each period, firms maximise their profits:

$$\Pi_{I,t} = p_{I,t}S_{I,t} - w_{I,t}N_{I,t} \tag{1.2}$$

where $p_{I,t}$ is the relative price of the intermediate good and $w_{I,t}$ is the real wage paid to workers in the intermediate sector.

The solution of the problem provides the optimal demand for intermediate labor:

$$w_{I,t} = p_{I,t}\alpha A N_{I,t}^{\alpha-1} \tag{1.3}$$

1.2.2 Final Firms

The final good sector is composed by a number j of producers, who operate in a monopolistically competitive market and face price adjustment costs à la Rotemberg (1982).
In our model households treat the goods sold through the two alternative channels as perfect substitutes; therefore, the choice of the optimal bundle of physical and online quantities is completely made by the firm. We make this assumption because we are specifically interested in the analysis of supply-side policies and in understanding the response of the firms to the pandemic shock.
Our approach is therefore different from Krueger et al. (2020), who posit that an exogenous fraction of goods in the consumption bundle is associated to a lower probability of infection than the rest of the bundle.

In order to produce their output, final firms have access to two different production functions:

$$S_{i,t}^{j} = \left[\left(\frac{N_{i,t}^{j}}{\tau_i} \right)^{\alpha_r} \left(S_{Ii,t}^{j} \right)^{1-\alpha_r} \right]^{\theta} \tag{1.4}$$

where $i \in \{rf, ro\}$ denotes the physical/contact-intensive and the online technologies, $S_{i,t}^{j}$ is final sectoral production, $N_{i,t}^{j}$ are the quantities of contact-intensive and online worked hours needed by the firms, while τ_i are the related production costs.
The parameter $\theta < 1$ characterises decreasing returns to scale in both technologies.

Finally, firm j total output S_t^j is equal to the sum of the physical and the online outputs:

$$S_t^j = S_{rf,t}^j + S_{ro,t}^j \tag{1.5}$$

Price rigidities

In each period firms maximise their profits Π_t^j subject to a price adjustment cost,
$\Phi_t = \frac{\gamma}{2}\left(\frac{P_t^j}{P_{t-1}^j} - 1\right)^2 S_t$, and to the demand function:

$$S_t^j = S_t \left(\frac{P_t^j}{P_t}\right)^{-\psi} \tag{1.6}$$

where the parameter ψ is the price elasticity of demand.

Firm j profit function is:

$$\Pi_t^j = \frac{P_t^j}{P_t} S_t^j - (1-\omega)\left[(1-\nu_t)\left(w_{ro,t}N_{ro,t}^j + p_{I,t}S_{Iro,t}^j\right) - \left(w_{rf,t}N_{rf,t}^j + p_{I,t}S_{Irf,t}^j\right)\right] - \Phi_t \tag{1.7}$$

Final producers receive from the public sector a subsidy ν_t, whose aim is providing an incentive to a larger utilisation of the online technology. The Ramsey planner intervenes in the economy to correct every type of inefficiency implied by the model structure. Since we want to isolate the planner's intervention targeting uniquely the negative impact of the shock, we need to clean the model from any other possible distortion. This is why we introduce a fixed public subsidy ω to the marginal costs, whose goal is offsetting the inefficiency implied by monopolistic competition in steady state. [1]

[1] Firms produce until, at the zero inflation steady state, it holds that $MC = \left(\frac{\psi-1}{\psi}\right)\frac{1}{(1-\omega)}$. Hence, we set ω

Cost minimisation

Firms optimally choose $N^j_{rf,t}, N^j_{ro,t}, S^j_{Irf,t}, S^j_{Iro,t}$ so that the physical and online labor demands are:

$$N^j_{rf,t} = \frac{\theta \alpha_r MC^j_t S^j_{rf,t}}{(1-\omega) w_{rf,t}} \tag{1.8}$$

$$N^j_{ro,t} = \frac{\theta \alpha_r MC^j_t S^j_{ro,t}}{(1-\omega)(1-\nu_t) w_{ro,t}} \tag{1.9}$$

while, the physical and online demands for intermediate inputs are:

$$S^j_{Irf,t} = \frac{\theta (1-\alpha_r) MC^j_t S^j_{rf,t}}{(1-\omega) p_{I,t}} \tag{1.10}$$

$$S^j_{Iro,t} = \frac{\theta (1-\alpha_r) MC^j_t S^j_{ro,t}}{(1-\omega)(1-\nu_t) p_{I,t}} \tag{1.11}$$

The first order conditions (1.8), (1.9), (1.10) and (1.11) show that the demands for factors are directly correlated with the sectoral output produced, while inversely related to the price of the factors.

According to the cost minimisation problem, we derive the firm's marginal cost:

$$MC_{i,t} = (1-\omega)\frac{1}{\theta}(1-\nu_t)(\tau_i)^{\alpha_r}\left(\frac{w_{i,t}}{\alpha_r}\right)^{\alpha_r}\left(\frac{p_{I,t}}{(1-\alpha_r)}\right)^{1-\alpha_r}(S_{i,t})^{\frac{1-\theta}{\theta}} \tag{1.12}$$

so that the effect of the markup is eliminated as if the market would be characterised by perfect competition.

The marginal cost is an increasing function of the factor prices (wages and intermediate good prices), of the production costs τ_i and of the sectoral production. In the same vein as the demands for productive factors, the marginal cost related to online production is affected by the subsidy ν_t.

Finally, cost minimisation implies that $MC_{rf,t} = MC_{ro,t}$. Moreover, using equation (1.5), the total marginal cost can be written as:

$$MC_t = (1-\omega)\frac{1}{\theta}(1-\nu_t)(\tau_{ro})^{\alpha_r}\left(\frac{w_{ro,t}}{\alpha_r}\right)^{\alpha_r}\left(\frac{p_{I,t}}{(1-\alpha_r)}\right)^{1-\alpha_r}\left(\frac{S_t}{1+\left[(1-\nu_t)\frac{(\tau_{ro})^{\alpha_r}}{(\tau_{rf})^{\alpha_r}}\left(\frac{w_{ro,t}}{w_{rf,t}}\right)^{\alpha_r}\right]^{\frac{\theta}{1-\theta}}}\right)^{\frac{1-\theta}{\theta}} \tag{1.13}$$

Optimal price setting

The solution to the optimal price setting problem yields the standard New Keynesian Phillips Curve:

$$(1-\psi)+\psi MC_t+\gamma\mathbb{E}_t\Lambda_t\left[(\pi_{t+1}-1)\pi_{t+1}\frac{S_{t+1}}{S_t}\right]=\gamma(\pi_t-1)\pi_t\frac{S_{t+1}}{S_t} \tag{1.14}$$ [2]

[2] The NKPC can be written also as:

$$(1-\psi)+\psi(1-\omega)MC_t^m+\gamma\mathbb{E}_t\Lambda_t\left[(\pi_{t+1}-1)\pi_{t+1}\frac{S_{t+1}}{S_t}\right]=\gamma(\pi_t-1)\pi_t\frac{S_{t+1}}{S_t}$$

where

$$MC_t^m = \frac{1}{\theta}(1-\nu_t)(\tau_{ro})^{\alpha_r}\left(\frac{w_{ro,t}}{\alpha_r}\right)^{\alpha_r}\left(\frac{p_{I,t}}{(1-\alpha_r)}\right)^{1-\alpha_r}\left(\frac{S_t}{1+\left[(1-\nu_t)\frac{(\tau_{ro})^{\alpha_r}}{(\tau_{rf})^{\alpha_r}}\left(\frac{w_{ro,t}}{w_{rf,t}}\right)^{\alpha_r}\right]^{\frac{\theta}{1-\theta}}}\right)^{\frac{1-\theta}{\theta}}$$

is the standard monopolistic competition marginal cost without the subsidy ω.

1.2.3 Households

Households preferences are defined over consumption C_t and labor effort, which can be divided in three different types: intermediate $N_{I,t}$, contact-intensive $N_{rf,t}$ and online $N_{ro,t}$. The representative households' lifetime utility function $U_t(C_t, N_{rf,t}, N_{ro,t}, N_{I,t})$ is akin to Moura (2018) and defined as:[3]:

$$\sum_{t=0}^{\infty} E_t \beta^t \left\{ \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{\alpha_t^N}{1+k} \left[\chi_1 \left(N_{rf,t} \right)^{1+\eta} + \frac{\chi_2}{\chi} \left(N_{ro,t} \right)^{1+\eta} + \chi_3 \left(N_{I,t} \right)^{1+\eta} \right]^{\frac{1+\kappa}{1+\eta}} \right\} \tag{1.15}$$

where β is the subjective discount factor, σ is the intertemporal elasticity of substitution, α^N is the pandemic shock, which will be discussed later in detail. The specification of the labor bundle implies reallocation rigidities, and hence imperfect labor mobility, when $\eta > 0$. This would introduce heterogeneity in wages and hours worked. The parameter κ measures the aggregate elasticity of labor supply and χ_1, χ_2 and χ_3 are weights attached respectively to the physical, online and intermediate labor.

We consider two different specifications of the utility function: the parameter χ is set equal to 1 for a shock that is symmetric to every labor type; otherwise, we set $\chi = (\alpha_t^N)^{\frac{1+\kappa}{1+\eta}}$. This latter case allows to model a scenario where the private sector internalises the benefits from avoiding contact-intensive activities. This is akin to Krueger et al. (2020), where households internalise -even in the market economy equilibrium- the different likelihood of being infected as a consequence of their consumption choices. More precisely, they describe a scenario with different infection probabilities according to the different sector, i.e. one sector is more contact-intensive and infectious than the other. With the second specification of equation (1.15) we are able to replicate this dynamic, even if our reallocation mechanism operates through the labor supply and not through consumer choices.

Nevertheless, we decided to consider the second case as an alternative and to use as benchmark the specification with $\chi = 1$, where the Covid-19 shock affects every type of labor. This because we envisage the pandemic not uniquely as a sectoral phenomenon, but instead as a shock dif-

[3]When the value of the parameter σ is equal to one, equation (1.15) is logarithmic in consumption C_t

fused to the entire economy. In this sense, also a less contact-intensive sector may be affected by negative spillovers coming from the interaction of its labor force with that of the other, more contagious, sectors, for example in a domestic environment. In order to sharpen our analysis and results, we also simulate the model considering the asymmetric shock (i.e., the second specification for χ) to check whether the market equilibrium optimally reallocates part of the production, without the intervention of the planner.

The budget constraint is:

$$C_t + b_t = \frac{R_{t-1} b_{t-1}}{\pi_t} + (1 - t_t)(w_{rf,t} N_{rf,t} + w_{ro,t} N_{ro,t} + w_{I,t} N_{I,t}) + \Pi_t \tag{1.16}$$

Where b_{t-1} is the real stock of government bond the households hold, R_{t-1} is the interest rate, b_t is the purchase of real public bonds, $w_{rf,t}$, $w_{ro,t}$ and $w_{I,t}$ are real wages paid for, respectively, physical ($N_{rf,t}$), online ($N_{ro,t}$) and intermediate ($N_{ro,t}$) labors. Finally, π_t is the inflation rate, t_t are distortionary income taxes and Π_t are firm real profits.
The households optimally choose, through maximisation of (1.15) subject to (1.16), the sequence of the allocation of $\left\{C_t, b_t, N_{rf,t}, N_{ro,t}, N_{I,t}\right\}_{t=0}^{\infty}$. This yields the Euler equation for consumption and the labor supplies for each sector:

$$\frac{1}{R_t} = \beta \left[\pi_{t+1}^{-1} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] \tag{1.17}$$

$$(1 - t_t) w_{rf,t} = \left(\alpha_t^N\right) \chi_1 (N_{rf,t})^{\eta} (C_t)^{\sigma} \tag{1.18}$$

$$(1 - t_t) w_{ro,t} = \left(\alpha_t^N\right) \chi_2 (N_{ro,t})^{\eta} (C_t)^{\sigma} \tag{1.19}$$

$$(1 - t_t) w_{I,t} = \left(\alpha_t^N\right) \chi_3 (N_{I,t})^{\eta} (C_t)^{\sigma} \tag{1.20}$$

The first order conditions (1.18), (1.19) and (1.20) highlight how the shock α^N directly affects the disutility of labor. Hence, as a consequence of the pandemic, households will be less willing to supply worked hours unless they receive an higher wage.

1.2.4 The Covid-19 shock

To mimick the effect and the endogenous persistence of the pandemic, we need to model a shock incorporating:

- an increase in the disutility of labor, i.e. households should be less willing to supply their labor in consequence of the pandemic, as in Corrado et al. (2021);
- an endogenous persistence mechanism, that should be related to the dynamics of contact-intensive activities. We therefore assume the following

$$\alpha_t^N = \left(\alpha_{t-1}^N\right)^\rho \left(\frac{S_{rf,t}}{S_{rf,t-1}}\right)^{\Delta(1-\rho)} \exp \varepsilon_t \tag{1.21}$$

where ρ is the exogenous persistence of the process, $\left(\frac{S_{rf,t}}{S_{rf,t-1}}\right)$ proxies the endogenous persistence channel, due to contact-intensive activities, and Δ allows to characterize the strength of this latter mechanism.
The underlying intuition is that the growth of contact-intensive retail trade raises possibility of getting infected, and therefore adversely affects the supply of worked hours. In consequence of this shock modelisation, the Ramsey planner is confronted with a trade-off between stabilising consumption and mitigating the shock through a contraction of economic activity.

1.2.5 Public sector

The government's budget constraint is:

$$v_t S_{ro,t} + \frac{R_{t-1} b_{t-1}}{\pi_t} + g_t = t_t (w_{rf,t} N_{rf,t} + w_{ro,t} N_{ro,t} + w_{I,t} N_{I,t}) + b_t \tag{1.22}$$

The public sector needs to levy taxes on the labor income and to issue new debt in order to repay interest on past debt, to finance the provision of the online subsidy and the supply of health services, g_t.
[4]

Public expenditure

In order to face the outbreak of Covid-19, the public sector needs to increase its expenditure on health and sanitary goods and services as documented in Mendoza et al. (2020). The demand for public expenditure is:

$$g_t = \bar{g} + \left[1 - \left(\frac{\bar{\alpha}^N}{\alpha_{t-1}^N} \right)^{\phi_g} \right] \tag{1.23}$$

where $\bar{g}$ is the steady state (or non-pandemic) level of public expenditure, $\bar{\alpha}^N$ is the steady state value of the shock and ϕ_g represents the elasticity to the shock α^N.
The supply of g_t is:

$$g_t = (S_{Ig,t})^{\alpha_g} \tag{1.24}$$

where $S_{Ig,t}$ is the fraction of intermediate input devoted to the production of the public good. The parameter α_g defines decreasing returns to scale, as it is lower than 1; moreover, we assume that this type of production presents returns that decrease faster with respect to those in the final good sector.
Assuming such a productive structure for g_t has an impact on the aggregation of the intermediate good $S_{I,t}$, as:

$$S_{I,t} = S_{Irf,t} + S_{Iro,t} + S_{Ig,t} \tag{1.25}$$

Following the shock, the increase in g_t will trigger an endogenous reallocation effect, because the relative price of intermediate goods will increase. Thus the the presence of Covid-related public expenditures will generate an additional supply-side effect.

[4] We posit that the government finances ω by means of lump-sum taxes. For the sake of simplicity, we remove from (1.22) both these subsidies and the revenues from lump-sum taxes that finance them.

1.2.6 Market clearing

An aggregate resource constraint closes the model, as aggregate production S_t has to cover not only the level of consumption C_t, but also needs to take into account the presence of public expenditure; hence:

$$S_t = C_t + g_t \tag{1.26}$$

1.2.7 Monetary and fiscal policies

The Ramsey planner

Provided the two available instruments to fight the pandemic, we evaluate the optimal level of these tools through a standard Ramsey problem. Hence, the Ramsey planner maximises households' expected utility of equation (1.15), subject to: the firms equilibrium conditions (1.1), (1.3), (1.4), (1.8) - (1.11), (1.13), (1.14), to the households equilibrium conditions, equations (1.17) - (1.20), to the public sector equilibrium conditions (1.22) - (1.24), (1.27), (1.28); to the market clearing condition (1.25) and to the aggregate resource constraint (1.26).

In our experiments, the planner always controls the online subsidy v_t as an instrument, and the planner can also optimally set labor taxation. In this sense, taxes are a proxy for administrative lockdown policies. We take here inspiration from Eichenbaum et al. (2022), who defines the lockdown by means of a tax on consumption.

