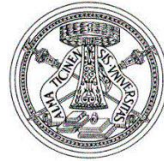


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The Role of Information into Matching Markets

From Theoretical Models to Practical Applications on
Real-World Hiring Procedure for Assistant Professors in
Italy

E. Quintilii

Supervisors: Prof. Fabio Tramontana (Università Cattolica del Sacro Cuore)

Prof. Giorgio Rampa (Università degli Studi di Pavia)

Reviewers: Prof. Antonio Romero-Medina (Universidad Carlos III de Madrid)

Prof. Matteo Triossi (Universidad Santiago de Chile)

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*To all the ones who believed
that changes are possible
no matter how big they are,
if changes start from us.*

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Abstract

The thesis is made up by three independent papers, logically linked by their validity for the analysis and redesigning proposal of the hiring procedure for Assistant Professors in Italy. All of them refer to the literature on matching theories and market design. The papers focus on the role played by information into the setting analyzed and their results are generalizable to similar job markets. The dissertation illustrates a complete work of market design, presented from the analysis of the current procedure to the proposal of a new structure. The research work started from formally demonstrating if the real-world market is failing, and, in the affirmative case, what is causing its failure. Then, the first paper introduces a decentralized matching model that proceeds by rounds, that formalizes this hiring procedure. The model shows what kind of assumptions and constraints are necessary for achieving stable results in this particular matching mechanism. On the reverse, it highlights which are the structure features that create unstable outcomes and allows the market designer to formally identify the causes of inefficiencies. The model is formalized in a setting of complete information, such that it makes clearer and easier its analysis and explanation. However, due to the unreality of this setting, I stressed the model into more realistic context where information is not always available. The second paper starts by removing only part of the information (the preferences' profiles of the other agents), and ends into an "uncertainty" scenario. I recalled the model of symmetric information (Roth and Rothblum 1999) and I demonstrated that the mechanism implements a Bayesian Nash Equilibrium strategy profile characterized by multiple stable equilibria. Then, I stressed the model by removing almost all the information, such that they do not know how many institutions will offer a job positions, which of them will open the vacancy and when. I also assumed that agents do not have lists of preferences but they follow their utility function. I set the assumption that any candidate beliefs that the round she is playing will be the last. I demonstrated that up to the real preference profile, for a shared common belief on the others' preference list, and

for a shared common belief over the state of the world, a truthfully revealing strategy profile is a Bayesian Nash Equilibrium.

Given the difficulty of managing a decentralized mechanism, I formalize the hiring procedure as a centralized market. I outline the advantages obtained by this redesigning policy, but I also show that there is still an aspect not addressed: the “meritocracy problem”. By the expression “meritocracy problem”, I refer to any contest whose winner is not actually better than all the others. The aim of the third paper is to transfer the meritocracy problem into the matching model. I recognize the so called “meritocracy problem” as a problem related to the decision rules and equivalent to an agency problem between the Institution and the committee. The focus is on the realization of the preference list. The meritocracy problem is given by a misalignment of interest – and preferences – between these two agents. It happens that the preferences’ lists submitted by the committees not always reflects the real preferences of the Institutions, i.e. commissioners misreport Institutions’ preferences. The results obtained after running the DA are stable up to the submitted preferences, but unstable up to what Institutions really desire. This instability is enough for stating the agency problem as a potential threat to the well-functioning of a matching market where at least one group of agents follow a semi-regulated decision process. I propose a solution focused on the control of commissioners’ actions.

Introduction

The thesis is made up by three independent papers, logically linked by their validity for the analysis and redesigning proposal of the hiring procedure for assistant professors in Italy. All of them refer to the literature on matching theories and market design whose presentation cover the first section of this thesis.

Since the first case of redesigning of a real-world market occurred (Roth 1984a, 2002; Roth and Peranson 2002), the way of looking at entry-level labor markets changed, and so did the role of the economist (Roth 2002). A hiring procedure is not just a procedure, but it is a complex economic space where resources are allocated on the basis of the agents' preferences, i.e. a matching market. It can be organized according to a centralized or decentralized structure, and its functioning is determined by clear rules, and by the matching mechanism which assigns the agents on one side of the market to the agents on the other side. Before implementing a procedure, it is fundamental to know deeply the market such as the mechanism chosen for the resources' allocation is the most suitable for it. Most of all, the right system should avoid the market to show inefficiencies that cause its inevitable failure (Roth 2007). For this reason, it is fundamental to have a deep knowledge of any single detail of the market, starting from its regulation till the behaviors of the agents (Roth 2010). If a market is recognized to be failing, it is necessary to identify the source/s of the bad-functioning at first, and then, to proceed with a proposal of fixing of redesigning policies – according to the what it is causing the failure – after a detailed and extensive analysis. This practice is defined as market design and constitutes a branch of the brand new field of economics addressed as Engineering Economics. This thesis reports the work of market design that I conducted on the hiring procedure for Assistant Professor in Italy.

The hiring procedure for Assistant Professor in Italy is based on a decentralized structure where each public university carries on an open competitive exam for assigning the job. The Law 240/2010 reports the mandatory guidelines for the procedure imposed by the Ministry of Education, University and Research. At first the opening of a position is publicly announced, and the university starts to gather the applications of the candidates till a declared deadline. Then, the academic profiles of the candidates are evaluated on the basis of the mandatory requirements imposed by the Law, and on the specific requirements detailed in the announcement by the single university. After this step, a part of the applications is rejected and a part of them is accepted to participate at the oral examination. This test is held at the university site and the candidate needs to physically be there, otherwise she is excluded from the procedure. Unfortunately, travel costs are not covered by the university, and it could happen that during the same day there is the oral examination at two different universities, such as the candidate is forced to renounce at least to one of them. Moreover, the market does not show the same thickness for all the procedures – even if of the same scientific sector – till the point that, more than one time, a sole applicant has been registered for a competition. The positions are continuously open during the year – i.e. there is not a specific hiring timing – and it is not possible to officially know how many institutions will offer a job and when. These few elements about the procedure could be enough to assert that probably the market is not working as good as possible. However, to affirm that this assignment system is inefficient is necessary a deeper analysis of its structure and functioning. At first, the procedure has to be formalized as a matching model, that allows an analysis at a theoretical level whose results will be concretized into policy advices later.