To better illustrate the distinct contributions of the two policy tools in the Ramsey-optimal plan, we also consider the possibility that the tax rate follows a simple rule aiming at control of public debt, as advocated in Schmitt-Grohé and Uribe (2004):

$$\frac{t_t}{\bar{t}} = \left(\frac{b_{t-1}}{\bar{b}}\right)^{\xi} \tag{1.27}$$

where $\bar{t}$ and $\bar{b}$ are respectively the steady state levels of income taxation and public debt. ξ defines the intensity of the reaction of taxation to debt accumulation.

Monetary policy

The monetary authority is assumed to follow a standard Taylor rule:

$$\frac{R_t}{\bar{R}} = (\pi_t)^{\theta_\pi} \left(\frac{S_t}{\bar{S}}\right)^{\theta_S} \tag{1.28}$$

where $\bar{S}$ is the steady-state level of output.

1.3 Results

1.3.1 Model calibration

Households preferences are quite standard, the discount factors β is 0.99 and the elasticity of intertemporal substitution σ is equal to 1; both parameters that characterize labor market frictions, κ and η, are assumed to be equal to 2, following Moura (2018).
We set the steady state ratio between physical and online production, $\frac{Srf}{Sro}$, equal to six; this indicates that the contact-intensive output accounts for the majority of total output and is in line with what observed from US Census Bureau (2022).
We opted for a conservative choice concerning the level of debt, which is equal to 80% of GDP, on annual basis. This pin downs a tax rate of 4%. The aggregate labor supply $N_{rf} + N_{ro} + N_I$ is assumed to be equal to 1 in steady state. We calibrate the three weights χ_1, χ_2, χ_3 attached to each labor type consequently.
The level of the intermediate productivity A is assumed to be equal to 1, and this yields a steady state value of the intermediate production S_I equal to one fifth of total production S.
Concerning the shock, ε is chosen in order to simulate an initial 10% drop in the aggregate production in the market equilibrium. The parameter $\Delta = 8$ defines the sensitivity of the shock to the share contact-intensive output. This implies that the higher is Δ, the stronger will be

the planner's incentive to intervene in the economy.
With respect to production and labor markets, the share of labor in the final production function is assumed to be $\alpha_r = 0.66$.
Intermediate and final productions are subject to decreasing returns to scale, as $\alpha = \theta = 0.87$; this follows Basu and Fernald (1997).
With respect to public expenditures, we set $\alpha_g = 0.7$ in order to obtain a persistent reaction of health services to the shock. We opted for this suggestive calibration to underline the importance of accounting for long-term Covid-19 expenses in the model (National Institute Health Care Excellence (2022)).
Monetary policy parameters are $\theta_\pi = 1.5$ and $\theta_S = 0.2$, in line with the priors form Christiano et al. (2014).
Finally, the steady state value of the online subsidy ν is zero.
Table 1.1 summarises the calibration.

(i) Parameter	Value	Definition
β	0.99	Households discount factor
σ	1	Elasticity of intertemporal substitution
κ	2	Aggregate elasticity of labor supply
η	2	Reallocation cost for labor
α	0.87	Returns to scale intermediate production
A	1	Productivity
α_r	0.66	Share of labor in final production
θ	0.87	Returns to scale final production
γ	18.5	Rotemberg menu cost
ψ	6	Price elasticity of demand
ξ	1.2	Intensity of tax reaction to debt accumulation
θ_π	1.5	Taylor rule: inflation
θ_S	0.2	Taylor rule: output
α_g	0.7	Persistence of health expenditures
ρ	0.9	Shock persistence
Δ	8	Sensitivity of the shock to variation in $S_{rf,t}$

(ii) Steady State	Value	Definition
$\frac{Srf}{Sro}$	6	Ratio physical to online output
$\frac{B}{S}$	80%	Debt to GDP ratio
t	4%	Labor income tax rate
ν	0	Online production subsidy

Table 1.1: *(i) Main parameters (ii) Steady state values*

1.3.2 Model dynamics

Our benchmark for policy analysis is the market equilibrium where there is no public intervention and taxes follow the simple heuristic rule of equation (1.27). We also consider the possibility that households internalise the benefits from supplying labor to the online sector (asymmetric pandemic shock). Finally, we consider the presence of health goods expenditure in the model.

We adopt a Ramsey-optimal approach [5], as described in section 1.2.7, in order to investigate how the planner would intervene in response to the pandemic. Thus, we simulate the Covid-19 shock affecting the economy and, in addition, we consider two different specifications of the set of instruments of the Ramsey planner (i.e., online subsidy alone and a combination of subsidy and labor taxation), in order to analyse the optimal supply-side fiscal policies.
We start presenting the results (Figures 1.2 and 1.3) for the simulation for a shock calibrated to reproduce a 10% drop in aggregate output, without health expenditure. We opt for this choice because we first want to focus on the impact of the two fiscal tools controlled by the planner. After that, we add public expenditure to investigate how the economy behaves with this additional fiscal instrument, which is not directly under the control of the planner.

Consider first the market equilibrium. According to condition (1.15), the shock affects every type of labor, making the households less willing to supply their labor. We observe a generalized contraction. Firms do not internalize the effect of contact-intensive retail trade on the shock persistence, so there is no incentive to reallocate towards online trade.
The Ramsey planner faces two different objectives: mitigating the economic recession, in order to stabilise consumption, while trying to contain the spread of the disease. When the planner only relies on the subsidy, she will try to induce labor and production reallocation towards the online industry.
The provision of a positive online subsidy is successful in achieving a reallocation of worked hours towards the online sector, while the contact-intensive and intermediate productions still decline. This shift dampens the fall in aggregate output and implies a reduction in the persistence of

[5] The Ramsey optimal policy is computed using a first-order approximation with Dynare 4.6.4

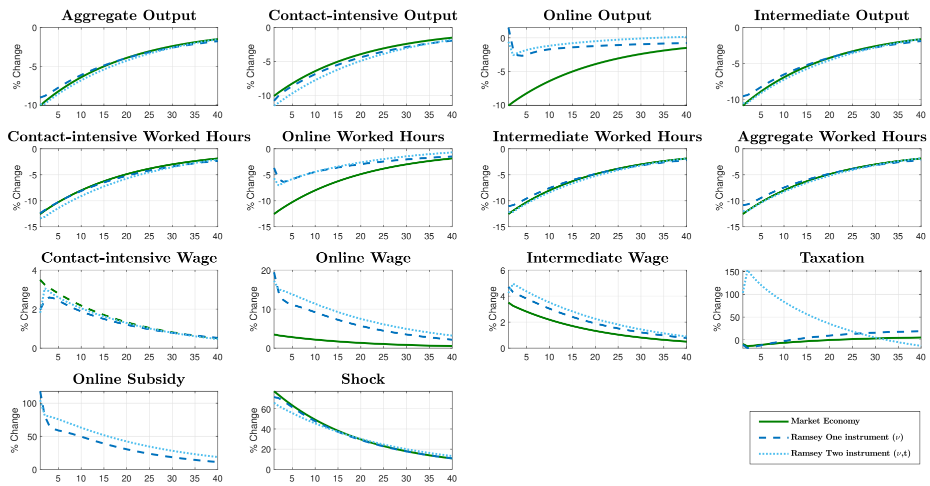

Figure 1.2: *IRFs to a pandemic shock, percentage deviations from steady state*

the shock, i.e. achieve a more effective containment of the pandemic.

Adding the labor income tax to the set of planner's policy tools allows to strengthen the shock mitigation effect. This is achieved by means of a relatively large and persistent increase in the tax rate. With the reallocation effect still valid, the economy experiences a contraction in aggregate output, due to a tighter fiscal regime: the increase in the level of taxation reduces the households purchasing power and consequently generates a reduction in consumption. This scenario confirms that, when an economy is in the middle of a pandemic, a contraction of output, necessary to obtain a more powerful mitigation effect, raises welfare.
This is also confirmed by the comparison of the welfare spreading from the different policies: as a matter of fact, we computed the consumption equivalents relative to the two Ramsey policies and we compare them with the baseline market economy. The results show that the scenario where the Ramsey planner intervenes in the economy with two instruments (online subsidy and the labor income taxation) is the one which attains the highest level of welfare. Moreover, the computations return a positive value for the consumption equivalent in both the Ramsey alter-

native policies (with one or two instruments), with respect to the market economy; specifically, households are willing to give up respectively 19% (for the Ramsey one-instrument case) and 20% (for the two- instruments case) of their consumption in order to remain in one of the two alternative regimes, instead of the market economy. Hence, even if the magnitude of this difference is not sizeable, we can still claim that two-instruments Ramsey scenario is the one providing the highest welfare benefit.

Finally, to further inspect the mechanism, note that the total number of worked hours, in the market economy benchmark, decreases; this is a consequence of the negative impact of the Covid shock, which directly affects contact-intensive production. Moreover, since firms do not internalize the role played by contact-intensive production on the propagation of the disease, they do not reallocate towards the online technology, that decreases accordingly. Finally, intermediate production accommodates this contraction by decreasing its supply to final firms. Overall, this effects implies a general contraction of the aggregate level of worked hours. It is interesting to notice how the contraction in total worked hours is less pronounced in the Ramsey scenario with one instrument (i.e., the online subsidy), because, in this case, firms have a strong incentive to reallocate and expand the online channel. To do this, they need to employ more labor force and this, at the aggregate level, translate into a lower decrease in total hours. This mechanisms is, instead, less pronounced in the two-instruments case, due to the stronger contraction in contact-intensive worked hours, which partially offsets the increase in online worked hours. Moreover, in the model, we have three different real wages (one for each type of differentiated labor). The negative impact of the pandemic generates an overall contraction in the level of output and households are less willing to supply their labor force, due to the effect of the disease. Hence, firms have to increase the level of wages for every type of production in order to try meet their labor demand. However, the results show that, when the planner intervenes in the economy, there is a larger incentive towards the exploitation of the online technology (and, accordingly, of the online labor), which translates into a stronger increase in the level of online wages, such that the online technology is relatively more attractive for the households' labor supply.

Asymmetric shock

In order to corroborate the previous analysis, here we discuss the results obtained assuming that Covid-19 hits asymmetrically the households' labor supply functions. It is the case (described in section 1.2.3) where the utility function (1.15) assumes a values for χ equal to $(\alpha_t^N)^{\frac{1+\kappa}{1+\eta}}$. Such a design implies that the shock affects only two labor supplies, the contact-intensive and the intermediate. Figure 1.3 presents the results.

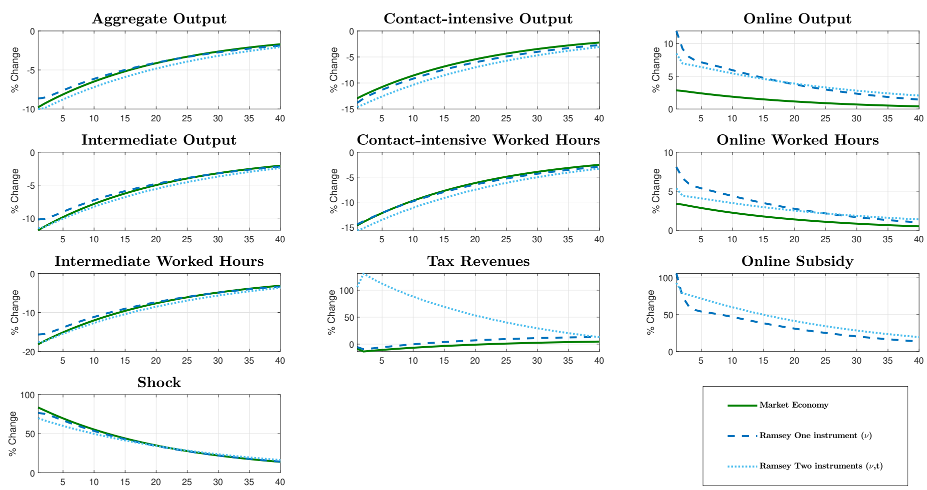

Figure 1.3: *IRFs to an asymmetric pandemic shock, percentage deviations from steady state*

The private sector now shifts the labor supply towards the online channel. Hence, the decentralised equilibrium now generates a relative expansion in the the online sector and a reduction in the intensity of the shock, since the economy is less dependent from contact-intensive activities.

The planner's intervention is coherent with the response to the symmetrical shock. This happens because the private sector does not internalise the impact on the persistence of shock implied by the contact-intensive trade.

1.3.3 Public expenditure for health services

We now presents the full version of the model, which features the presence of health expenditures. Figures 1.4 and 1.5 show the results.

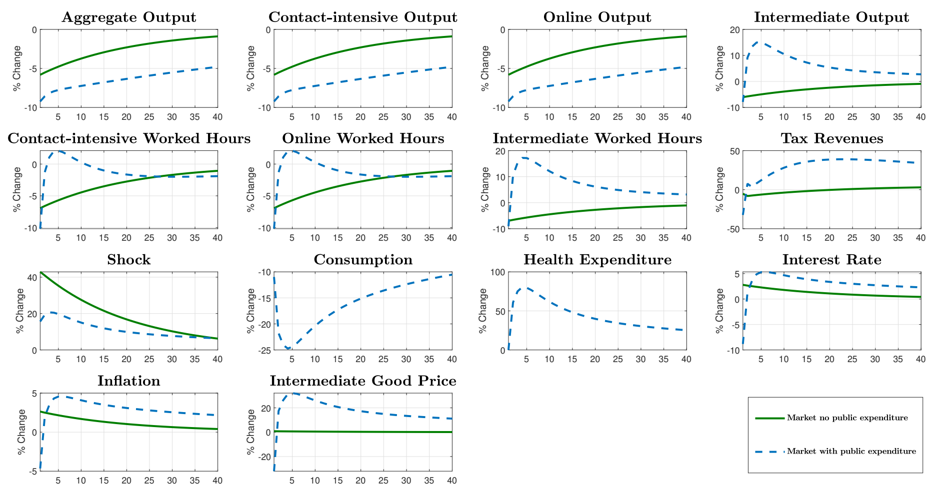

Figure 1.4: *IRFs to a pandemic shock, market economy with endogenous public health expenditures, percentage deviations from steady state*

The comparison between the two different market outcomes (Figure 1.4) clearly shows that the main mechanism driving this version of the model is based on the fact that, as a reaction to the pandemic outbreak, the public sector increases health expenditures (panel 11). As a consequence, the anticipation of higher future taxes induces households to reduce their private consumption (panel 10).
This additional reduction in consumption makes the economic recession to be more pronounced in the model embedding public expenditure. The crucial consequence is a more effective containment of the pandemic, due to a stronger contraction of productive activities.
There are other channels through which the increase in public expenditure affects the dynamics of the economy; first, if g_t raises, so does taxation that target public debt, causing an additional labor supply distortion which further depresses output.
Furthermore, fewer worked hours are allocated on contact-intensive and

online productions, in favour of the intermediate sector, creating a contractionary effect on final production. This creates a strong difference with respect to the case without g_t.
Finally, we also observe a monetary channel; as a matter of fact, in the simulation including public expenditure, both inflation and the interest rate grow more than in the other scenario. The increase in g_t boosts intermediate production, raising the relative price of the intermediate good (Panel 14).
The monetary contraction leads to a further demand fall, strengthening the economic recession, but also contributing to a more powerful mitigation of the pandemic.
Summing up, in the market equilibrium the endogeneity of public health expenditures triggers a deeper contraction because households anticipate higher future taxes. One unintended consequence is that this dampens the shock.

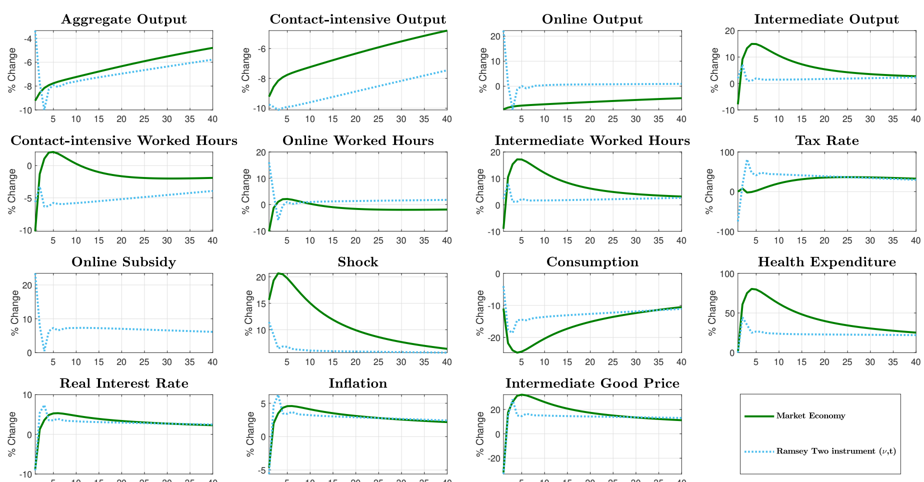

Figure 1.5: *IRFs to a pandemic shock, Ramsey two instruments with public expenditures, percentage deviations from steady state*

Figure 1.5 compares the Ramsey planner's equilibrium (with two instruments) with the market equilibrium with endogenous public expenditure. Considering the presence of g_t does not change the fact that the private

sector does not internalise the endogenous infection risk; this means that the Ramsey planner still has an incentive in intervening in the economy. Hence, the planner provides the online subsidy in order to reallocate the production and increase labor taxation to achieve the necessary drop in aggregate output.
The planner's intervention mitigates the shock, and reduces health expenditures. This consequently implies a lower drop of private consumption. In spite of the sharp acceleration in the containment of the shock, due to the penalisation of contact-intensive activities, the persistence of health expenditures slows down the overall adjustment process. When the reduction of contact-intensive activities has drastically reduced the shock, the planner generates tax revenues that match health expenditures dynamics. This is obtained by keeping the tax rate above steady state. The persistently high online subsidy reduces marginal costs in the retail sector, compensating for the inflationary effect of the higher tax rate.

1.4 Conclusions

In our model economy, the market equilibrium is inefficient because agents do not internalize the endogenous persistence of the shock, i.e. the market does not reallocate retail trade towards the online sector and away from contact-intensive activities. By including endogenous health expenditures, we highlight a hitherto unexplored channel that magnifies the contraction in economic activity. The model allows investigating how supply-side fiscal policies can affect the health-vs-economy trade-off that has characterized the debate about the optimal policy responses to the Covid-19 pandemic. The Ramsey planner improves the trade-off between macroeconomic stabilisation and infection mitigation, engineering a reallocation away from contact-intensive. In this regard, a production subsidy for online trade turns out to be very effective. To sharpen the analysis, we have followed a heuristic approach and opted for a highly stylized model. Our proposed set of policy interventions should be verified in models that integrate a richer macroeconomic structure with a more realistic characterization of the epidemiological aspects of the Covid-19 shock. We leave this for future research.

The Ramsey planner improves the trade-off between macroeconomic stabilisation and shock mitigation, engineering a reallocation away from contact-intensive activities, obtained with a subsidy provided to the online technology. Moreover, adding labor income taxation (a proxy for administrative lockdowns) to the planner's tools allows to achieve a stronger mitigation of the Covid-19 shock. The results show that a contraction in overall economic activities is necessary when the system is in the middle of a pandemic and hence a strong macroeconomic stimulus focused on boosting aggregate demand may not represent the best policy to fight the spread of Covid-19. In this spirit, the work sheds light on the importance of investigating supply-side policies oriented to a potential reallocation towards less contagious sectors. Considering public expenditure in the form of health services as a weapon to react to the pandemic generates a more effective virus containment; this happens mainly thanks to the reduction in private consumption. This implies a drop in the level of production, especially the contact-intensive one and, consequently, reduces the severity of the pandemic. In addition, combining the planner's intervention with the provision of health services leads to a stronger and more effective containment of the spread of Covid-19, without the exacerbation of output losses.

Finally, our paper provides a methodological contribution to the design of the Covid-19 shock in macroeconomic non-SIR models. As a matter of fact, in a Ramsey-optimal framework, our proposal successfully creates a trade-off for the planner, who has to decide the optimal policy to fight both the health effect of the spread of the disease and the economic recession. In this framework, thinking of Covid-19 as an sectoral adverse productivity shock (in line with Guerrieri et al. (2022)) fails in generating the sanitary/economic trade-off, leading to a non-intervention of the planner in the economy. We think that our proposal could be well-suited to investigate public policies with a Ramsey-optimal approach.

1.5 Appendix to Chapter 1

1.5.1 The role of labor reallocation frictions

In this section we simulate the model for different values of the parameter η, which controls the reallocation cost for labor. Figure 1.6 and 1.7 respectively present the results for the market equilibrium and for the Ramsey optimal policy (two instruments case).

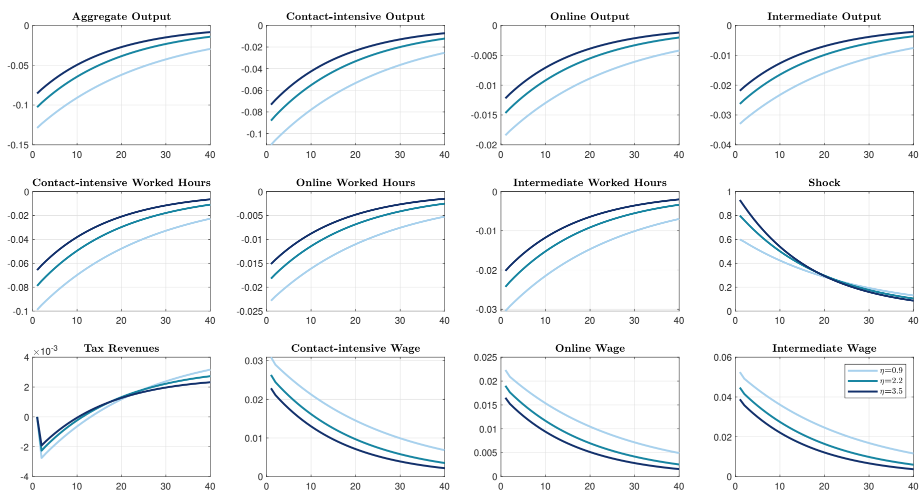

Figure 1.6: *IRFs to a pandemic shock for different levels of η, market economy*

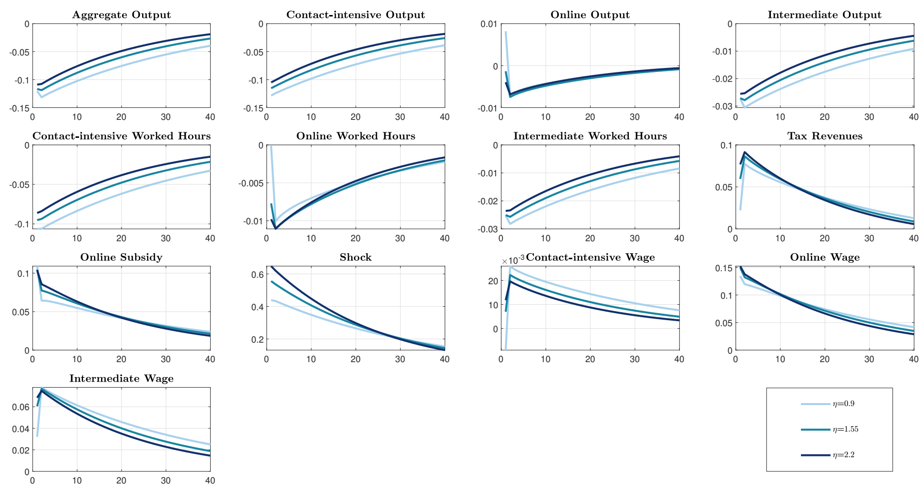

Figure 1.7: *IRFs to a pandemic shock for different levels of* η*, Ramsey two instruments*

The shock to the disutility of labor triggers a rise in the level of wages, for every type of labor, in order to compensate the households' lower propensity to supply worked hours. Firms demand less labor. Overall, this effect leads to a reduction in the number of worked hours. To understand how labor reallocation frictions affect the mechanism at play, recall the labor supplies of equations 1.18, 1.19 and 1.20. We can write them as[6]:

$$N_{rf,t} = \left(\frac{(1-t_t)w_{rf,t}}{\left(1+\alpha_t^N\right)\chi_1\left(S_t\right)^{\sigma}} \right)^{\frac{1}{\eta}} \tag{1.29}$$

An increase in labor market flexibility, i.e. a lower η, translates into a stronger effect of the impact of the shock on the labor supply.
As a consequence, this effect also leads to a more severe economic recession, which mitigates the impact of the shock on the economy. As

[6]We present here only the equation for the contact-intensive labor supply, but the same argument holds for the other types of labor

a matter of facts, for lower degrees of the reallocation frictions, the decentralised equilibrium is relatively more effective in managing the virus spread, through the stronger contraction in worked hours and aggregate output. As a consequence, the Ramsey planner decides for a weaker intervention. Our qualitative results are nevertheless confirmed.

1.5.2 Adverse productivity shock

In this section, we present evidence that, when adopting a Ramsey-optimal approach, designing the pandemic shock uniquely as an adverse productivity shock à la Guerrieri et al. (2022) fails in generating the trade-off between mitigating the pandemic and dampening the economic contraction and the planner's incentives are quite different.
In line with Guerrieri et al. (2022), we assume that physical production suffers an increase in its production cost, τ_{rf}, while online trade is unaffected. The shock thus takes the following form:

$$\tau_{rf,t} = (1-\rho)\bar{\tau}_{rf} + \rho\,\tau_{rf,t-1} + \varepsilon_{\tau} \tag{1.30}$$

where ε_{τ} is a white noise exogenous shock to the physical production cost and ρ indicates the persistence of the shock. The shock is calibrated in order to have a 10% drop in the aggregate output. Figure 1.8 presents the results.

All simulations show that the planner's intervention produces almost the same effects of those obtained through the market mechanism; as a matter of fact, the increase in labor cost triggers a contraction in the production of the contact-intensive output, because firms are less willing to employ contact-intensive worked hours. The possibility to shift towards the online technology partially contains the recession, but this transition mechanism is not strong enough to avoid the fall of aggregate output. The mechanism here is that firms, facing a purely economic shock, move their production towards the unaffected technology, since it is now characterised by a lower marginal cost.
It is worth noting the response of the planner to the shock, in panels 7 and 9. The variation in the levels of the two instruments is very small due to the planner's willingness to correct the inefficiency implied by the presence of price stickiness. The shock triggers an increase in the production costs and hence also the inflation rises accordingly. The monetary policy chosen by the central authority is not tight enough to counteract the

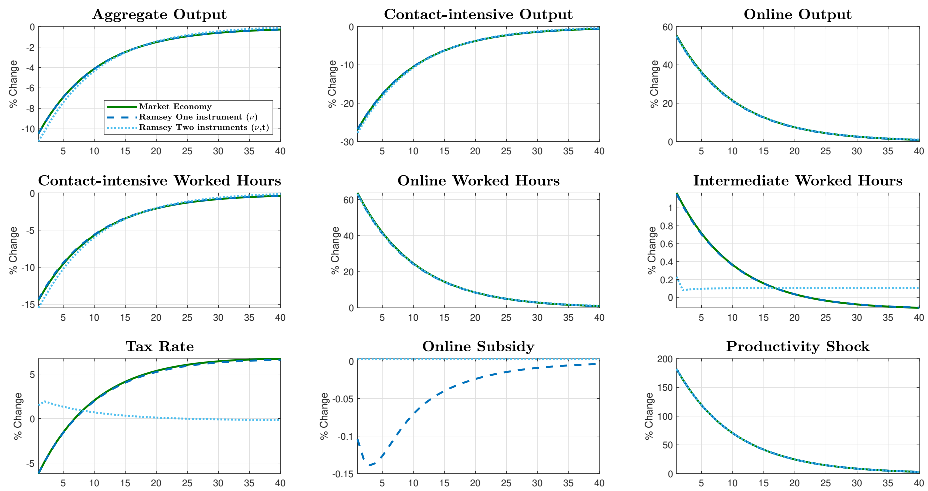

Figure 1.8: *IRFs to an adverse productivity shock, percentage deviations from steady state*

inflation, i.e. the increase in the interest rate is not high enough. Hence, the planner needs to intervene: through the negative online subsidy, in order to make this sector less productive and fight the increase in prices. Or, in the two instruments scenario, the intervention is obtained through taxation: in fact, the decreased overall productivity requires less labor effort.

1.5.3 Derivation of key equations

In this section the full derivation of the key equations is presented

Intermediate Firm

The problem of the intermediate firm is:

$$\max_{S_{I,t}} \Pi_{I,t} = p_{I,t}S_{I,t} - w_{I,t}N_{I,t}$$

$$s.t. \tag{1.31}$$

$$S_{I,t} = AN_{I,t}^{\alpha}$$

$\frac{\partial L}{\partial S_{I,t}} = 0$

$$\lambda_t = p_{I,t}$$

Recall that the Lagrangean multiplier λ_t can be seen as the marginal cost. Hence, $\lambda_t = MC_{I,t}$. This yields to the standard relation for perfect competition:

$$MC_{I,t} = p_{I,t} \tag{1.32}$$

Through cost minimisation, the demand for intermediate labor, $N_{I,t}$, is obtained as follows:

$$\max_{N_{I,t}} \Pi_{I,t} = p_{I,t}S_{I,t} - w_{I,t}N_{I,t}$$

$$s.t. \tag{1.33}$$

$$S_{I,t} = AN_{I,t}^{\alpha}$$

$\frac{\partial L}{\partial N_{I,t}} = 0$

$$w_{I,t} = p_{I,t}\alpha AN_{I,t}^{\alpha-1} \tag{1.34}$$

Final Firms

The problem of the final producer is:

$$\max_{N^j_{rf,t},N^j_{ro,t},N^j_{I,t},S^j_{Irf,t},S^j_{Iro,t},p^j_t} \Pi^j_t = \frac{P^j_t}{P_t} S^j_t - (1-\nu_t)\left(w_{ro,t}N^j_{ro,t} + p_{I,t}S^j_{Iro,t}\right)$$

$$-w_{rf,t}N^j_{rf,t} - p_{I,t}S^j_{Irf,t} - \frac{\gamma}{2}\left(\frac{P^j_t}{P^j_{t-1}} - 1\right)^2 S_t$$

$$s.t. \tag{1.35}$$

$$S^j_{rf,t} = \left[\left(\frac{N^j_{rf,t}}{\tau_{rf}}\right)^{\alpha_r} (S^j_{Irf,t})^{1-\alpha_r}\right]^{\theta}$$

$$S^j_{ro,t} = \left[\left(\frac{N^j_{ro,t}}{\tau_{ro}}\right)^{\alpha_r} (S^j_{Iro,t})^{1-\alpha_r}\right]^{\theta}$$

$$S^j_t = S^j_{rf,t} + S^j_{ro,t}$$

$$S^j_t = S_t \left(\frac{P^j_t}{P_t}\right)^{-\psi}$$

The Lagrangean is:

$$L = \frac{P^j_t}{P_t} S_t - (1-\nu_t)\left(w_{ro,t}N^j_{ro,t} + p_{I,t}S^j_{Iro,t}\right) - w_{rf,t}N_{rf,t} - p_{I,t}S^j_{Irf,t} - \frac{\gamma}{2}\left(\frac{P^j_t}{P^j_{t-1}} - 1\right)^2 S_t$$

$$-\frac{P^j_t}{P_t} MC_t \left[S^j_t - \left[\left(\frac{N^j_{rf,t}}{\tau_{rf}}\right)^{\alpha_r} (S^j_{Irf,t})^{1-\alpha_r}\right]^{\theta} - \left[\left(\frac{N^j_{ro,t}}{\tau_{ro}}\right)^{\alpha_r} (S^j_{Iro,t})^{1-\alpha_r}\right]^{\theta}\right]$$

The first order conditions are:

$\frac{\partial L}{\partial N^j_{rf,t}} = 0$

$$N_{rf,t}^{j}=\frac{\theta\alpha_r MC_t S_{rf,t}^{j}}{(1-\nu_t)\,w_{rf,t}} \tag{1.36}$$

$\frac{\partial L}{\partial N_{ro,t}^{j}}=0$

$$N_{ro,t}^{j}=\frac{\theta\alpha_r MC_t S_{ro,t}^{j}}{(1-\nu_t)\,w_{ro,t}} \tag{1.37}$$

$\frac{\partial L}{\partial S_{Irf,t}^{j}}=0$

$$S_{Irf,t}^{j}=\frac{\theta(1-\alpha_r)MC_t S_{rf,t}^{j}}{p_{I,t}} \tag{1.38}$$

$\frac{\partial L}{\partial S_{Iro,t}^{j}}=0$

$$S_{Iro,t}^{j}=\frac{\theta(1-\alpha_r)MC_t S_{ro,t}^{j}}{(1-\nu_t)\,p_{I,t}} \tag{1.39}$$

$\frac{\partial L}{\partial p_t^{j}}=0$

$$(1-\psi)\,S_t\frac{P_t^{j-\psi}}{P_t^{1-\psi}}-\gamma\left(\frac{P_t^{j}}{P_{t-1}^{j}}-1\right)\frac{1}{P_{t-1}^{j}}S_t+\gamma\left(\frac{P_{t+1}^{j}}{P_t^{j}}-1\right)\frac{1}{P_t^{j2}}S_{t+1}-\psi MC_t S_t\frac{P_t^{j-\psi-1}}{P_t^{-\psi}}=0$$