The first paper introduces a brand-new decentralized matching model that proceeds by rounds, by starting from the formalization of the hiring procedure for Assistant Professors in Italy. It differentiates from what has been presented so far in the literature, and constitutes a powerful tool of analysis of the real market mentioned above. At the same time, it formalizes the “contest mechanism”, widely used in the public sector, but it keeps also the strength of being implemented in

the more general job market. The game is made up by a finite number of rounds, and each round has the same structure: 1) some institutions offer a job position; 2) candidates apply to the job positions; 3) each institutions pick one candidate and order the others into a waiting list; 4) temporary matchings are realized. Institutions play only once, while candidates have no restrictions on the number of applications they can send. Moreover, institutions commit to the temporary matchings but candidates have the right to break the current assignment when accepted by a new institution. Then, for the first time it is introduced the concept of “institutional acceptability” (or “costs-constraint” for companies) as a limit on the possibility of rejecting candidates up to the institutional regulation (or due to the costs for companies of carrying on a long hiring procedure). The model shows what kind of assumptions and constraints are necessary for achieving stable results in these particular matching mechanism. On the reverse, it highlights which are the structure features that create unstable outcomes and allows the market designer to formally identify the causes of inefficiencies. The model is formalized in a setting of complete information, such that it makes clearer and easier its analysis and explanation. It marks the starting point of the “redesigning journey” of the real Italian case-study that this thesis represents, and it constitutes the benchmark model for the following paper.

Even if some conclusions about the real market can already be deducted from the analysis of the model in a complete information setting, it hardly suffers of unreality. Most of all, the information plays a key role into the procedure, since according to how many information candidates have, their strategies can differ. For this reason, it seemed worthy to stress the model under this point view, and to carry on an analysis of what could happen as more and more information are removed.

The second paper brings the decentralized multiple-round mechanism in a more realistic environment where less and less information are available. It starts by showing the consequences of removing only part of the information (the preferences’ profiles of the other agents), and ends into an “uncertainty” scenario introduced for the very first time in the matching literature. The aim of this setting

is to reproduce the real informative conditions as good as possible, and to test the functioning of the model. I recalled the model of symmetric information (Roth and Rothblum 1999) and I demonstrated that in a setting of incomplete information, the mechanism implements a Bayesian Nash Equilibrium strategy profile characterized by multiple stable equilibria. Their outcomes are always stable until the numerical asymmetry between the two sets of the market occurs. and gives stable outcomes. Even though for any candidate revealing a truncation of her preferences stochastically dominates to report the entire list, this kind of strategies acquires more and more risk as information are removed, and as the number of candidates into the market increases. Then, I stressed the model by removing almost all the information, such that they do not know how many institutions will offer a job positions, which of them will open the vacancy and when. I also assumed – under the light of a realistic reproduction – that agents do not have lists of preferences that are ready to be used for defining their actions, on the contrary, they have a utility function that helps them decide what to do at each round, and their actions define the *a posteriori* preferences' list. For the analysis in the setting of uncertainty – or strong incomplete information – I set the assumption that – since they do not have any information on the total job offer – any candidate beliefs that the round she is playing will be the last. Then, I demonstrated that up to the real preference profile, for a shared common belief on the others' preference list, and for a shared common belief over the state of the world, a truthfully revealing strategy profile is a Bayesian Nash Equilibrium. Even if the results could appear intuitive, and not so innovative, it is necessary to remember that 1) nobody has ever stressed a decentralized model so much; 2) for the first time, agents do not have prepared list of preferences over the other set but only a “utility profile” defined by their utility function; 3) the strength of the model is the conclusions it allows to deduct about real markets. The example of the practical application on the hiring procedure of Assistant Professors in Italy, clears the point. Confused, chaotic, far from transparent, the hiring procedure has so many failing features that it could be hard to identify which are the most important, and which of them can and have to be addressed by a redesigning policy. The huge gap of information

suffered from the market could then drive a researcher or analyst into a trap, a smock curtain that hides the real problems. But, as long as the model achieves desirable results also when traditional basic information (as the own preference profile) are removed, then it is easier to focus on the main determinant of inefficiency: the timing.

The timing is then found to be the core of the market failure, and one of the most fascinating aspects into a matching mechanism. Hard to be directly included into the model, and just explored a little in this study, it paves the way for further studies. Analyzing if and how Institutions can use the timing of their entrance into the market – i.e. the choice of when offering the job position – as a strategy for attracting more preferable candidates, will be the focus of future investigations. However, the studies of the first two papers show that the real decentralized mechanism can be improved anyway – at least a little - by acting on its timing-management. Unfortunately, some adjustments would require too much effort and would distort the decentralized nature of the system. As long as this “inefficiency threshold” is reached, it becomes worthless to work on a system whose efficiency has been compromised in favor of stable outcomes. Moreover, by imposing new strict mandatory rules it could also generate a reverse effect on the system, causing a development of “unofficial communications” that would be able to game the mechanism by imposing “unofficial rules” for the agents. For example, assume that all the institutions that offer into the same period are now requested by law to reveal the results of the contest on the same day and to give fifteen days to winner-candidates for accepting. The system would be unable to face out-of-the-game arrangements, such that an institution forces the candidate to accept the offer before the official day. It could also trigger the institutions to offer the job sooner and sooner as the contest starts, making the selection based less on a serious analysis of the candidates, and more on fast-judging characteristics (e.g. previous work-relationships, network, ..). Since this is not something completely new in the Italian case, it is a quite credible consequence. The results obtained so far can be generalized for the job market too. Of course the policy considerations are not applicable into a wide – and uncontrollable – environment, but they still hold for

“closed” hiring procedure, e.g. the internships’ assignment of students to companies managed by the college itself (usual for Master courses in Italy).

The difficulty of managing a decentralized mechanism clearly foster the need of thinking a new system, able to solve the inefficiencies and to guarantee a sufficient control over the procedure. It comes natural, then, to follow one of the most famous real-world examples in the entry-level job market: The American Market for Physicians (Roth 1984a, 1991a; Roth and Peranson 2002) and to propose a centralization of the market. More invasive than a restyling policy, the reorganization would manage the current failures, ensure stable outcomes and control over the timing of the hiring procedure, in addition to the simplicity of aligning all institutions to same rules (as asked by Law). So, I formalize the hiring procedure as a centralized market. I show how it would be structured and how it would work by implementing the deferred-acceptance algorithm (Gale and Shapley 1962) as matching mechanism. Then, I outline the advantages obtained by this redesigning policy, but I also show that even if this procedure is better than the current one, there is still an aspect that is not addressed: the “meritocracy problem”. By the expression “meritocracy problem”, I refer to any situation where the candidate selected as winner of the contest does not meet one or more mandatory requirements of the job announcement, or she is not the one who satisfy them at best. Equivalently, I refer to any contest whose winner is not actually better than all the others. So far we consider institutions as non-strategic, following the dominant strategy of choosing who they prefer the most. The assumption behind the preferences’ lists of institutions was that the order was given by the level of suitability of the candidates to the job, in the decentralized as in the centralized structure. Real events negate this assumption, such that institutions are picking who they prefer, but the one they prefer does not match the one who is the best for the position. Unfortunately, this is a contradiction. Indeed, the institutions state in the job announcements the requirements on which the evaluation of candidates will be based. Consequently, it is rational to assume that the preference list of an institution will report the order of candidates according to this evaluation, such that the first is better than all the others. At this point the

natural question to rise is: why should the institution assign the job position to someone who is not the best for it – and build a non-rational preference list?

The aim of the third paper is to transfer the meritocracy problem into the matching model for highlighting the consequences it has on the functioning of the mechanism (mostly on the stability of the results), and for demonstrating that with a centralized structure a solution is possible. At first, I recognize the so called “meritocracy problem” as a problem related to the decision rules or process implemented while building up the preferences’ lists. Then, I briefly categorize different kinds of decision process that agents follow – based on real matching studies – as a) private, b) regulated and c) semi-regulated. The latter is characterized for being a two-steps decision process, carried out by two different agents who act as being one, or better by a principal and a representative. Thanks to this analysis, it is clear that the meritocracy problem is equivalent to an agency problem (Eisenhardt 1989; Gjesdal 1982; Ross 1973a) between the Institution – that gives guidelines into its regulations and in the job announcement – and the committee made up by professors – who practically select the candidates. For the first time the famous information problem of agency is treated into a matching model. The focus is then on the realization of the preference list. In fact, the Institution has preferences over the qualities of the candidates, but not directly over individuals, such that it is able to identify group of equivalent likelihood but not to express a strict preference relation. This is the reason why for completing the task it need a committee whose role should be only to break the ties into the indifference groups in order to build up the list. The meritocracy problem is then given by a misalignment of interest – and preferences – between these two agents. Real cases show that sometimes the member of the commission prefer candidates they already know (because of personal or work motives), instead of the best ones. From a matching point of view, it happens that the preferences’ lists submitted by the committees not always reflects the real preferences of the Institutions. This means that commissioners are actually misreporting the Institutions’ preferences by moving candidates over positions of the list. The results obtained after running the DA are surely stable up to the preferences’ orders submitted, but unstable up

to what Institutions really desire. As demonstrated by Roth (Roth 1984b), it is sufficient only the misrepresentation of one agent's preferences for activating a domino effect over the entire system. The instability of the results caused by the misalignment of interest is enough for stating the agency problem as a potential threat to the well-functioning of a matching market where at least one group of agents follow a semi-regulated decision process. I propose a solution focus on the control of commissioners' actions, easily implementable by having an electronical procedure of selection. The paper contributes to the literature by identifying a hidden potential – and highly dangerous – cause of inefficiency of a matching market, never explored so far.

The conclusions briefly recap the most important steps of this work, by highlighting its strengths and its weaknesses, and mostly the most relevant contributions to the literature and to the real-world market. Finally, I mention the interesting future developments that could arise from this work, and starting points for further studies.

Section I

Theoretical framework

According to Roth “*matching is one of the most important functions of markets*” (Roth 2008). Markets of any sort are characterized by a peculiar mechanism of resource allocations. In the so-called matching markets, payments and prices are not involved, and the process of allocation relies on the reciprocal choice of the agents: they can be coupled if and only if both have signaled the other in their respective lists of preferences. During the 1950's L. Shapley started to analyze this kind of markets by focusing on the outcomes' characteristics, such as stability and efficiency, obtained by the process of allocation. In 1962, D. Gale and L. Shapley published the paper “**College Admissions and the Stability of Marriage**”, where concepts stemmed from mathematical matching problems and game theory, were theoretically applied to markets for the first time. The authors demonstrated how it was possible to build up processes of allocation (matching mechanisms) able to guarantee stable outcomes and efficient results in terms of allocation. An outcome or matching is defined stable if it is not blocked by any individual or by any pair of agents, i.e. each agent is acceptable to her mate, and there are no two agents who would prefer each other to their current mates. Stability is one of the elements of the successful functioning of a matching market. It also implies the Pareto Efficiency (Abdulkadiroglu and Sönmez 2010) of the final assignment, such that a stable matching makes all participants as better off as possible. The work started by Gale and Shapley has been carried on by different authors, but the one who mostly contributed to the definition of the theoretical basis of this new field was A. E. Roth. The theorems postulated by these three main authors established the robust basis for the development of Matching Theories in Economics, and for the new branch called *Economic Engineering*. The practice of **Market Design** (Roth

2010, 2007) is one of its main activities: economists are called not only to analyze matching markets, but also to re-design the structure of the inefficient ones (Roth 2002). This area of study represents the practical realization of what Gale and Shapley proposed from a theoretical point of view. Roth has been recognized the pioneer of this new branch whose starting point can be identified in the first proposal of re-organization occurred in 1984 for the market of American Physicians (Roth 1984a). Issues from Microeconomics and Game Theory are the main tools implemented by a *market designer*, together with the theories of matching (Roth 2000). Particularly, from the Game Theory stemmed the idea of building up efficient markets that rely on a set of detailed rules. The successful applications on real-world matching markets (Jackson 2013), designed by Roth in collaboration with other authors (Abdulkadiroğlu, Pathak, and Roth 2005; Abdulkadiroğlu, Pathak, Roth, et al. 2005; Roth 1984; Roth et al. 2005), revealed the practical value of the field. But mostly, they highlighted the great impact on the human welfare of a tailored re-structuring of inefficient matching markets. These real cases are integral part of the literature that is made up by theoretical and practical studies, where the development of new theories is as much important as their application on real-world markets.