Consider a symmetric equilibrium where $P_t^j=P_t$ and set $\frac{P_t^j}{P_{t-1}^j}=\pi_t$

$$(1-\psi)\,S_t\frac{1}{P_t}-\psi MC_t S_t\frac{1}{P_t}-\gamma(\pi_t-1)\frac{1}{P_{t-1}}S_t+\gamma(\pi_{t+1}-1)\frac{1}{P_t^2}S_{t+1}=0$$

Now multiply for P_t and divide for S_t

$$(1-\psi)-\psi MC_t-\gamma(\pi_t-1)\pi_t+\gamma(\pi_{t+1}-1)\pi_{t+1}\frac{S_{t+1}}{S_t}=0$$

Hence, by considering the anti-monopolistic subsidy ω_t, we finally obtain:

$$(1-\psi)+\psi(1-\omega_t)MC_t+\gamma\mathbb{E}_t\Lambda_t\left[(\pi_{t+1}-1)\pi_{t+1}\frac{S_{t+1}}{S_t}\right]=\gamma(\pi_t-1)\pi_t\frac{S_{t+1}}{S_t} \tag{1.40}$$

Marginal cost

In order to derive the equation for the marginal cost, we consider the cost minimisation problem of the final firm:

$$\max_{N^j_{ro,t},S^j_{ro,t}}\Pi^j_t=\tfrac{P^j_t}{P_t}S^j_t-(1-\nu_t)\left(w_{ro,t}N^j_{ro,t}+p_{I,t}S^j_{Iro,t}\right)-w_{rf,t}N^j_{rf,t}-p_{I,t}S^j_{Irf,t}-\tfrac{\gamma}{2}\left(\tfrac{P^j_t}{P^j_{t-1}}-1\right)^2S_t$$

For the online production, we will have that the first order condition with respect to $N_{ro,t}$ is:

$$N_{ro,t}=\frac{\theta\alpha_r\lambda_{ro,t}S_{ro,t}}{(1-\nu_t)w_{ro,t}} \tag{1.41}$$

while the first order condition with respect to $S_{Iro,t}$ is:

$$S_{Iro,t}=\frac{\theta(1-\alpha_r)\lambda_{ro,t}S_{ro,t}}{(1-\nu_t)p_{I,t}} \tag{1.42}$$

Equating equations (1.41) and (1.42) (through the $\lambda_{ro,t}$) yields:

$$\frac{N_{ro,t}(1-\nu_t)w_{ro,t}}{\theta\alpha_rS_{ro,t}}=\frac{S_{Iro,t}(1-\nu_t)p_{I,t}}{\theta(1-\alpha_r)S_{ro,t}}$$

$$\frac{N_{ro,t}w_{ro,t}}{\alpha_r} = \frac{S_{Iro,t}\,p_{I,t}}{(1-\alpha_r)}$$

Express $S_{Iro,t}$ as a function of $N_{ro,t}$

$$S_{Iro,t} = (N_{ro,t})\,w_{ro,t}\frac{(1-\alpha_r)}{\alpha_r p_{I,t}} \tag{1.43}$$

Now, recall that the equation for the total (online) costs is:

$$TC_{ro,t} = (1-\nu_t)\,(w_{ro,t}N_{ro,t} + p_{I,t}SI_{ro,t}) \tag{1.44}$$

Now, substitute equation (1.43) into equation (1.44):

$$TC_{ro,t} = (1-\nu_t)w_{ro,t}N_{ro,t} + (1-\nu_t)p_{I,t}\,(N_{ro,t})\,w_{ro,t}\frac{(1-\alpha_r)}{\alpha_r p_{I,t}}$$

$$TC_{ro,t} = (1-\nu_t)w_{ro,t}N_{ro,t}\left(1+\frac{1-\alpha_r}{\alpha_r}\right)$$

$$TC_{ro,t} = \frac{(1-\nu_t)w_{ro,t}N_{ro,t}}{\alpha_r} \tag{1.45}$$

this is the real production cost.

Substitute again equation (1.43) into the online production function:

$$S_{ro,t} = \left[N_{ro,t} \left(\frac{1}{\tau_{ro}} \right)^{\alpha_r} \left(w_{ro,t} \frac{1-\alpha_r}{\alpha_r p_{I,t}} \right)^{1-\alpha_r} \right]^{\theta}$$

Therefore

$$N_{ro,t} = \frac{(S_{ro,t})^{\frac{1}{\theta}}}{\left(\frac{1}{\tau_{ro}} \right)^{\alpha_r} \left(w_{ro,t} \frac{1-\alpha_r}{\alpha_r p_{I,t}} \right)^{1-\alpha_r}}$$

Therefore

$$TC_{ro,t} = (1-\nu_t)(\tau_{ro})^{\alpha_r} \left(\frac{w_{ro,t}}{\alpha_r} \right)^{\alpha_r} \left(\frac{p_{I,t}}{(1-\alpha_r)} \right)^{1-\alpha_r} (S_{ro,t})^{\frac{1}{\theta}}$$

Now we can obtain the marginal cost by taking the partial derivative of total cost with respect to the quantity produced:

$$MC_{ro,t} = \frac{\partial TC_{ro,t}}{\partial S_{ro,t}}$$

This yields:

$$MC_{ro,t} = \frac{1}{\theta}(1-\nu_t)(\tau_{ro})^{\alpha_r} \left(\frac{w_{ro,t}}{\alpha_r} \right)^{\alpha_r} \left(\frac{p_{I,t}}{(1-\alpha_r)} \right)^{1-\alpha_r} (S_{ro,t})^{\frac{1-\theta}{\theta}} \tag{1.46}$$

but it must hold that:

$$MC_{ro,t} = MC_{rf,t}$$

hence:

$$\frac{1}{\theta}(1-\nu_t)(\tau_{ro})^{\alpha_r} \left(\frac{w_{ro,t}}{\alpha_r} \right)^{\alpha_r} \left(\frac{p_{I,t}}{(1-\alpha_r)} \right)^{1-\alpha_r} (S_{ro,t})^{\frac{1-\theta}{\theta}} = \frac{1}{\theta} \left(\tau_{rf} \right)^{\alpha_r} \left(\frac{w_{rf,t}}{\alpha_r} \right)^{\alpha_r} \left(\frac{p_{I,t}}{(1-\alpha_r)} \right)^{1-\alpha_r} \left(S_{rf,t} \right)^{\frac{1-\theta}{\theta}}$$

and finally:

$$\frac{S_{rf,t}}{S_{ro,t}} = \left[(1-\nu_t)\left(\frac{\tau_{ro}}{\tau_{rf}}\right)^{\alpha_r}\left(\frac{w_{ro,t}}{w_{rf,t}}\right)^{\alpha_r}\right]^{\frac{\theta}{1-\theta}}$$

In addition, considering the aggregation condition of equation (1.5) yields:

$$S_t = \left[(1-\nu_t)\left(\frac{\tau_{ro}}{\tau_{rf}}\right)^{\alpha_r}\left(\frac{w_{ro,t}}{w_{rf,t}}\right)^{\alpha_r}\right]^{\frac{\theta}{1-\theta}} S_{ro,t} + S_{ro,t}$$

$$S_t = \left[1 + \left[(1-\nu_t)\left(\frac{\tau_{ro}}{\tau_{rf}}\right)^{\alpha_r}\left(\frac{w_{ro,t}}{w_{rf,t}}\right)^{\alpha_r}\right]^{\frac{\theta}{1-\theta}}\right] S_{ro,t}$$

$$S_{ro,t} = \frac{S_t}{1 + \left[(1-\nu_t)\left(\frac{\tau_{ro}}{\tau_{rf}}\right)^{\alpha_r}\left(\frac{w_{ro,t}}{w_{rf,t}}\right)^{\alpha_r}\right]^{\frac{\theta}{1-\theta}}}$$

Substituting this last result into equation (1.12) yields the final marginal cost equation:

$$MC_t = \frac{1}{\theta}(1-\nu_t)(\tau_{ro})^{\alpha_r}\left(\frac{w_{ro,t}}{\alpha_r}\right)^{\alpha_r}\left(\frac{p_{I,t}}{(1-\alpha_r)}\right)^{1-\alpha_r}\left(\frac{S_t}{1+\left[(1-\nu_t)\frac{(\tau_{ro})^{\alpha_r}}{\left(\tau_{rf}\right)^{\alpha_r}}\left(\frac{w_{ro,t}}{w_{rf,t}}\right)^{\alpha_r}\right]^{\frac{\theta}{1-\theta}}}\right)^{\frac{1-\theta}{\theta}} \tag{1.47}$$

Households

The households problem assumes the following form:

$$\max_{C_t,b_t,N_{rf,t},N_{ro,t},N_{I,t}} U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{\alpha_t^N}{1+k}\left[\chi_1\left(N_{rf,t}\right)^{1+\eta} + \chi_2\left(N_{ro,t}\right)^{1+\eta} + \chi_3\left(N_{I,t}\right)^{1+\eta}\right]^{\frac{1+\kappa}{1+\eta}} \right\}$$

$$s.t. \tag{1.48}$$

$$C_t + b_t = \frac{R_{t-1}b_{t-1}}{\pi_t} + (1-t_t)\left[w_{rf,t}N_{rf,t} + w_{ro,t}N_{ro,t} + w_{I,t}N_{I,t}\right]$$

The Lagrangean of the problem is:

$$\text{L=E}_t\beta^t \left\{ \begin{array}{c} \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{\alpha_t^N}{1+\kappa}\left[\chi_1(N_{rf,t})^{1+\eta} + \frac{\chi_2}{\chi}(N_{ro,t})^{1+\eta} + \chi_3(N_{I,t})^{1+\eta}\right]^{\frac{1+\kappa}{1+\eta}} \\ -\lambda_t\left[C_t + b_t - \frac{R_{t-1}b_{t-1}}{\pi_t} - (1-t_t)\left[w_{rf,t}N_{rf,t} + w_{ro,t}N_{ro,t} + w_{I,t}N_{I,t}\right]\right] \end{array} \right\}$$

The first order conditions are:

$I.\ \frac{\partial L}{\partial C_t} = 0$

$$\lambda_t = C_t^{-\sigma}$$

$II.\ \frac{\partial L}{\partial B_t} = 0$

$$-\lambda_t\beta^t + \lambda_{t+1}\beta^{t+1}\frac{R_t}{\pi_{t+1}} = 0$$

$III.\ \frac{\partial L}{\partial N_{rf,t}} = 0$

$$\left(\alpha_t^N\right)\left[\chi_1(N_{rf,t})^{1+\eta} + \chi_2(N_{ro,t})^{1+\eta} + \chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}\chi_1(N_{rf,t})^{\eta} = \lambda_t(1-t_t)w_{rf,t}$$

$IV.\ \frac{\partial L}{\partial N_{ro,t}} = 0$

$$\left(\alpha_t^N\right)\left[\chi_1(N_{rf,t})^{1+\eta} + \chi_2(N_{ro,t})^{1+\eta} + \chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}\chi_2(N_{ro,t})^{\eta} = \lambda_t(1-t_t)w_{ro,t}$$

$V.\ \frac{\partial L}{\partial N_{I,t}} = 0$

$$\left(\alpha_t^N\right)\left[\chi_1(N_{rf,t})^{1+\eta}+\chi_2(N_{ro,t})^{1+\eta}+\chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}\chi_3(N_{I,t})^{\eta}=\lambda_t(1-t_t)w_{I,t}$$

Plug I into II to obtain the Euler equation

$$C_t^{-\sigma}\beta^t=C_{t+1}^{-\sigma}\beta^{t+1}\frac{R_t}{\pi_{t+1}}$$

$$R_t=\pi_t\frac{C_t^{-\sigma}}{C_{t+1}^{-\sigma}}\frac{\beta^t}{\beta^{t+1}}$$

$$\frac{1}{R_t}=\beta\left[\pi_{t+1}^{-1}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right] \tag{1.49}$$

Plug I into III, IV and V to obtain the three labor supplies:

$$(1-t_t)w_{rf,t}=\left(\alpha_t^N\right)\chi_1\left[\chi_1(N_{rf,t})^{1+\eta}+\chi_2(N_{ro,t})^{1+\eta}+\chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}(N_{rf,t})^{\eta}(C_t)^{\sigma} \tag{1.50}$$

and

$$(1-t_t)w_{ro,t}=\left(\alpha_t^N\right)\chi_2\left[\chi_1(N_{rf,t})^{1+\eta}+\chi_2(N_{ro,t})^{1+\eta}+\chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}(N_{ro,t})^{\eta}(C_t)^{\sigma} \tag{1.51}$$

and

$$(1-t_t)w_{I,t}=\left(\alpha_t^N\right)\chi_3\left[\chi_1(N_{rf,t})^{1+\eta}+\chi_2(N_{ro,t})^{1+\eta}+\chi_3(N_{I,t})^{1+\eta}\right]^{\frac{\kappa-\eta}{1+\eta}}(N_{I,t})^{\eta}(C_t)^{\sigma} \tag{1.52}$$

1.5.4 List of equations

This section present the full set of equations.

- Euler equation
$$\frac{1}{R_t} = \beta \left[\pi_{t+1}^{-1} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right]$$
- Contact-intensive labor supply
$$(1-t_t) w_{rf,t} = \left(\alpha_t^N \right) \chi_1 (N_{rf,t})^{\eta} (C_t)^{\sigma}$$
- Online labor supply
$$(1-t_t) w_{ro,t} = \left(\alpha_t^N \right) \chi_2 (N_{ro,t})^{\eta} (C_t)^{\sigma}$$
- Intermediate labor supply
$$(1-t_t) w_{I,t} = \left(\alpha_t^N \right) \chi_3 (N_{I,t})^{\eta} (C_t)^{\sigma}$$
- Intermediate production function
$$S_{I,t} = A N_{I,t}^{\alpha}$$
- Intermediate labor demand
$$w_{I,t} = p_{I,t} \alpha A N_{I,t}^{\alpha-1}$$
- Contact-intensive intermediate input demand
$$S_{Irf,t}^{j} = \frac{\theta(1-\alpha_r) MC_t^j S_{rf,t}^j}{p_{I,t}}$$
- Online intermediate input demand
$$S_{Iro,t}^{j} = \frac{\theta(1-\alpha_r) MC_t^j S_{ro,t}^j}{(1-\nu_t) p_{I,t}}$$
- Contact-intensive labor demand
$$N_{rf,t}^{j} = \frac{\theta \alpha_r MC_t^j S_{rf,t}^j}{w_{rf,t}}$$

- Online labor demand

$$N^j_{ro,t} = \frac{\theta \alpha_r MC^j_t S^j_{ro,t}}{(1-\nu_t) w_{ro,t}}$$

- Marginal cost

$$MC_t = \frac{1}{\theta}(1-\nu_t)(\tau_{ro})^{\alpha_r}\left(\frac{w_{ro,t}}{\alpha_r}\right)^{\alpha_r}\left(\frac{p_{I,t}}{(1-\alpha_r)}\right)^{1-\alpha_r}\left(\frac{S_t}{1+\left[(1-\nu_t)\frac{(\tau_{ro})^{\alpha_r}}{\left(\tau_{rf}\right)^{\alpha_r}}\left(\frac{w_{ro,t}}{w_{rf,t}}\right)^{\alpha_r}\right]^{\frac{\theta}{1-\theta}}}\right)^{\frac{1-\theta}{\theta}}$$

- Contact-intensive output

$$S^j_{rf,t} = \left[\left(\frac{N^j_{rf,t}}{\tau_{rf}}\right)^{\alpha_r}\left(S^j_{Irf,t}\right)^{1-\alpha_r}\right]^{\theta}$$

- Online output

$$S^j_{ro,t} = \left[\left(\frac{N^j_{ro,t}}{\tau_{ro}}\right)^{\alpha_r}\left(S^j_{Iro,t}\right)^{1-\alpha_r}\right]^{\theta}$$

- NKPC

$$(1-\psi)+\psi(1-\omega_t)MC_t+\gamma\mathbb{E}_t\Lambda_t\left[(\pi_{t+1}-1)\pi_{t+1}\frac{S_{t+1}}{S_t}\right] = \gamma(\pi_t-1)\pi_t\frac{S_{t+1}}{S_t}$$

- Taylor rule

$$\frac{R}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\theta_\pi}\left(\frac{S_t}{\bar{S}}\right)^{\theta_S}$$

- Intermediate inputs clearing

$$S_{I,t} = S_{Irf,t} + S_{Iro,t} + S_{Ig,t}$$

- Health goods demand function

$$g_t = \bar{g} + \left[1 - \left(\frac{\bar{\alpha}^N}{\alpha^N_{t-1}}\right)^{\phi_g}\right]$$

- Health goods supply function

$$g_t = (S_{Ig,t})^{\alpha_g}$$

- Aggregate resource constraint

$$S_t = C_t + g_t$$

- Government budget constraint

$$v_t S_{ro,t} + \frac{R_{t-1} b_{t-1}}{\pi_t} + g_t = t_t (w_{rf,t} N_{rf,t} + w_{ro,t} N_{ro,t} + w_{I,t} N_{I,t}) + b_t$$

- Tax rule

$$\frac{t_t}{\bar{t}} = \left(\frac{b_{t-1}}{\bar{b}} \right)^{\xi}$$

- Pandemic shock

$$\alpha_t^N = \left(\alpha_{t-1}^N\right)^{\rho} \left(\frac{S_{rf,t}}{S_{rf,t-1}} \right)^{\Delta(1-\rho)} \exp \varepsilon_t$$

Chapter 2

Covid-19 and online trade: a firm dynamics perspective

Emanuele Colombo Azimonti

Abstract This work develops a model featuring endogenous firm dynamics in response to a pandemic shock. All firms can exploit a contact-intensive retail channel, but only a subset can shift part of its production by entering the online retail trade that is inherently less contagious. The shock triggers endogenous firms entry in the online sector and combination of an online retail subsidy and a tax on contact-intensive production further stimulates this mechanism. This mitigates both the fall in private consumption and the persistence of the pandemic.