The matching markets were categorized as two-sided and one-sided accordingly to the number of groups of agents strategically involved into the game. Then, the model of a **one-sided matching market** involves a group of strategic players who has preferences over the resources that represent the other side of the market. Resources have not preferences or strategies, and they already belong to the agents on the other side of the market. However, any agent would like to own a resource different from the one of the initial endowment. So, they want to re-allocate the available resources among them, on the basis of their preferences. A typical example is the “Housing Market”, used also by L. Shapley and H. Scarf (1974) to introduce the **Top-Trading Cycle Algorithm**. This is an efficient re-allocative procedure that provide a stable outcome, and makes the truthful revealing of real preferences a dominant strategy for all the agents (Roth 1982a).

The most famous – and successful – practical application of these theoretical achievements in the one-sided matching field, is the Kidney Exchange (Roth et al. 2004; Roth, Sönmez, and Ünver 2005; Roth, Sönmez, and Utku Ünver 2005). Thanks to the implementation of an ad hoc algorithm based on the TTC, the Kidney Exchange clearinghouse of New England¹ has reached an amount of 38 transplants per year² – versus the 4 transplants of 2003 and 2004 (when the matchings were manually realized)³.

However, the studies of this dissertation deal only with **two-sided matching markets**. This structure entails two finite set of agents who strategically participate to the allocation mechanism, that assigns players of one side of the market to the ones of the other side, on the basis of their preferences. The model differs according to the number of agents a single player can be paired to, and the outcome can be a one-to-one correspondence – the marriage model – or a many-to-one assignment – the college admission model (Gale and Shapley 1962). In addition to these traditional structure, it has been formalized the many-to-many matching model (Echenique and Oviedo 2004; Martinez et al. 2004; Roth and Sotomayor 1992; Sotomayor 1999) that found a practical application in labor market settings (Hatfield and Kominers 2016; Kominers 2012; Sotomayor 2004). The dissertation will focus on the one-to-one assignment structure, where one agent of one set is coupled with exactly one agent of the other set. The procedures of matching can be completely centralized or decentralized, according to the market structure. In the first case, the agents only interact with a central body – a clearinghouse – that manages the matching mechanism by submitting a list of preferences; in the second case, agents directly interact in a sequential game of proposal and acceptances/rejections.

A **decentralized matching procedure** describes very common real situations, mostly in college admission and labor market settings – imagine Colleges that

¹ The New England Program for Kidney Exchange - NEPK, established in 2004.

² Based on the data from Kidney Exchange Connection website, www.kidneyexchangeconnection.org.

³ Data from « The evolution of a successful Kidney Paired Donation Program » (Hanto et al. 2010).

send the letters where they offer a seat to students and then wait for their response, or Firms that make a job offer to a worker. As above-mentioned, the model implies a sequential dynamic game, where the preferences of the agents are revealed through their actions – they do not submit a list – i.e. a non-revelation game. The decentralized situations have been later and less explored than the centralized procedures. Introduced for the first time by Roth and Vande Vate (1990, 1991), all the following papers were committed in analyzing how specific features can affect the functioning of the model or the strategies of the agents. The first two papers of this dissertation belong to this current of the matching branch. They aspire to enrich the existing literature by introducing a new kind of decentralized mechanism that describes a hiring procedure, and by exploring it into incomplete information settings. The literature on decentralized structure have been mainly focused on college admission problem and hiring procedures. In 1997 Blum et al. considered the dynamic game of offers and reaction into the senior-level labor market – while usually literature focuses on the assignments of entry-level job positions. The decentralized procedure helps a previous stable market to return to stability, i.e. as a vacancy is created by a retirement, the market needs to fulfill again the position by direct offers (vacancy chains literature). The study presents a decentralized mechanism that starts from an existing assignment that mimic the Deferred Acceptance algorithm, and it is called Decentralized DA. Even if its structure is different from the model presented in the following papers – called DMR – the results are very close. In both cases the equilibrium strategy for the Recruiters' side is to reveal their true preferences, and the outcome is stable for the true preferences, both if workers reveal their true preferences or just a truncation. Moreover, as in the DMR, a differentiation between workers occurs, since some results change according to the “matching state” of the agent. This paper inspired the identification of the temporary-matched candidates and the need of distinguish the strategies' analysis. In both cases, the workers who are still unmatched, can beneficiate of misreporting their true preferences into the game. On the contrary, once they are matched they are no longer able to profit of a deviation from a truthfully revealing strategy. In the DMR mechanism these results

are obtained only if the following assumptions on the game holds: no costs and no restrictions on applications, “institutional acceptability” constraint, and no commitment for the candidates’ side to the temporary assignment obtained at the end of each round. It is not unusual that stability or equilibrium strategies are achieved thanks to assumptions or constraint. Alcalde et al. (1998) demonstrated that their sequential mechanism in which firms propose and workers react, implements in SPE the Firms’ optimal outcome when agents’ preferences are additive. It is also possible to obtain or not the same results when the roles are inverted, and instead of firms the workers make the offers. The first trial of this kind is attributed to Alcalde and Romero-Medina in 2000. They demonstrated that when students play simultaneously, the exchange of role does not affect the mechanism. However, the same result cannot be demonstrated for a model where students sequentially propose (Alcalde and Romero-Medina 2005). The Decentralized Deferred Acceptance mechanism is not the only one that mimics the deferred acceptance algorithm. In 2011 Haeringer and Wooders introduced a sequential model whose functioning recalls the DA, with the exception that the agents’ actions are all sequentially – even if they demonstrated that the simultaneous actions would not affect their results. It is quite close to the DMR mechanism, and on a first view the DMR could seem just the reverse of their model, but this is not the case. In their model, at each stage, all the firms that have a vacancy sequentially send a direct offer to one workers; and sequentially each worker reacts to this offer; if a worker accepts then the matching is formed and the couple is removed from the market. At the following stage, all the firms that have not been matched in the previous stage, send a direct offer and so on. Basically, at the first stage, all firms send a direct offer, and then the mechanism goes on for trying to match the ones that have been rejected by workers. In the DMR, the entire set of recruiters is permuted over a finite set of rounds, so the same number of job positions offered during the first stage in Haeringer and Wooders, in the DMR is offered in multiple rounds. Candidates can send applications only to the recruiter that plays that specific round, and then the recruiter will offer the position to one among the ones who applied. If we assume the exchange the roles of firms and workers in

Haeringer and Wooders, we would have that in the first stage all workers sequentially send a proposal to one firm and there are no limits on which this firm can be. Instead, in the DMR mechanism, candidates can address only a sub-set of firms at each round and cannot respect the preference order – as it would be in the other model. However, the results achieved by both models are very close: there is a unique Subgame Perfect Equilibrium outcome that is optimal for one of the two sides of the market⁴. The similarity of results could be attributed to the fact that both models share features of the D-A algorithm. As already mentioned, the second paper contained into this dissertation explores how information affect the functioning of the mechanism. In 2009, Niederle and Yariv noticed that, in a setting of aligned preferences, a decentralized mechanism could generate unstable outcomes under uncertainty – and in presence of market frictions. In order to drive outcomes to stability then it is necessary to add strong assumptions on the richness of the economy. Previously – in 2008 – Païs considered the lack of information into a decentralized mechanism considering ordinal preferences and implementing the notion of Ordinal Bayesian Nash Equilibrium. The same notion was introduced in 2007 by Ehlers and Massò for exploring the Ordinal Bayesian Nash Equilibrium into a centralized structure that runs the D-A. However, the study of how partial information can affect a – centralized – mechanism has been firstly introduced by Roth and Rothblum in 1999. Interesting are also the laboratory experiments carried out by Niederle and Featherstone (2009) and by Païs et al. (2012) whose results gave a solid real-world basis to some of the statements contained in the papers on DMR. It is easy to notice that both the literature on decentralized mechanism, and mostly on incomplete information in decentralized mechanisms, needs to be enriched and developed. The two papers presented into this dissertation give a first answer to this need.

On the contrary, most of the famous real designs have the same **centralized structure**: a central clearing-house manages the preferences of the agents that

⁴ In Haeringer-Wooders it is Worker-Optimal - workers react to offers made by firms. In the DMR it is Candidate-Optimal – candidates send an application and institutions react by selecting the candidate they like and ordering the others into a waiting list.

strategically play into the market, by implementing a matching algorithm for the matching process. The algorithm varies by the needs of the market. It is necessary to remember that a centralized structure could be the basis of many kinds of assignment procedures. The first matching studies started on a centralized mechanism, on the inspiration of the mathematical problem of matching. D. Gale and L. Shapley transferred the mathematical approach to the matching problem, onto economic spaces with a resource allocation problem. Thanks to this new point of view, they were able to formalize a procedure, called Deferred Acceptance Algorithm whose outcome guarantees the Pareto Optimum. The algorithm was presented for the first time in 1962, and tested for two kinds of two-sided matching markets: the Stability of Marriage (one-to-one assignment), and the College Admission Problem (many-to-one assignment). Some years later, other authors started to study the properties of the algorithm and of the outcome it was able to achieve. The most prominent theorems were developed between the 1970's and the 1990's, even if the research work on the algorithms is still active. Among all, the one who mostly contributed to the matching literature on centralized mechanism – and on the study of the D-A – was A. E. Roth (Roth 1982b, 1985, 2008; Roth and Sotomayor 1992). The deferred-acceptance (D-A) algorithm has been proved to be the one that guarantees the best results: stable outcomes, efficiency, and it makes a dominant strategy to reveal the true preferences for the proposing side of the market⁵. The probability of manipulation of a matching algorithm through the misrepresentation of preferences (Roth 1984b), i.e. how much the algorithm is strategy-proof, determines the well-functioning of the market: if the outcomes are based on false declarations, they cannot be stable and so the market will fail – moreover, due to the domino effect it is sufficient the misconduct of one agent for compromising the final result (Roth 1984b). A recent study has presented a ranking of matching mechanisms on the basis of how easily they can be manipulated (Pathak and Sönmez 2011) and the D-A is showed to be the most robust. The third paper included into this dissertation, recall the

⁵ One of the most famous characteristics of the D-A algorithm, is achieving the best outcome possible for the proposing side, this is the reason why they do not need to misrepresent in order to achieve their most desired mate.

manipulation problem but from a new point of view. Even if the manipulation taken into account is still the misstatement of preferences, so far into the literature, agents are considered to voluntarily report false preferences. However, the paper illustrates that it could happen, that they are not aware of the misrepresentation of their preferences. This is possible when the agent is not a single person, but two subjects (in this case, an Institution and a Committee) connected through an Agency Relation. Introduced for the very first time, the agency relation is certainly a critical aspect of matching market, especially if not taken into account. In fact, to solve a misrepresentation caused by an agency problem – where the two subjects have different interests and preferences – it is not sufficient to implement the D-A algorithm, and to pose that side of the market as the proposing side. In this case, it is necessary to act on the relation between the two subjects, and to implement the (adapted) properties of Affirmative Actions and Responsiveness (Martinez Ruth et al. 2000) introduced by Abdulkadiroglu in 2005. Recently implemented in real-world College Admission problems (Aygün and Bo 2014; Bo 2016), they have been fundamental for formalizing a typical governance problem into a matching one. Typically, they have been used for introducing into the matching model the racial quotas that have to be respected, up to some regulation, when assigning the seats (Aygün and Bo 2014). In a recent work by I. Bo, it has been demonstrated that in some of these cases, the deferred-acceptance algorithm alone could not be sufficient for obtaining the optimal matches (Bo 2016). The final paper of the dissertation goes into this direction, aiming to specify that the designing the reorganizational of a market it is not simply made up creating a centralized structure, and then running the D-A.