2.1 Introduction

The pandemic crisis creates the opportunity for the expansion of online trade towards new firms and this is likely to involve a shift from the contact-intensive to the online retail (OECD (2020)). In this sense, policymakers should implement adequate strategies in order to incentive this transformation and to facilitate the access to the online technology.

This work proposes a theoretical macroeconomic model of endogenous firm dynamics in response to a pandemic shock. In the model, all firms have access to a contact-intensive retail channel, whose exploitation contributes to a stronger persistence of the contagion. A subset of firms can exploit also an online trade channel, conditional to the entry into the online sector. The entry decision ultimately depends on the idiosyncratic level of firm productivity and on the effects of the Covid-19 shock, which we design as an unconventional supply shock. The paper offers policy implications in terms of containment of the pandemic, discussing fiscal policies that stimulate sectoral reallocation.

There is a growing number of papers addressing the issue of understanding and guiding the online transition. Cong et al. (2021) investigate whether the pandemic has encouraged digital technology adoption in China. They find that the pandemic both accelerates the digital transformation of the incumbents and the access of new entrants in the online retail. Beckers et al. (2021) find similar results for Belgium. Andrews et al. (2021) underline how technology adoption is key to improve firms resilience, in particular when firms do not to merely adjust to new market conditions, but when they seize new growth opportunities (for example, the adoption of online commerce). In a similar vein, Barrero et al. (2021) and Bloom and Van Reenen (2007) highlight that high productivity firms could more effectively accommodate their business models towards new practices and technologies, that are necessary to mitigate the effect of Covid-19. Finally, these works show that such firms have also been able to better capitalise on policy support measures available.
In our model, we rationalize these facts in a theoretical model featuring endogenous firm dynamics and idiosyncratic online productivity, where specific supply-side fiscal policies help firms to adopt a new and less contagious technology (online commerce) in order to fight the pandemic.

The literature has so far offered several proposals concerning the introduction and the analysis of Covid-19 within the business cycle framework. For example, Guerrieri et al. (2022) and Buera et al. (2021) simulate Covid-19 as a sectoral shutdown and they show how this can either allow for a reallocation away from contact-intensive activities or, if there are strong complementarities among productions, generate a Keynesian supply shock. We follow Corrado et al. (2021) and envisage the pandemic as a symmetric labor supply shock, with the addition of an endogenous component, so that the persistence of the shock (and its mitigation) depends on the share of contact-intensive activities on total economic activity. Second, part of the firms can reallocate production towards an online (and less contagious) retail. This happens only as a consequence of the entry decision in the online sector. To design this mechanism, we introduce endogenous firm dynamics following Hopenhayn (1992), Piersanti et al. (2020) and Barbaro et al. (2022). In our setup, firms can enter the online market upon the payment of a fixed cost, if they are still profitable enough. The pandemic affects the entry decision and consequently the share of online and contact-intensive firms. Finally, we investigate the role of supply-side fiscal policies both on firms entry and on the mitigation of the pandemic. Specifically, we focus on an online subsidy and a contact-intensive tax, whose aim is creating an incentive to reallocate to the online retail, so that the economy relies less on contagious activities.

Our results in synthesis. In the market equilibrium the pandemic triggers a drop in aggregate production and consumption. However, it also stimulates an expansion of the online sector, in a twofold perspective: incumbents are more likely to survive and more new firms enter the online market. Overall, this implies reallocation of production towards the online channel, which helps to contain the economic recession and the spread of the virus. The public fiscal intervention achieves a further increase in the online market participation, by making also relatively less efficient firms enter the online sector. The additional reallocation implies a stronger contraction of the contact-intensive sector and, consequently, a substantial mitigation of the pandemic.

We also contribute to the debate about the most effective policies to fight Covid-19. Under this perspective, most of the literature focuses on demand-side policies. Bayer et al. (2020), Elenev et al. (2020), Faria-e Castro (2021), for example, study the impact of different fiscal tools,

such as a decrease in income taxation, or an expansion of unemployment insurance. All these studies share the concern for the stabilisation objective that guide macroeconomic policies. Another strand investigate supply-side interventions: Hubbard and Strain (2020a), Hubbard and Strain (2020b) and Hanson et al. (2020) suggest the most appropriate mix of financial aid to avoid firms bankruptcies; this takes the form of grants to small firms and of loans to big ones, whose liquidity problems can be attenuated by greater internal resources and access to financial markets.
We share with this last strand the attention for supply-side fiscal policies; however, we do not focus on temporary financial measures, but we propose an alternative scope for supply-side interventions, as we show the advantages, both in terms of shock mitigation and of economic recovery, of providing an incentive to online trade.

Another strand merges macroeconomic and epidemiological models. Farboodi et al. (2021), Eichenbaum et al. (2020) and Alvarez et al. (2021) focus on the optimal lockdown policy to control the spread of Covid-19 while minimizing the economic costs of the lockdown. In a similar vein, Krueger et al. (2020) claim that the composition of the households' consumption bundle endogenously tilts towards less contagious sectors.
Even if our methodology differs, we share with these works the fact that individual responses to the shock do not fully internalise their effects on the persistence of the pandemic shock. However, while these works prescribe a tight initial lockdown, our model offers the view that the reallocation towards online trade may require less severe lockdowns.

The rest of the chapter is organized as follows: section 2.2 describes the model, section 2.3 presents calibration and results; finally, section 2.4 concludes.

2.2 The model

The model incorporates firm dynamics, with the endogenous entry decision in the online sector. More precisely, firms can enter the this sector after the payment of a fixed cost, if they can still achieve a non-negative profit.
There are five agents in the economy: households, base good firms, in-

termediate firms, retailers and the public sector. The model embeds frictions, both nominal and real: there are price stickiness and labor reallocation costs.

Households consume, save through government bonds and supply labor. Base good firms operate in a perfectly competitive market and sell their goods to intermediate firms. In the model there are two retail channels, one is contact-intensive (and more contagious), the other one is online. Households are indifferent between the two channels. Perfectly competitive intermediate goods producers can choose how much to produce for the contact-intensive (online) retail channel. Importantly, these producers are identically efficient when producing for the contact-intensive retail sector. By contrast, idiosyncratic firm efficiency and a symmetrical fixed cost characterize technology in the online channel. To introduce nominal rigidities in the model, we assume that the same set of retailers operates in each retail channel. Each retailer buys the intermediate good produced for contact-intensive (online) retail and turns it into contact-intensive (online) differentiated good. Given households' indifference between the two retail channels, the price of each good variety is unique. This implies that each monopolistic retailer is prepared to pay the same price for the intermediate input. As a result, the market price for intermediate inputs must be unique. Due to Rotemberg nominal rigidities all retail goods have the same price.

A pandemic shock hits the economy; its persistence is endogenous to the share of contact-intensive productive activities. In response to the shock, the public sector intervenes in the economy with two different fiscal tools: an online subsidy which boosts the online production; and a tax on the contact-intensive production, in order to discourage the utilisation of this retail channel. Moreover, the government levies labor income taxation and issues public debt.

Figure 2.1 summarises the model economy.

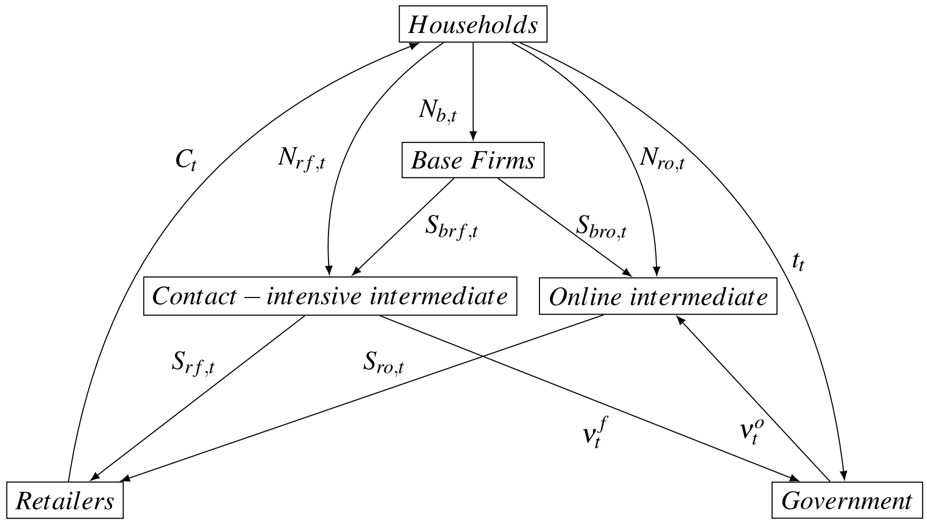

Figure 2.1: Flow diagram of the economy

2.2.1 Base good firms

The production function of base firms is:

$$S_{b,t} = A_b N_{b,t}^{\alpha} \tag{2.1}$$

where $S_{b,t}$ is the intermediate output, $N_{b,t}$ is intermediate labor and A_b defines productivity. Intermediate firms are subject to decreasing returns to scale $\alpha < 1$.
In each period, firms maximise their profits:

$$\Pi_{b,t} = p_{b,t} S_{b,t} - w_t N_{b,t} \tag{2.2}$$

where $p_{b,t}$ is the price of the intermediate good in units of consumption good and w_t is the real wage paid to workers.
The solution of the problem provides the optimal demand for intermediate labor:

$$w_t = p_{b,t} \alpha A_b N_{b,t}^{\alpha-1} \tag{2.3}$$

2.2.2 Intermediate good firms

All perfectly competitive intermediate firms can exploit the contact-intensive retail, but only a subset can shift part of the production by entering the online retail trade.

Contact-intensive sector

Firms in the contact-intensive sector have mass equal to 1. Their production function is:

$$S^j_{rf,t} = \left[\left(\frac{N^j_{rf,t}}{\tau_{rf}} \right)^{\alpha_r} (S^j_{brf,t})^{1-\alpha_r} \right]^{\theta} \tag{2.4}$$

where $S^j_{rf,t}$ defines firm j contact-intensive production, $N^j_{rf,t}$ is the quantity of contact-intensive worked hours, while τ_{rf} is a labor productivity shifter. $\theta < 1$ defines decreasing returns to scale.
Contact-intensive profits are:

$$\Pi^j_{rf,t} = p^{INT}_t S^j_{rf,t} - \left(1 + \nu^f_t\right) \left(w_t N^j_{rf,t} + p_{b,t} S^j_{brf,t}\right) \tag{2.5}$$

where p^{INT}_t is the consumption price charged by intermediate firms. The public sector levies a tax ν^f_t to discourage the exploitation of the contact-intensive retail.

Factor demands are:

$$N^j_{rf,t} = \frac{\theta \alpha_r p^{INT}_t S^j_{rf,t}}{(1+\nu^f) w_t} \tag{2.6}$$

$$S^j_{brf,t} = \frac{\theta (1-\alpha_r) p^{INT}_t S^j_{rf,t}}{(1+\nu^f_t) p_{b,t}} \tag{2.7}$$

According to the cost minimisation problem, we derive the firm's marginal cost for the contact-intensive sector. Moreover, due to perfect competition, we have that $MC_t = p_t^{INT}$. This yields:

$$p_t^{INT} = \frac{1}{\theta}\left(1+\nu_t^f\right)\left(\tau_{rf}\right)^{\alpha_r}\left(\frac{w_t}{\alpha_r}\right)^{\alpha_r}\left(\frac{p_{b,t}}{(1-\alpha_r)}\right)^{1-\alpha_r}\left(S_{rf,t}\right)^{\frac{1-\theta}{\theta}} \quad (2.8)$$

Online sector

A mass $\eta_{f,t} < 1$ of firms operate both in the contact-intensive and online retails. Hence, $\eta_{f,t}$ is:

$$\eta_{f,t} = NE_t + INC_t \quad (2.9)$$

where INC_t are incumbent firms, while NE_t defines potential new entrants in the online sector. Online firms are heterogeneous in the level of their productivity, when operating online. The variable A_t^j captures this feature.

Firm j's online production function is:

$$S_{ro,t}^j = A_t^j\left[\left(\frac{N_{ro,t}^j}{\tau_{ro}}\right)^{\alpha_r}(S_{bro,t}^j)^{1-\alpha_r}\right]^{\theta} \quad (2.10)$$

where $S_{ro,t}^j$ defines firm j online production, $N_{ro,t}^j$ is the quantity of online worked hours demanded by the firms, while τ_{ro} is a labor productivity shifter. The presence of decreasing returns to scale θ guarantees that, when the firm can switch between contact-intensive and online, it would not move the entire production towards the more efficient technology.

Online profits are:

$$\Pi_{ro,t}^{j} = p_t^{INT} S_{ro,t}^{j} - (1-\nu_t^o)\frac{\left(w_t N_{ro,t}^{j} + p_{b,t} S_{bro,t}^{j}\right)}{(\alpha_t^N)^{\theta_\alpha}} - c \tag{2.11}$$

The public sector provides a subsidy ν_t^o. The term c is a fixed entry cost in the online sector. α_t^N is the pandemic shock, which creates an asymmetry between the contact-intensive and online cost functions, such that, after the outbreak of Covid-19, firms face higher costs for the contact-intensive production.

Factor demands are:

$$N_{ro,t}^{j} = \frac{\theta \alpha_r p_t^{INT} S_{ro,t}^{j} (\alpha_t^N)^{\theta_\alpha}}{(1-\nu^o) w_t} \tag{2.12}$$

$$S_{bro,t}^{j} = \frac{\theta (1-\alpha_r) p_t^{INT} S_{ro,t}^{j} (\alpha_t^N)^{\theta_\alpha}}{(1-\nu_t^o) p_{b,t}} \tag{2.13}$$

The online marginal cost is:

$$p_t^{INT} = \frac{1}{\theta}\frac{(1-\nu_t^o)}{(\alpha_t^N)^{\theta_\alpha}} (\tau_{ro})^{\alpha_r} \left(\frac{w_t}{\alpha_r}\right)^{\alpha_r} \left(\frac{p_{b,t}}{(1-\alpha_r)}\right)^{1-\alpha_r} (S_{ro,t})^{\frac{1-\theta}{\theta}} \tag{2.14}$$

Considering (2.8) and (2.14), cost minimisation for online firms implies that, in both sectors, firms operate at the same marginal cost, i.e. until when the contact-intensive and the online marginal costs are equal.

Moreover, we can define:

$$p_t^{BUN} = \left(\frac{w_t \tau_{ro}}{\alpha_r}\right)^{\alpha_r} \left(\frac{p_b}{(1-\alpha_r)}\right)^{1-\alpha_r} \tag{2.15}$$

where p_t^{BUN} is the price of the bundle of productive factors.