But, what is the practice of **Market Design**? The Market Design is considered to be the core of the new research field called Engineering Economics (Roth 2002). The name of the branch assimilates the figure of the economist to the engineer due to the new task the field assigns to her. In a Market Design project, the economist has the responsibility of re-designing and re-structuring inefficient matching markets, after have studied and analyzed them. The re-structure means

the passage from a de/centralized structure to the other, but the work of the market designer entails also to identify the most suitable matching mechanism for any situation. Despite the properties of the D-A, there is not a single algorithm that can be considered the universal solution for every inefficient market and the same is for the design, since they depend on the market conditions. The choice of the mechanism must reflect the characteristics of the market, and be aligned to the regulation in force, as well as be able to front the complexities that can arise in that assignment procedure. The task of the market designer is to be able to identify the matching mechanism that is the best response aligned to the design purposes. In order to achieve this goal, it is fundamental to have a complete knowledge of the economic environment to fix, even a small detail could have consequences on the process of re-structuring. A. E. Roth is considered the pioneer of this field, and his first real-world market design project (Roth 1984a) ratified the starting point. The literature of Market Design has been enriched of many real-world projects, whose success foster the studies into this direction. However, also works based on real case studies – without being then really realized – constitutes an important part of the literature – as this dissertation. Many authors focused their work on the algorithms' peculiarities (Abdulkadiroglu et al. 2008; Kojima and Manea 2010; Kojima and Ünver 2013; Miralles 2009) underlying the advantages they allow to achieve in determined situations through theoretic examples, while others offered real situations as basis for their research works (Bhatia et al. 2015; Chen and Onur 2013). Moreover, new features has been gaining relevance during the analysis of matching mechanisms and the authors have started to deal, for example, with issues as contracts (Hatfield and Kojima 2010; Hatfield and Milgrom 2005), waiting times (Dimakopoulos and Heller 2014), maxi-min preferences (Jiao and Tian 2015). The attention for real markets has revealed also situation never mentioned in the previous literature as the case of hybrid mechanisms (Veski et al. 2014). The markets at the center of the two-sided centralized structure research have usually been college/school admissions, and entry-level labor markets, the branch to which also the final paper belongs. Behavioral theories and Game Theory are considered fundamental tools of the practice of Market Design. The firsts are

implemented for achieving a better understanding of the agents – in addition to experimental economics techniques (Kagel and Roth 2000) that allow to confirm the conjectures when a real application. The second permeate the domain of the matching theories, but it also taken as an inspirational guide for the definition of “the rules of the game” when establishing a new design. The market design is the practical realization of the theoretic studies on Matching theories. The results obtained from the theoretical models and analysis are implemented on real structure, aiming to improve the agents’ well-being.

Matching Theories and Market Design is then an indissoluble duo. Any market design project starts from an analysis of the current situation from an empirical point of view, and it is then brought on a model (and theoretical) level. The theoretic efficiency looks at the stability of the assignment – a Pareto optimal allocation – and the practical efficiency aims to avoid the three identified causes of failure of any matching market (Roth 2007): 1. Lack of thickness; 2. Congestion; 3. Safety, i.e. guaranteeing to agents the chance of revealing their true preferences. Theoretical and Practical studies and aims are merged in a sole final solution of a new design for the inefficient market (Roth 2010). The dissertation work is based on these very same principles, both theoretical and practical, and it is structured following the points of work of any market design project.

Section II

The Hiring Procedure for Assistant Professors in Italy

The second section is entirely dedicated to the recruitment process for the Assistant Professors positions in the Italian Academic Market. It starts with a presentation of the current system together with a brief reference at the regulative background, in order to offer a sufficient knowledge of the environment where the project of redesign is placed. This section concentrates the work of analysis carried out on the current system both from a legislative and an empirical point of view. At first, it presents the most important Italian Law that regulates the procedure, together with the European guidelines they should be aligned to. Then, it describes the procedures bringing to light some of the most critical features supported by data. The data catch three different aspects of the recruitment process, and offer a window on the trend of the contests in the Economic scientific sector over the period 2004 – 2011, on the winners' profiles of some of these contests, and on the satisfaction level and personal contests of some of the people who passed through this procedure and are currently working as Assistant Professors. The first set of data has been acquired on the website of the Ministry of Education, University and Research (MIUR⁶); the second is represented by an investigation on individual profiles; and the last is made up by the results of a survey I conducted at the end of the year 2014. This section, then, constitutes the very first part of a Market Design project that entails the deep knowledge of the procedure we are asked to reform. All the information reported in the next paragraphs represent the fundamental pillar on which the redesigning project will be based.

⁶ Italian acronym for the Ministry, Ministero dell'Istruzione, Università e Ricerca.

The choice of this market relies on the relative easiness to find data and information; it is treated, simultaneously, as a representative and an exploratory case-study.

2.1 The Italian Academic Job Market

The term “University” denotes the “Institutes of high culture” (art. 33, Italian Constitution) that owns the aim of providing courses of high education. Universities can be public or private institutions, but in the following study, with the generic term “University”, I will refer exclusively to Public Institutions. The Assistant Professor (or researcher) represents the position of entrance in the Academic job market, and it is the starting point of the academic career path. The process of recruitment is defined by the Article 18 comma 1(b) of the Law n. 240/2010⁷. The current recruitment procedure has a decentralized structure, i.e. each university makes an announcement for vacancies and elects among the candidates who participate at the public contests. The contest is announced on the website of the Universities, of MIUR, on the official journal called “La Gazzetta Ufficiale”⁸ and of European Union. In the announcement there must be defined the scientific-sector of the job position, as well as the job characteristics and the selection criteria. These criteria are partly imposed by the Law and mandatory (e.g. candidates must hold a PhD degree or an equivalent foreign academic degree), partly defined by the University regulation, and partly specific for that vacancy. Usually, the regulation trace the Law 240/2010 and add just few rules, while the specific requirements only define preferential classes of candidates (e.g. age limits or years of experience). Universities control the call of professors of first and second level by their own regulations in the respect of the Law n. 168/1989 and of the European Charter for Researchers 2005. The proposal of call must be approved by the absolute majority of first level professors for a full professor position, and of first and second level

⁷ The entire text in the original language (Italian) is reported into the Appendix.

⁸ Gazzetta Ufficiale is the official source of knowledge on Italian legislation, and tool of information and diffusion of legislative text, both private and public, and it is published in collaboration with the Ministry of Justice.

professors for an associate professor position, and in both cases by the Board of Directors. The number of calls are constrained to the amount of financial resources available for the University, and the proposal of call must receive the approval from the MIUR before being announced. It has not been provided a specific timing period for that first process, such that actually Universities do not know in advance how long after the request they will receive an answer. A local committee will judge the applicants on the basis of the value of their research works, and other skills specifically indicated into the announcement. The evaluation process is divided into two phases, and at the end the committee will declare the winner of the competition. The selection is based on the evaluation of scientific publications, personal curricula, teaching activity, knowledge of a foreign language and other skills that could be requested.

The recruitment procedure has experienced different adjustments during the last fifteen years. In particular, the final action in 2010, with the law L. 240/2010 known as Riforma Gelmini, has completely renewed the job characteristics of this academic position, and it provided some minor modifications to solve the inefficiencies showed by the previous recruitment process. Formally, universities can stipulate fixed-term contracts with researchers in order to conduct research programs and activities of teaching, integrative teaching and of service to students. Nowadays, there are two kinds of fixed-term contract, type A and B, where the second is reserved to the ones who already availed the first. The “fixed-contract” form gives to the Departments the possibility to terminate or to confirm the job relation with the researchers after an analysis of their academic work⁹. The change from a permanent to a fixed-term contract, is actually the most important point – and the core – of Gelmini’s Reform, that did not cause fundamental changes in the recruitment procedure. The aim of the new Law was to subsidize the academic market, by avoiding the “stagnation” of professors in entry-level positions¹⁰. The

⁹ A type A researcher is evaluated by the University after 3 years; on the basis of the result of the evaluation, the contract can be confirmed for other 2 years or not. On the contrary, a type B contract lasts for 3 years and there are no chances of being renewed.

¹⁰ The permanent contract did not push some professors to proceed in their academic path, and to not carry out a valuable research work. The consequence was that many entry-level position were occupied by old professors with low rate of research productivity.

continuum of reforms¹¹, that have interested this recruitment process, highlights the willingness of solving evident problems, and, sometimes, the missing of the objective. The measures provided by any reform regulated the composition of the committees (with a particular regard to family affiliation between the candidates and commissioners), the content of the announcements, the level of academic education (from 2015 the PhD title is essential¹²), the examinations, and so on. No reforms intervened on the structure of the system or, on the steps of the hiring procedure so far. Even if some aspects have been better off by the Riforma Gelmini, the market still shows a great numbers of inefficiencies that have not been addressed by the Law. Moreover, the Law 240/2010 acts as a set of mandatory rules, but at the same time, a certain degree of freedom is still left to Universities, and the MIUR seems to exercise only a slight control over them. However, it is important to underline that Universities are Public Institutions, and so part of the Public Administration and supported by Public Expenditure. This entails they should respect the principles of transparency and clarity, and that their autonomy is submitted to MIUR legislative acts. Moreover, the law in force recognizes the European Charter as the legal reference point for all universities that have to follow its measures when realizing the announcements of selection and to respect its guidelines. The document is made up by a set of general principles and requirements which specifies the role, responsibilities and entitlements of researchers as well as of employers¹³. It is based on the principles of transparency of the recruitment process and equality of regard for all the candidates.

Due to the decentralized structure, and to the high level of autonomy that Universities exercise during the hiring procedure, the assignment of Assistant Professors is hardly recognized as a “market”. This misconception is probably one of the reason why this assignment procedure has not been studied as a matching

¹¹ Decree of the President of the Republic n. 382/1980; Law n. 210/1998; Decree of the President of the Republic n. 117/2000; Law n. 230/2005; Law n. 240/2010

¹² Art.29 (13), Law n.240/2010

¹³ Ibidem.

market yet, even if this is exactly what should be done for making it efficient. The next sub-paragraph details the hiring procedure as reported by the Law 240/2010.

2.1.1 The Academic Entry-Level Recruitment Procedure

As said above, the Law 240/2010 is currently in force and it assigns the guidelines for the researchers' recruitment¹⁴. The Ministry did not define a specific time period for publishing a vacancy announcement, so Universities have the chance to offer a job position at any time of the year¹⁵. The same holds for the duration of the hiring procedure that relies on the University decision, and it is not regulated by the Law.

The call is made by the Department where the vacation is offered, and the Department will carry on the procedure¹⁶. The announcement contains all the information about the role, the requirements to participate and specifies the SSD (disciplinary scientific sector) of the position to which candidates will have to demonstrate to belong to, through their scientific production. When the deadline for sending applications has passed, the committee is selected. The composition of the committee has to be aligned to the guidelines of the European Charter¹⁷ integrated into the Law 240/2010: three members, one of the Department (and of the same SSD) that offers the job position, and two randomly selected from different universities – better if one of them belongs to a foreign university. No one commissioner must have a family relation – up to the fourth grade – with an applicant. The preliminary evaluation of the candidates involves the academic titles – PhD degree or an equivalent academic title is a requisite of access from 2015 – curriculum vitae and scientific production (each university will establish a maximum number of papers that can be presented), including the PhD's thesis. Internationally recognized criteria and parameters are implemented, even though

¹⁴ Art. 24, Law 240/2010.

¹⁵ Once the proposal of announcing a new position has been approved by MIUR.

¹⁶ The term “university” is used in a general way to indicate the institution, so sometimes it will be used also in substitution of the term Department.

¹⁷ European Charter for Researchers – The Code of Conduct for the Recruitment for Researchers 2005

the bibliometrics references are actually applied mainly by Scientific Departments and less considered in the Social Sciences and Liberal Arts' sectors¹⁸. The best candidates¹⁹ are convened to a public dissertation of their scientific production and the knowledge of a foreign language (English) is tested – if required in the announcement of the university also other languages could be tested. The oral examination is carried out at the University site, and no exception is admitted²⁰. Then, the committee writes a brief judgment of all the attendees, and select the winner who will be proclaimed with an official document. If the winner renounces to the job, the position will be offered to the second best candidates by the committee. All the documents related to the procedure have to be published on the website of the university, and of the MIUR, so that anyone have the possibility of consulting them.²¹ Timing periods for each step of the hiring process are not defined, and they vary by contest.

Since any University opens vacancies at different timings, and contests last at least 6-8 months, we register an over-crossing of the various competitions. For having a clear picture of the recruitment process, it is necessary to understand that all institution follows these steps, but at different timings and with different durations.