The firm's online supply function is:

$$S_{ro,t}^{j} = \left(A_t^j\right)^{\frac{1}{1-\theta}} \left(\frac{\theta\left(\alpha_t^N\right)^{\theta_\alpha}}{\left(1-\nu_t^o\right)} \frac{p_t^{INT}}{p_t^{BUN}}\right)^{\frac{\theta}{1-\theta}} \tag{2.16}$$

From (2.16) and (2.11), the value function $V_t\left(A_t^j\right)$ can be written recursively as:

$$V_t\left(A_t^j\right) = (1-\theta)\left(A_t^j \frac{\theta^\theta\left(\alpha_t^N\right)^{\theta_\alpha}}{\left(1-\nu_t^o\right)^\theta} \frac{p^{INT}}{\left(p_t^{BUN}\right)^\theta}\right)^{\frac{1}{1-\theta}} - c + \beta E_t\left\{\Lambda_{t+1} V_{t+1}\left(A_t^j\right)\right\} \tag{2.17}$$

where the first term on the right considers online profits, while $\beta E_t\left\{\Lambda_{t+1} V_{t+1}\left(A_t^j\right)\right\}$ is the firm continuation value.
We can identify the idiosyncratic productivity threshold $\hat{A}_t$, which defines the mass of firms that operates online, through the condition:

$$V\left(\hat{A}_t\right) = 0 \tag{2.18}$$

Equation (2.17) shows how the threshold reacts to current and expected economic conditions: an increase in the price p_t^{INT} raises the firm value and lowers the efficiency requirement needed to participate to the online sector. On the other hand, an increase in the input price p_t^{BUN} or in the fixed entry cost c generates the opposite effect. Finally, firms also care about their future value, represented by the last term of the equation.

New Entrants

At the period $t-1$ there is a mass $(1-\eta_{f,t-1})$ of firms operating exclusively in the contact-intensive sector. Potential new entrants belong to this mass and at the beginning of period t they have the chance of accessing the online sector by drawing a fortunate productivity level A_t^j from the following Pareto distribution:

$$f_t(A_t) = \int_z^{+\infty} \frac{\xi_p (z)^{\xi_p}}{\left(A_t^j\right)^{\xi_p+1}} d\left(A_t^j\right) = 1 \tag{2.19}$$

Firms enter the online market if they meet the zero-profit condition. Therefore, the mass of new entrants is:

$$NE_t = \left(1 - \eta_{f,t-1}\right)\left(\frac{z}{\hat{A}_t}\right)^{\xi_p} \tag{2.20}$$

where z defines the technological frontier and ξ_p controls the shape of the Pareto distribution. $\hat{A}_t$ defines the productivity threshold related to the zero profit condition of equation (2.18). Finally, the term $\left(\frac{z}{\hat{A}_t}\right)^{\xi_p}$ is the value of the integral (2.19) considering $\hat{A}_t$.

Incumbents

At the beginning of period t, the $\eta_{f,t-1}$ incumbent firms draw their productivity from the Pareto distribution:

$$f_t(A_t) = \int_{\hat{A}_{t-1}(1-\delta)}^{+\infty} \frac{\xi_p \left(\hat{A}_{t-1}(1-\delta)\right)^{\xi_p}}{\left(A_t^j\right)^{\xi_p+1}} d\left(A_t^j\right) = 1 \tag{2.21}$$

Incumbents, on average, deplete their knowledge capital; we capture this by setting $(1-\delta) < 1$, following Liu et al. (2021) and Barbaro et al. (2022). Hence, the mass of incumbents firms, INC_t is defined by:

$$INC_t = H_t \eta_{t-1} \tag{2.22}$$

where

$$H_t = \int_{\hat{A}}^{+\infty} \frac{\xi_p \left(\hat{A}_{t-1}(1-\delta)\right)^{\xi_p}}{\left(A_t^j\right)^{\xi_p+1}} d\left(A_t^j\right) = \left(\frac{\hat{A}_{t-1}(1-\delta)}{\hat{A}_t}\right)^{\xi_p} \tag{2.23}$$

is the endogenous survival probability in period t for the $\eta_{f,t-1}$ firms.

Threshold

In this section, we derive the the efficiency threshold associated to the intertemporal zero profit condition (2.18).
First, for operative firms it holds that:

$$E_t\left\{V_t\left(A_t^j\right)\right\} = \int_{\widehat{A}_{t+1}}^{+\infty} V_{t+1}\left(A_{t+1}^j\right) \frac{\xi_p \left(\widehat{A}_{t+1}\right)^{\xi_p}}{\left(A_{t+1}^j\right)^{\xi_p+1}} d\left(A_{t+1}^j\right) = E_t\left\{H_{t+1}V_{t+1}^{av}\right\} \tag{2.24}$$

where V_{t+1}^{av} defines the continuation value of the η_f firms conditional to survival in $t+1$. Combining equations (2.17), (2.18) and (2.24) the

following condition identifies the productivity threshold:

$$\hat{A}_t = \left[\frac{c - \beta E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right\}}{\theta^\theta(1-\theta)}\right]^{1-\theta} \frac{(1-\nu_t^o)^\theta}{(\alpha_t^N)^{\theta_\alpha}} \frac{\left(p_t^{BUN}\right)^\theta}{p^{INT}} \tag{2.25}$$

Equation (2.25) clearly shows how an increase in the entry cost c and in the price of the bundle of productive factors p_t^{BUN} leads to a consequent increase in the productivity threshold, i.e. accessing the online market becomes more difficult.
On the contrary, an higher current or future profitability, respectively p_t^{INT} and $\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right\}$, make relatively less efficient firms enter the online sector.

The pandemic α_t^N creates an incentive towards the online retail, as it lowers the entry threshold. Finally, the provision of the subsidy ν_t^o makes the threshold decrease and this translates into a further online incentive.

Intermediate sector aggregation

In order to obtain $S_{ro,t}$, we combine equations (2.16), (2.19) and (2.21); this yields:

$$S_{ro,t} = \frac{\xi_p (1-\theta)}{\xi_p (1-\theta) - 1} \left[\frac{\theta (\alpha_t^N)^{\theta_\alpha}}{(1-\nu_t^o)} \frac{p_t^{INT}}{p_t^{BUN}} \right]^{\frac{\theta}{1-\theta}} \eta_{f,t} \hat{A}_t^{\frac{1}{1-\theta}} \tag{2.26}$$

while, from equation (2.8), we can obtain $S_{rf,t}$:

$$S_{rf,t} = \left(\frac{\theta}{(1+\nu_t^f)} \frac{p_t^{INT}}{p_t^{BUN}} \right)^{\frac{\theta}{1-\theta}} \tag{2.27}$$

Equations (2.26) and (2.27) show how an increase in p_t^{INT} translates into an increase in the price-cost margin, i.e. the term $\frac{p_t^{INT}}{p_t^{BUN}}$. This also loosens the zero-profit condition (2.18), allowing the access to the online sector to relatively less-efficient firms.

2.2.3 Retailers

Each monopolistic retailer acquires the intermediate contact-intensive or online good and turns it into a differentiated good. Due to the presence of price rigidities à la Rotemberg (1982), retail goods have the same price. More specifically, the price adjustment cost is:

$$\Phi_t = \frac{\gamma}{2} \left(\frac{P_t^j}{P_{t-1}^j} - 1 \right)^2 S_t \tag{2.28}$$

Aggregate output is:

$$S_t = S_{rf,t} + S_{ro,t} \tag{2.29}$$

which, considering equations (2.26) and (2.27), is:

$$S_t = \left[\frac{\xi_p (1-\theta)}{\xi_p (1-\theta) - 1} \left[\frac{(\alpha_t^N)^{\theta_\alpha}}{(1-\nu_t^o)} \right]^{\frac{\theta}{1-\theta}} \eta_{ft} \hat{A}_t^{\frac{1}{1-\theta}} + \left[\frac{1}{(1+\nu_t^f)} \right]^{\frac{\theta}{1-\theta}} \right] \left(\frac{\theta p_t^{INT}}{p_t^{BUN}} \right)^{\frac{\theta}{1-\theta}} \tag{2.30}$$

Finally, the solution of the optimal price setting problem yields the standard NKPC:

$$(1-\psi) + \psi p_t^{INT} + \gamma \mathbb{E}_t \Lambda_t \left[(\pi_{t+1} - 1)\pi_{t+1} \frac{S_{t+1}}{S_t} \right] = \gamma(\pi_t - 1)\pi_t \frac{S_{t+1}}{S_t} \tag{2.31}$$

2.2.4 Households

Households preferences are defined over consumption C_t and labor effort N_t. The representative households' lifetime utility function is akin to Moura (2018) [1]:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{\alpha_t^N}{1+\eta} \left[\chi_1 (N_t)^{1+\eta} \right] \right\} \tag{2.32}$$

where N_t is the labor bundle defined as:

$$N_t = N_{rf,t} + N_{ro,t} + N_{b,t} \tag{2.33}$$

[1] When the value of the parameter σ is equal to one, equation (2.32) is logarithmic in consumption C_t

The parameter β is the subjective discount factor, σ is the intertemporal elasticity of substitution, α^N is the pandemic shock. The specification of the labor bundle implies reallocation rigidities, and hence imperfect labor mobility, when $\eta > 0$.
The households' budget constraint is:

$$C_t + b_t = \frac{R_{t-1} b_{t-1}}{\pi_t} + (1 - t_t) w_t N_t + \Pi_t \tag{2.34}$$

where b_{t-1} is the real stock of government bond the households hold, R_{t-1} is the interest rate, b_t is the purchase of real public bonds, w_t is the real wage paid in the three different sectors. Finally, t_t are distortionary income taxes and Π_t are firm real profits.
The households maximise (2.32) with respect to C_t, b_t and N_t. This

yields the Euler equation for consumption:

$$\frac{1}{R_t} = \beta \left[\pi_{t+1}^{-1} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] \tag{2.35}$$

and the labor supply, that, considering (2.33), is:

$$(1 - t_t) w_t = \left(\alpha_t^N \right) \chi_1 (N_{rf,t} + N_{ro,t} + N_{b,t})^{\eta} (C_t)^{\sigma} \tag{2.36}$$

2.2.5 The Covid-19 shock

To reproduce the effect and the endogenous persistence of the pandemic, we need to model a shock incorporating:

- an increase in the disutility of labor, i.e. households should be less willing to supply their labor in consequence of the pandemic, as in Corrado et al. (2021);

- an endogenous persistence mechanism, that should be related to the dynamics of contact-intensive activities. We therefore assume the following:

$$\alpha_t^N = \left(\alpha_{t-1}^N\right)^{\rho} \left(\frac{S_{rf,t}}{S_{rf,t-1}}\right)^{\Delta(1-\rho)} \exp \varepsilon_t \tag{2.37}$$

where ρ is the exogenous persistence of the process, $\left(\frac{S_{rf,t}}{S_{rf,t-1}}\right)$ proxies the endogenous persistence channel, due to contact-intensive activities, and Δ allows to characterize the strength of this latter mechanism.
The underlying intuition is that the growth of contact-intensive retail trade raises possibility of getting infected, and therefore adversely affects the supply of worked hours.

2.2.6 Public sector

The government budget constraint is:

$$v_t^o S_{ro,t} + \frac{R_{t-1} b_{t-1}}{\pi_t} = t_t w_t (N_{rf,t} + N_{ro,t} + N_{b,t}) + v_t^f S_{rf,t} + b_t \tag{2.38}$$

The public sector levies taxes both on the labor income and on the contact-intensive production and issues new debt in order to repay past debt interests and to finance the provision of the online subsidy.

2.2.7 Market clearing

An aggregate resource constraint closes the model. Aggregate production S_t has to cover not only the level of consumption C_t, but also needs to take into account the presence of the fixed entry cost, weighted for the share of firms operative in the online sector. Hence:

$$S_t = C_t + \eta_{f,t} c \tag{2.39}$$

2.2.8 Monetary and fiscal policies

To mitigate the effects of the pandemic, the public sector can deploy two fiscal tools, the online subsidy v_t^o and the contact-intensive tax v_t^f. They follow two simple rules:

$$v_t^o = 1 - \left(\frac{C_t}{\bar{C}}\right)^{\phi_{v^o}} \tag{2.40}$$

and

$$v_t^f = 1 - \left(\frac{\bar{\alpha}^N}{\alpha_t^N}\right)^{\phi_{v^f}} \tag{2.41}$$

Equations (2.40) and (2.41) show that the public sector's goal is twofold: the online subsidy creates an incentive to reallocate towards the less contagious retail; the contact-intensive tax directly targets the pandemic shock and discourages the use of the more contagious channel.

Labor income taxation follows a simple rule aiming at the control of public debt, as in Schmitt-Grohé and Uribe (2004):

$$\frac{t_t}{\bar{t}} = \left(\frac{b_{t-1}}{\bar{b}}\right)^{\xi} \tag{2.42}$$

where $\bar{t}$ and $\bar{b}$ are respectively the steady state levels of income taxation and public debt. ξ defines the intensity of the reaction of taxation to debt accumulation.

The monetary authority follows a standard Taylor rule:

$$\frac{R_t}{\bar{R}} = (\pi_t)^{\theta_\pi} \left(\frac{S_t}{\bar{S}}\right)^{\theta_S} \tag{2.43}$$

where $\bar{S}$ is the steady-state level of output.

2.3 Results

2.3.1 Calibration

Table 2.1 describes the calibration. Households parameters assume the standard literature values, i.e. the discount factors β is 0.99 and the elasticity of intertemporal substitution σ is equal to 1; the parameter η, i.e. the reallocation cost for labor, is equal to 2, following the calibration of Moura (2018).

We set the steady state ratio between contact-intensive and online intermediate productions, $\frac{Srf}{Sro}$, equal to six; this indicates that the contact-intensive output accounts for the majority of total output and is in line with what observed from US Census Bureau (2022).

We set the debt-to-GDP ratio equal to 80% , on annual basis. This pins down a income tax rate equal to 0.04. The aggregate labor supply is assumed to be equal to one in steady state ($N = N_{rf} + N_{ro} + N_b = 1$). The value of the parameter χ_1 guarantees this relation.

The level of productivity in the base good sector, A_b, is assumed to be equal to 1, and this yields a steady state value of the base production S_b equal to one fifth of total production S.

Concerning the shock, ε is is chosen in order to have an initial 10% drop in aggregate in the market equilibrium; moreover, $\Delta = 8$ defines the sensitivity of the shock to the share of contact-intensive output. This implies that the higher Δ, the stronger will be the necessity of a public intervention in the economy. Finally, the parameter θ_α is calibrated to match the nearly 6% increase in the online trade. This is in line with what observed for the United States according to OECD (2020).

We set the share of labor in the intermediate production function equal to $\alpha_r = 0.66$.

Moreover, both in base and intermediate productions, we set firms returns $\alpha = \theta = 0.87$, at the upper bound of Basu and Fernald (1997) estimates.

Concerning firm dynamics, we set the share of intermediate firms operating in the online sector η_f equal to 24%, in line with IBIS World (2022). We set the tail index of the Pareto distribution $\xi_p = 15$ and the firm entry rate in the online sector, $\frac{NE}{\eta_f} = 2.5\%$, which accounts for a 10% average yearly entry rate. These values are in line with Barbaro et al. (2022). The technological frontier of the distribution of new entrants, z, and the depreciation rate of firms efficiency, δ, are calibrated accordingly.
Monetary policy parameters are $\theta_\pi = 1.5$ and $\theta_S = 0.2$, in line with Christiano et al. (2014).
To compute the values of the fiscal policies parameters $\phi_{\nu o}$ and $\phi_{\nu f}$ we rely on the welfare analysis of section 2.3. Hence, we set $\phi_{\nu o} = 0.03$ and $\phi_{\nu f} = 0.15$, which are the values that guarantee the maximum individual welfare. Finally, the steady state values of the online subsidy ν^o and of the contact-intensive tax ν^f are zero.

(i) Parameter	Value	Definition
β	0.99	Households discount factor
σ	1	Elasticity of intertemporal substitution
η	2	Reallocation cost for labor
α	0.87	Returns to scale base production
A_b	1	Base firms productivity
α_r	0.66	Share of labor in intermediate production
θ	0.87	Returns to scale intermediate production
ξ_p	15	Pareto tail index
$\frac{NE}{\eta_f}$	0.025	Rate of entry in the online sector
δ	0.004	Firm efficiency depreciation
γ	18.5	Rotemberg menu cost
ψ	6	Price elasticity of demand
ξ	1.2	Intensity of tax reaction to debt accumulation
θ_π	1.5	Taylor rule: inflation
θ_S	0.2	Taylor rule: output
ρ	0.9	Shock persistence
Δ	8	Sensitivity of the shock to variation in $S_{rf,t}$
θ_α	0.068	Elasticity of the cost asymmetry
ϕ_{vo}	0.03	Policy parameter: online subsidy
ϕ_{vf}	0.15	Policy parameter: contact-intensive tax

(ii) Steady State	Value	Definition
$\frac{Srf}{Sro}$	6	Ratio contact-intensive to online output
$\frac{B}{S}$	0.8	Debt-to-GDP ratio
t	0.04	Labor income tax
η_f	0.24	Share of online firms
ν^o	0	Online production subsidy
ν^f	0	Contact-intensive tax

Table 2.1: *(i) Main parameters (ii) Steady state values*

2.3.2 Model dynamics

The benchmark for the policy experiment is the market equilibrium with no public intervention. Thus, we simulate the impact of the pandemic on the economy and on firm dynamics and we investigate the role of supply-side fiscal policies in this framework.

Figure 2.2 presents the results of a shock reproducing a 10% drop in aggregate output.

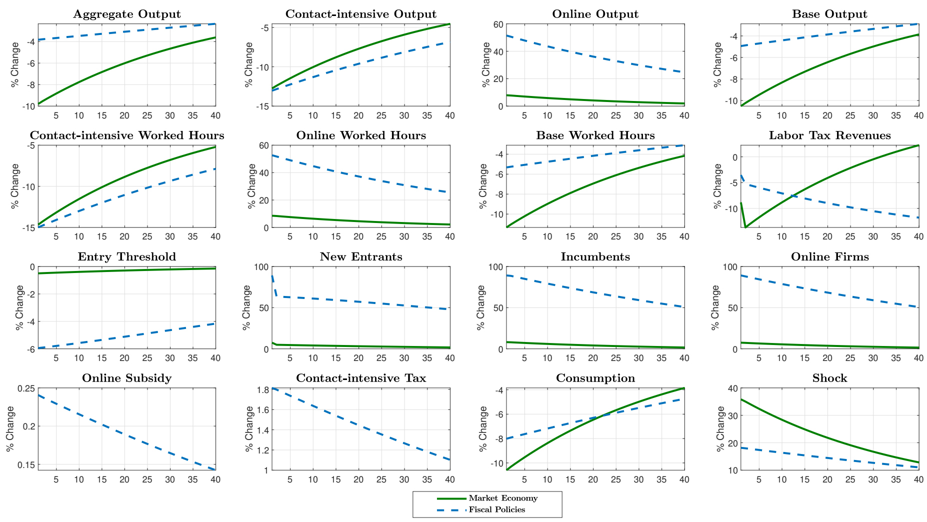

Figure 2.2: *IRFs to a pandemic shock, percentage deviations from steady state*

The shock has a twofold effect: one the one hand, it affects the labor disutility, leading to a decrease in the labor supply and hence to a contraction in aggregate output and consumption. On the other, it triggers an increase in the contact-intensive production costs (equation (2.11)). In the decentralised equilibrium, the online retail channel becomes relatively more efficient and thus firms endogenously enter the online sector and reduce the share of contact-intensive activities. More precisely, incumbent firms are more likely to survive and more firms are able to join the online market, due to the decrease in the level of the entry threshold. The public sector's intervention bolster this dynamic. As a matter of facts, it achieves a larger participation in the online sector thanks to the online subsidy and the tax on contact-intensive production. These two fiscal tools lower the online productivity requirement more than in the market equilibrium, so that also relatively less efficient firms can enter the market. In a similar vein, more incumbent firms survives after the shock.

Overall, the public intervention makes firms reallocate more, with respect to the market solution; this implies a stronger mitigation of the persistence of Covid-19 and a substantial dampening of the drop in aggregate output, allowing the households to maintain higher levels of consumption.

2.3.3 Welfare analysis

In this section, we compute and confront the individual welfare obtained when testing for different values of the two fiscal policies parameters ϕ_{v_o} and ϕ_{v_f} in equations (2.40) and (2.41), i.e. for different levels of intensity of the online subsidy and the contact-intensive tax. We proxy welfare through the following function:

$$W_t = U_t + \beta W_{t+1} \tag{2.44}$$

where W_t indicates welfare and U_t is the households utility function (2.32). Figure 2.3 presents the results.

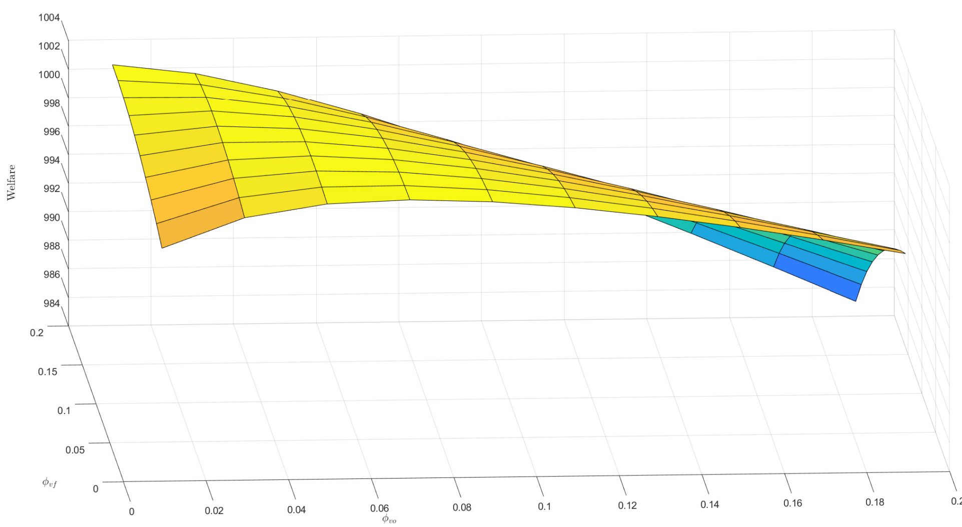

Figure 2.3: *Welfare analysis for different values of ϕ_{v_o} and ϕ_{v_f}*

The welfare analysis shows that the maximum level of welfare coincides with low values of both policy parameters and then clearly decreases as the intensity of the two instruments increases. This signals a trade-off for the public sector: even if a stronger fiscal stimulus may help to control the pandemic, the growing financing costs (through taxation) create a non-negligible distortion for the economy. In addition, the maximum welfare corresponds to a point where the level of the contact-intensive tax is higher than that of the online subsidy. Hence, for the public sector it is optimal, in a welfare-maximisation perspective, to strongly discourage the contact-intensive production and to reallocate towards the online and mitigate the pandemic.

2.4 Conclusions

We analyze the effect of supply-side fiscal policies on firms entry and exit from online trade after the outbreak of Covid-19. In our macroeconomic model, we design the pandemic as an unconventional supply shock, which takes the form of a labor disutility shock with endogenous persistence and dependent on the share of contact-intensive activities in the economy. This mimicks the fact that contact-intensive sectors are the major culprits for the diffusion of the virus, due to physical interaction.

The results show that, in the market equilibrium, the pandemic outbreak stimulates an expansion of the online sector, as more firms are able to enter the market or are more likely to survive. This holds despite the negative labor supply effect (which necessarily generates an economic downturn), as Covid-19 implies an increase in the production cost for the contact-intensive sector. This reallocative process makes the economy less reliant upon physical interaction and hence mitigates the persistence of the shock.
Though the market outcome implies a partial control of the pandemic, there is still room for fiscal policies. As a matter of facts, the public intervention, with the online subsidy and the tax on the contagious production, triggers a stronger reallocation towards the online retail, thanks to a more pronounced decrease of the the entry threshold, allowing a larger survival rate for incumbents and entry also to relatively less efficient firms. Overall, the supply-side fiscal policies are successful in achieving a stronger online reallocation, a less severe economic recession and a substantial containment of the disease.

2.5 Appendix to Chapter 2

2.5.1 Derivation of key equations

In this section we present the full derivation of the key equations of the model.

Base Firm

The problem of the base good firm is:

$$\max_{S_{b,t}} \Pi_{b,t} = p_{b,t}S_{b,t} - w_t N_{b,t}$$
$$s.t. \tag{2.45}$$
$$S_{b,t} = A_b N_{b,t}^{\alpha}$$

$\frac{\partial L}{\partial S_{b,t}} = 0$

$$\lambda_t = p_{b,t}$$

Recall that the Lagrangean multiplier λ_t can be seen as the marginal cost. Hence, $\lambda_t = MC_{b,t}$. This yields to the standard relation for perfect competition:

$$MC_{b,t} = p_{b,t} \tag{2.46}$$

Through cost minimisation, the demand for base labor, $N_{b,t}$, is:

$$\max_{N_{b,t}} \Pi_{b,t} = p_{b,t}S_{b,t} - w_t N_{b,t}$$
$$s.t. \tag{2.47}$$
$$S_{b,t} = A_b N_{b,t}^{\alpha}$$

$\frac{\partial L}{\partial N_{b,t}} = 0$

$$w_t = p_{b,t}\alpha A_b N_{b,t}^{\alpha-1} \tag{2.48}$$

Intermediate Firms - Contact-intensive sector

The problem for the intermediate contact-intensive retail is:

$$\max \Pi^{j}_{rf,t} = p^{INT}_{t} S^{j}_{rf,t} - \left(1+\nu^{f}_{t}\right)\left(w_t N^{j}_{rf,t} + p_{b,t} S^{j}_{brf,t}\right) \tag{2.49}$$

$$s.t. \tag{2.50}$$

$$S^{j}_{rf,t} = \left[\left(\frac{N^{j}_{rf,t}}{\tau_{rf}}\right)^{\alpha_r} (S^{j}_{brf,t})^{1-\alpha_r}\right]^{\theta}$$

Which can be written as:

$$\Pi^{j}_{rf,t} = p^{INT}_{t}\left[\left(\frac{N^{j}_{rf,t}}{\tau_{rf}}\right)^{\alpha_r} (S^{j}_{brf,t})^{1-\alpha_r}\right]^{\theta} - \left(1+\nu^{f}_{t}\right)\left(w_t N^{j}_{rf,t} + p_{b,t} S^{j}_{brf,t}\right)$$

The first order conditions are:

$$\frac{\partial \Pi^{j}_{rf,t}}{\partial N^{j}_{rf,t}} = 0$$

$$N^{j}_{rf,t} = \frac{\theta \alpha_r p^{INT}_{t} S^{j}_{rf,t}}{\left(1+\nu^{f}_{t}\right) w_t} \tag{2.51}$$

$$\frac{\partial \Pi^{j}_{rf,t}}{\partial S^{j}_{brf,t}} = 0$$

$$S^{j}_{brf,t} = \frac{\theta(1-\alpha_r) p^{INT}_{t} S^{j}_{rf,t}}{\left(1+\nu^{f}_{t}\right) p_{b,t}} \tag{2.52}$$

Intermediate Firms - Online sector

The problem for the intermediate online retail is:

$$\max \Pi^{j}_{ro,t} = p^{INT}_{t} S^{j}_{ro,t} - (1-\nu^{o}_{t}) \frac{\left(w_t N^{j}_{ro,t} + p_{b,t} S^{j}_{bro,t}\right)}{(\alpha^{N}_{t})^{\theta_\alpha}} \tag{2.53}$$

$$s.t.$$

$$S^j_{ro,t} = A^j_t \left[\left(\frac{N^j_{ro,t}}{\tau_{ro}} \right)^{\alpha_r} (S^j_{bro,t})^{1-\alpha_r} \right]^{\theta} \tag{2.54}$$

The first order conditions are:

$$\frac{\partial \Pi^j_{ro,t}}{\partial N^j_{ro,t}} = 0$$

$$N^j_{ro,t} = \frac{\theta \alpha_r p^{INT}_t S^j_{ro,t} (\alpha^N_t)^{\theta\alpha}}{(1-\nu^o_t)\, w_t} \tag{2.55}$$

$$\frac{\partial \Pi^j_{ro,t}}{\partial S^j_{bro,t}} = 0$$

$$S^j_{bro,t} = \frac{\theta (1-\alpha_r) p^{INT}_t S^j_{ro,t} (\alpha^N_t)^{\theta\alpha}}{(1-\nu^o_t)\, p_{b,t}} \tag{2.56}$$

Moreover, we can define the price of the bundle of productive factors as
:

$$p^{BUN}_t = \left(\frac{w_t \tau_{ro}}{\alpha_r} \right)^{\alpha_r} \left(\frac{p_b}{(1-\alpha_r)} \right)^{1-\alpha_r} \tag{2.57}$$

To derive the firm's online supply function, we plug equations (2.55) and (2.56) into equation (2.54):

$$S^j_{ro,t} = A^j_t \left[\left(\frac{\theta \alpha_r p^{INT}_t S^j_{ro,t} (\alpha^N_t)^{\theta\alpha}}{(1-\nu^o_t)\, w_t} \frac{1}{\tau_{ro}} \right)^{\alpha_r} \left(\frac{\theta (1-\alpha_r) p^{INT}_t S^j_{ro,t} (\alpha^N_t)^{\theta\alpha}}{(1-\nu^o_t)\, p_{b,t}} \right)^{1-\alpha_r} \right]^{\theta}$$

$$S^j_{ro,t} = \left(A^j_t \right)^{\frac{1}{1-\theta}} \left[\left(\frac{\alpha_r}{w_t} \frac{1}{\tau_{ro}} \right)^{\alpha_r} \left(\frac{(1-\alpha_r)}{p_{b,t}} \right)^{1-\alpha_r} \right]^{\frac{\theta}{1-\theta}} \left[\frac{\theta p^{INT}_t (\alpha^N_t)^{\theta\alpha}}{(1-\nu^o_t)} \right]^{\frac{\theta}{1-\theta}}$$

Finally, considering (2.57) yields:

$$S^j_{ro,t} = \left(A^j_t\right)^{\frac{1}{1-\theta}} \left[\frac{\theta(\alpha^N_t)^{\theta_\alpha}}{(1-\nu^o_t)} \frac{p^{INT}_t}{p^{BUN}_t}\right]^{\frac{\theta}{1-\theta}} \tag{2.58}$$

Entry threshold

In this section we derive the entry threshold related to the intertemporal zero profit condition $V\left(\hat{A}_t\right) = 0$.

We define the firm's value function:

$$V_t\left(A^j_t\right) = \Pi^j_{ro,t} - c + \beta E_t\left\{\Lambda_{t+1} V_{t+1}\left(A^j_t\right)\right\} \tag{2.59}$$

We begin inserting equations (2.55), (2.56) and (2.58) into equation (2.53):

$$\Pi^j_{ro,t} = p^{INT} S^j_{ro,t} - \frac{(1-\nu^o_t)}{(\alpha^N_t)^{\theta_\alpha}} w_t \frac{\theta\alpha_r p^{INT}_t (\alpha^N_t)^{\theta_\alpha}}{(1-\nu^o_t) w_t} \left(A^j_t\right)^{\frac{1}{1-\theta}} \left(\frac{\theta(\alpha^N_t)^{\theta_\alpha}}{(1-\nu^o_t)} \frac{p^{INT}_t}{p^{BUN}_t}\right)^{\frac{\theta}{1-\theta}} -$$

$$- \frac{(1-\nu^o_t)}{(\alpha^N_t)^{\theta_\alpha}} p_{b,t} \frac{\theta(1-\alpha_r) p^{INT}_t (\alpha^N_t)^{\theta_\alpha}}{(1-\nu^o_t) p_{b,t}} \left(A^j_t\right)^{\frac{1}{1-\theta}} \left(\frac{\theta(\alpha^N_t)^{\theta_\alpha}}{(1-\nu^o_t)} \frac{p^{INT}_t}{p^{BUN}_t}\right)^{\frac{\theta}{1-\theta}}$$

$$\Pi^j_{ro,t} = p^{INT} \left(A^j_t\right)^{\frac{1}{1-\theta}} \left(\frac{\theta(\alpha^N_t)^{\theta_\alpha}}{(1-\nu^o_t)} \frac{p^{INT}_t}{p^{BUN}_t}\right)^{\frac{\theta}{1-\theta}} \left[1 - \theta\alpha_r - \theta(1-\alpha_r)\right]$$

$$\Pi^j_{ro,t} = \left(p^{INT}\right)^{\frac{1}{1-\theta}} \left(A^j_t\right)^{\frac{1}{1-\theta}} \left(\frac{\theta(\alpha^N_t)^{\theta_\alpha}}{(1-\nu^o_t)} \frac{1}{p^{BUN}_t}\right)^{\frac{\theta}{1-\theta}} \left[1-\theta\right]$$

$$\Pi^j_{ro,t} = \left(p^{INT} A^j_t\right)^{\frac{1}{1-\theta}} \left(\frac{\theta(\alpha^N_t)^{\theta_\alpha}}{(1-\nu^o_t)} \frac{1}{p^{BUN}_t}\right)^{\frac{\theta}{1-\theta}} \left[1-\theta\right]$$

$$\Pi^{j}_{ro,t} = (1-\theta)\left(A^{j}_{t}\frac{\theta^{\theta}(\alpha^{N}_{t})^{\theta_{\alpha}}}{(1-\nu^{o}_{t})^{\theta}}\frac{p^{INT}}{\left(p^{BUN}_{t}\right)^{\theta}}\right)^{\frac{1}{1-\theta}}$$

We can insert the last results into equation (2.59):

$$V_t\left(A^{j}_{t}\right) = (1-\theta)\left(A^{j}_{t}\frac{\theta^{\theta}(\alpha^{N}_{t})^{\theta_{\alpha}}}{(1-\nu^{o}_{t})^{\theta}}\frac{p^{INT}}{\left(p^{BUN}_{t}\right)^{\theta}}\right)^{\frac{1}{1-\theta}} - c + \beta E_t\left\{\Lambda_{t+1}V_{t+1}\left(A^{j}_{t}\right)\right\}$$

Note that we can write, for operative firms:

$$E_t\left\{V_t\left(A^{j}_{t}\right)\right\} = \int_{\widehat{A}_{t+1}}^{+\infty} V_{t+1}\left(A^{j}_{t+1}\right)\frac{\xi\left(\widehat{A}_{t+1}\right)^{\xi}}{\left(A^{j}_{t+1}\right)^{\xi+1}} d\left(A^{j}_{t+1}\right) = E_t\left\{H_{t+1}V^{av}_{t+1}\right\}$$

Hence:

$$V_t\left(A^{j}_{t}\right) = (1-\theta)\left(A^{j}_{t}\frac{\theta^{\theta}(\alpha^{N}_{t})^{\theta_{\alpha}}}{(1-\nu^{o}_{t})^{\theta}}\frac{p^{INT}}{\left(p^{BUN}_{t}\right)^{\theta}}\right)^{\frac{1}{1-\theta}} - c + \beta E_t\left\{\Lambda_{t+1}H_{t+1}V^{av}_{t+1}\right\}$$

Considering the intertemporal zero-profit condition yields:

$$(1-\theta)\left(A^{j}_{t}\frac{\theta^{\theta}(\alpha^{N}_{t})^{\theta_{\alpha}}}{(1-\nu^{o}_{t})^{\theta}}\frac{p^{INT}}{\left(p^{BUN}_{t}\right)^{\theta}}\right)^{\frac{1}{1-\theta}} - c + \beta E_t\left\{\Lambda_{t+1}H_{t+1}V^{av}_{t+1}\right\} = 0$$

$$\left(A^{j}_{t}\right)^{\frac{1}{1-\theta}} = \frac{c - \beta E_t\left\{\Lambda_{t+1}H_{t+1}V^{av}_{t+1}\right\}}{(1-\theta)\left(\frac{\theta^{\theta}(\alpha^{N}_{t})^{\theta_{\alpha}}}{(1-\nu^{o}_{t})^{\theta}}\frac{p^{INT}}{\left(p^{BUN}_{t}\right)^{\theta}}\right)^{\frac{1}{1-\theta}}}$$

$$\hat{A}_t = \left[\frac{c - \beta E_t \left\{ \Lambda_{t+1} H_{t+1} V_{t+1}^{av} \right\}}{(1-\theta) \left(\frac{\theta^{\theta} (\alpha_t^N)^{\theta\alpha}}{(1-\nu_t^o)^{\theta}} \frac{p^{INT}}{\left(p_t^{BUN}\right)^{\theta}} \right)^{\frac{1}{1-\theta}}} \right]^{1-\theta}$$

Finally, the threshold is:

$$\hat{A}_t = \left[\frac{c - \beta E_t \left\{ \Lambda_{t+1} H_{t+1} V_{t+1}^{av} \right\}}{\theta^{\theta} (1-\theta)} \right]^{1-\theta} \frac{(1-\nu_t^o)^{\theta}}{(\alpha_t^N)^{\theta\alpha}} \frac{\left(p_t^{BUN}\right)^{\theta}}{p^{INT}} \tag{2.60}$$

Intermediate Sector Aggregation

To obtain $S_{ro,t}$, we integrate the supply function (2.58) for the productivity level A_t^j:

$$S_{ro,t} = \left[\frac{\theta (\alpha_t^N)^{\theta\alpha}}{(1-\nu_t^o)} \frac{p_t^{INT}}{p_t^{BUN}} \right]^{\frac{\theta}{1-\theta}} \left[\int_{\hat{A}_t}^{+\infty} \left(A_t^j\right)^{\frac{1}{1-\theta}} \frac{\xi_p z^{\xi_p}}{\left(A_t^j\right)^{\xi_p+1}} d(A_t^j) + \int_{\hat{A}_t}^{+\infty} \left(A_t^j\right)^{\frac{1}{1-\theta}} \frac{\xi_p \left(\hat{A}_{t-1}(1-\delta)\right)^{\xi_p}}{\left(A_t^j\right)^{\xi_p+1}} d(A_t^j) \right]$$

$$S_{ro,t} = \frac{\xi_p (1-\theta)}{\xi_p (1-\theta) - 1} \left[\frac{\theta (\alpha_t^N)^{\theta\alpha}}{(1-\nu_t^o)} \frac{p_t^{INT}}{p_t^{BUN}} \right]^{\frac{\theta}{1-\theta}} \eta_{f,t} \hat{A}_t^{\frac{1}{1-\theta}} \tag{2.61}$$

To obtain $S_{rf,t}$, we exploit equation (2.8):

$$S_{rf,t} = \left(\frac{\theta p_t^{INT}}{(1+\nu_t^f) p_t^{BUN}} \right)^{\frac{\theta}{1-\theta}} \tag{2.62}$$

Retailers

Retailers aggregate contact-intensive and online quantities to produce the final bundle S_t:

$$S_t = S_{rf,t} + S_{ro,t}$$

$$S_t = \left[\frac{\xi_p (1-\theta)}{\xi_p (1-\theta) - 1} \left[\frac{(\alpha_t^N)^{\theta_\alpha}}{(1-\nu_t^o)} \right]^{\frac{\theta}{1-\theta}} \eta_{f,t} \hat{A}_t^{\frac{1}{1-\theta}} + \left[\frac{1}{(1+\nu_t^f)} \right]^{\frac{\theta}{1-\theta}} \right] \left(\frac{\theta p_t^{INT}}{p_t^{BUN}} \right)^{\frac{\theta}{1-\theta}} \tag{2.63}$$

Retailers optimally set prices and their problem is:

$$\max \Pi_t^R = \frac{p_t^j}{p_t} \left(S_{rf,t} + S_{ro,t} \right) - p_t^{INT} \left(S_{rf,t} + S_{ro,t} \right) - \frac{\gamma}{2} \left(\frac{p_t^j}{p_{t-1}^j} - 1 \right)^2 S_t$$

$$s.t. \tag{2.64}$$

$$S_t^j = S_{rf,t} + S_{ro,t}$$

$$S_t^j = S_t \left(\frac{p_t^j}{p_t} \right)^{-\psi}$$

We insert the two constraints and compute the derivative $\frac{\partial \Pi_t^R}{\partial p_t^j} = 0$:

$$(1-\psi) S_t \frac{p_t^{j-\psi}}{p_t^{1-\psi}} - \psi p_t^{INT} S_t \frac{p_t^{j-\psi-1}}{p_t^{-\psi}} - \gamma \left(\frac{p_t^j}{p_{t-1}^j} - 1 \right) \frac{1}{p_{t-1}^j} S_t + \gamma \left(\frac{p_{t+1}^j}{p_t^j} - 1 \right) \frac{1}{p_t^{j,2}} S_{t+1} = 0$$

Consider a symmetric equilibrium where $p_t^j = p_t$ and define $\frac{p_t^j}{p_{t-1}^j} = \pi_t$:

$$(1-\psi) S_t \frac{1}{p_t} - \psi p_t^{INT} S_t \frac{1}{p_t} - \gamma(\pi_t - 1) \frac{1}{p_{t-1}} S_t + \gamma(\pi_{t+1} - 1) \frac{1}{p_t^2} S_{t+1} = 0$$

Now multiply for p_t and divide for S_t to get the NKPC:

$$(1-\psi)+\psi p_t^{INT}-\gamma(\pi_t-1)\pi_t+\gamma(\pi_{t+1}-1)\pi_{t+1}\frac{S_{t+1}}{S_t}=0 \quad (2.65)$$

Households

The households problem assumes the following form:

$$\max_{C_t,b_t,N_t} U_t = E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{(C_t)^{1-\sigma}}{1-\sigma}-\frac{\alpha_t^N}{1+\eta}\chi_1(N_t)^{1+\eta}\right\}$$
$$s.t. \quad (2.66)$$
$$C_t+b_t=\frac{R_{t-1}b_{t-1}}{\pi_t}+(1-t_t)w_tN_t$$

The Lagrangean of the problem is:

$$\text{L=E}_t\beta^t\left\{\ \frac{(C_t)^{1-\sigma}}{1-\sigma}-\frac{\alpha_t^N}{1+\eta}\chi_1(N_t)^{1+\eta}-\lambda_t\left[C_t+b_t-\frac{R_{t-1}b_{t-1}}{\pi_t}-(1-t_t)w_tN_t\right]\ \right\}$$

The first order conditions are:

$I.\ \frac{\partial L}{\partial C_t}=0$

$$\lambda_t=C_t^{-\sigma}$$

$II.\ \frac{\partial L}{\partial B_t}=0$

$$-\lambda_t\beta^t+\lambda_{t+1}\beta^{t+1}\frac{R_t}{\pi_{t+1}}=0$$

$III.\ \frac{\partial L}{\partial N_t}=0$

$$\alpha_t^N \chi_1 (N_t)^\eta = \lambda_t (1 - t_t) w_t$$

Plug I into II to obtain the Euler equation

$$C_t^{-\sigma} \beta^t = C_{t+1}^{-\sigma} \beta^{t+1} \frac{R_t}{\pi_{t+1}}$$

$$R_t = \pi_t \frac{C_t^{-\sigma}}{C_{t+1}^{-\sigma}} \frac{\beta^t}{\beta^{t+1}}$$

$$\frac{1}{R_t} = \beta \left[\pi_{t+1}^{-1} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] \tag{2.67}$$

Plug I into III, IV and V to obtain the labor supply:

$$(1 - t_t) w_t = \alpha_t^N \chi_1 (N_t)^\eta C_t^\sigma \tag{2.68}$$

Now consider

$$N_t = N_{rf,t} + N_{ro,t} + N_{b,t}$$

This finally yields

$$(1 - t_t) w_t = \alpha_t^N \chi_1 (N_{rf,t} + N_{ro,t} + N_{b,t})^\eta C_t^\sigma \tag{2.69}$$

2.5.2 Complete list of equations

This section present the full set of equations.

- Euler equation

$$\frac{1}{R_t} = \beta \left[\pi_{t+1}^{-1} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right]$$

- Labor supply

$$(1-t_t)w_t = \left(\alpha_t^N\right) \chi_1 (N_{rf,t} + N_{ro,t} + N_{b,t})^{\eta} (C_t^{\sigma})$$

- Base production function

$$S_{b,t} = A_b N_{b,t}^{\alpha}$$

- Base labor demand

$$w_t = p_{b,t} \alpha A_b N_{b,t}^{\alpha-1}$$

- Contact-intensive base good demand

$$S_{brf,t}^{j} = \frac{\theta(1-\alpha_r) p_t^{INT} S_{rf,t}^{j}}{(1+\nu_t^f) p_{b,t}}$$

- Online base good demand

$$S_{bro,t}^{j} = \frac{\theta(1-\alpha_r) p_t^{INT} S_{ro,t}^{j} (\alpha_t^N)^{\theta_\alpha}}{(1-\nu_t) p_{b,t}}$$

- Contact-intensive labor demand

$$N_{rf,t}^{j} = \frac{\theta \alpha_r p_t^{INT} S_{rf,t}^{j}}{(1+\nu_t^f) w_t}$$

- Online labor demand

$$N_{ro,t}^{j} = \frac{\theta \alpha_r p_t^{INT} S_{ro,t}^{j} (\alpha_t^N)^{\theta_\alpha}}{(1-\nu_t^o) w_t}$$

- Contact-intensive output

$$S_{rf,t} = \left(\frac{\theta p_t^{INT}}{(1+\nu_t^f) p_t^{BUN}} \right)^{\frac{\theta}{1-\theta}}$$

- Online output

$$S_{ro,t} = \frac{\xi_p (1-\theta)}{\xi_p (1-\theta) - 1} \left[\frac{\theta (\alpha_t^N)^{\theta_\alpha}}{(1-\nu_t^o)} \frac{p_t^{INT}}{p_t^{BUN}} \right]^{\frac{\theta}{1-\theta}} \eta_{ft} \hat{A}_t^{\frac{1}{1-\theta}}$$

- Aggregate output

$$S_t = \left[\frac{\xi_p (1-\theta)}{\xi_p (1-\theta) - 1} \left[\frac{(\alpha_t^N)^{\theta_\alpha}}{(1-\nu_t^o)} \right]^{\frac{\theta}{1-\theta}} \eta_{ft} \hat{A}_t^{\frac{1}{1-\theta}} + \left[\frac{1}{(1+\nu_t^f)} \right]^{\frac{\theta}{1-\theta}} \right] \left(\frac{\theta p_t^{INT}}{p_t^{BUN}} \right)^{\frac{\theta}{1-\theta}}$$

- Price of factor bundle

$$p_t^{BUN} = \left(\frac{w_t \tau_{ro}}{\alpha_r} \right)^{\alpha_r} \left(\frac{p_b}{(1-\alpha_r)} \right)^{1-\alpha_r}$$

- NKPC

$$(1-\psi) + \psi + p_t^{INT} + \gamma \mathbb{E}_t \Lambda_t \left[(\pi_{t+1} - 1) \pi_{t+1} \frac{S_{t+1}}{S_t} \right] = \gamma (\pi_t - 1) \pi_t \frac{S_{t+1}}{S_t}$$

- Taylor rule

$$\frac{R}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}} \right)^{\theta_\pi} \left(\frac{S_t}{\bar{S}} \right)^{\theta_S}$$

- Base good clearing

$$S_{b,t} = S_{brf,t} + S_{bro,t}$$

- Aggregate resource constraint

$$S_t = C_t + \eta_{f,t} c$$

- Government budget constraint

$$\nu_t^o S_{ro,t} + \frac{R_{t-1} b_{t-1}}{\pi_t} + g_t = t_t w_t (N_{rf,t} + N_{ro,t} + N_{b,t}) + \nu_t^f S_{rf,t} + b_t$$

- Tax rule

$$\frac{t_t}{\bar{t}} = \left(\frac{b_{t-1}}{\bar{b}}\right)^{\xi}$$

- Pandemic shock

$$\alpha_t^N = \left(\alpha_{t-1}^N\right)^{\rho} \left(\frac{S_{rf,t}}{S_{rf,t-1}}\right)^{\Delta(1-\rho)} \exp \varepsilon_t$$

- Incumbents

$$INC_t = \eta_{f,t-1} \left(\frac{\hat{A}_{t-1}(1-\delta)}{\hat{A}_t}\right)^{\xi_p}$$

- New entrants

$$NE_t = \left(1-\eta_{f,t-1}\right)\left(\frac{z}{\hat{A}_t}\right)^{\xi_p}$$

- Online firms

$$\eta_{f,t} = INC_t + NE_t$$

- Entry threshold

$$\hat{A}_t = \left[\frac{c-\beta E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right\}}{\theta^{\theta}(1-\theta)}\right]^{1-\theta} \frac{(1-\nu_t^o)^{\theta}}{(\alpha_t^N)^{\theta_\alpha}} \frac{\left(p_t^{BUN}\right)^{\theta}}{p^{INT}}$$

- Continuation value

$$V_{t+1}^{av} = \Pi_{ro,t+1}^{av} - c + \beta E_{t+1}\left\{\Lambda_{t+2}H_{t+2}V_{t+2}^{av}\right\}$$

- Average online profits

$$\Pi_{ro,t}^{av} = \left(S_{ro,t}^{av}\right)\left(1-\theta p_t^{INT}\right)$$

- Average online output

$$S_{ro,t}^{av} = \left[\frac{p_t^{INT}}{p_t^{BUN}} \frac{\theta(\alpha_t^N)^{\theta_\alpha}}{(1-\nu_t^o)}\right]^{\frac{\theta}{1-\theta}} \frac{\xi(1-\theta)}{\xi(1-\theta)-1}\hat{A}_t$$

- Survival probability

$$H_t = \left(\frac{\hat{A}_{t-1}(1-\delta)}{\hat{A}_t}\right)^{\xi}$$

- Stochastic discount factor

$$\Lambda_t = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}$$

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