2.2 The Problems of the Current System

The Italian Academic job market, due to its decentralized structure, results to be fragmented and it probably would be necessary to conduct a local analysis of any recruitment process. Sometimes, it appears to be more a peculiar procedure of each university rather than a process of the University as a system. This is

¹⁸ Sometimes, the announcement, or the Department Regulation, reports the rank of Journals that will be considered during the evaluation process – and it could differ from the International rank.

¹⁹ Their number have to be between 10-20% of the total candidates and in any case not less than six, they are all admitted when the total number is less than seven.

²⁰ This feature of the hiring procedure has recently been at the center of the academic debate. At first, different universities asked for conducting the oral examination at a subsidiary office in the United Kingdom, but the MIUR denied the request because it has to be carried out onto National territory. During the current year (2016), the University of Bologna, for the first time, took part to the International Market for Economist by participating at the Conference in Bilbao. In fact, the Law establishes that the official oral examination has to be placed at the University site, but it does not impede to previously interview candidates at different sites.

²¹ Previously, also a written test was required to candidates now removed.

definitely the opposite of a clear and “transparent” market, as required by the legislation. In this subsection I illustrate some of the inefficiencies that characterize the system in its entirety.

First of all, **the regulations** of universities about the recruitment process sometimes appear to be incomplete or ambiguous and not completely aligned with the law in force. The composition of the committee is often the point of dissonance: they are build up in many different ways²². This element solo creates a system not clear, not fair and not controlled. Having a committee with more than one professor from the university that announce the vacancy, could be perceived as inequitable and the ones who are already working for the Department at the moment of the call – called “internal candidates” – could be advantaged by this situation.

The second problem is just related to a sort of “advantage” someone profited by. Cases of so called “**influence peddling**” have been registered (Durante et al. 2011; Perotti 2008) during the years, highlighting a meritocratic problem. The Law establishes a relation over the fourth grade of family affiliation between a candidate and a commissioner, but Perotti illustrates as a network of friends can easily overcome it, by simply recurring to mutual favors. The works of Perotti are surrounded by a lot of critiques and to demonstrate these actions is more difficult than it could appear, above all because the access at the public documents is not as easy as it should be (and they are not at disposal on the university websites as prescribed by the Law). However, the research works of Perotti seemed to have robust basis. A confirm of his work arrived also this year (2016), when the Dep. of Economics of the University of Salerno – one of the institution analyzed into his works – was repeatedly reported at the Administrative Regional Court (TAR)²³ for “piloted contests”. The advantage given by the family relationship is not the only one that could be find in this market. The above mentioned “**internal candidates**” are the main characters of this “scandal”. Most of the times, they complete their academic path in the same university (PhD degree, research fellowship, post-Doc

²² See Appendix for the detailed texts of the regulations of some randomly picked universities.

²³ TAR is the acronym for Tribunale Amministrativo Regionale – Regional Administrative Court.

programs) and they are working there also during the recruitment procedure. The consolidated relation with the members of the academic staff seems to give them a preferential treatment. The data gathered from the survey revealed that almost the 35% of the respondents, consider the presence of internal candidates as a factor that negatively impacts on the choices of the contests²⁴. Real facts show the reason. During the hiring process for a job position in Political Economics at Università dell'Insubria, the winner of the contest was the only one with zero publications but with a robust relation with the president of the committee. The complaint to the TAR executed by the other candidates was accepted and the contest was nullified²⁵, under the attention of the media and of all the academic community. The data of the survey showed that the 12% renounced to apply for more preferred contests because of the presence of internals, and for "*understanding of internal mechanisms*"²⁶.

It is important to notice at this point, that the hiring process is **expensive** for candidates – as well as for Institutions. Applicants suffer costs in terms of time and money. The first is related to the time spent for searching contests all over the year, preparing the documentation that can differ by university, waiting for receiving the admission to the oral examination and then for the final results. Moreover, if the candidate is admitted to the interview phase, she has to present herself at the University site at her own expenses without any chance of being reimbursed. It could happen to participate in just one contest – and winning the job position – but also to participate in 25 contest – as happened to a respondent of the survey.²⁷ The cost is not the only limit candidates should overcome. It could happen that different Institutions choose the same day for holding the interviews

²⁴ Data obtained from 150 answers – the respondents were currently Assistant Professors. More information on the survey sample can be found into the Appendix.

²⁵ Notice that, any time that there is a complain to the TAR the duration of the contest increases. If the contest is nullified, it has to be repeated from its very first step. The TAR has not the power to substitute the winner of the competition, so it could only recognize if the hiring process respected the legislation or not. Moreover, the claim to the TAR has also a monetary cost for the candidate who appeals.

²⁶ See the Appendix for the complete quote of some comments related to « internal candidates ».

²⁷ The average number of contests respondents dealt with is 2.25, however they belong to different departments and we have no detailed information on their contest (e.g. the number of applicants).

at their sites. Since applicants cannot be present simultaneously at both Universities, it means that they must withdraw from one of the contests.

Indeed, the **lack of a clear time-line** of the job announcements could be considered as a problem of this system, as well as not knowing in advance the **total job offer** per SSD during the year. There is not a predefined timing for the announcement of vacancies, neither it is known in advance the total number of job positions that will be open, and by which institution²⁸. One of the possible consequences is that applicants do not respond only to the ones they are really interested in, due to the anxiety of staying unemployed. It could be useful to compare this system to the unofficial procedure followed by the Institutions that choose to go on the International Market for Economists. There are no written rules that then rely more on habits, but it is possible to identify a clear timing structure of the hiring process. Most of all, the Universities that hire candidates through the International Market open their job positions during the same two months, and inform the winners during the same one month. This structure allows the candidates to concentrate on the steps of the hiring procedure, instead of keep applying while preparing an interview, while waiting for a response from someone else. The clearness of a timing structure also reduces the uncertainty of the process, and the anxiety of the waiting time that exist between the hiring steps and the contests.

In some cases, there was a long period of time between two contests for the same disciplinary scientific-sector (SSD)²⁹. These happenings increase the incentive for applying to the available contests, and it does not matter what their real preferences are. The system cause problems of **congestion** because the absence of an offered job position in a determined sector for a long period of time let candidates of different years amass, by covering post-doc or fellowship positions

²⁸ Usually, not all the universities offer a job position in a same SSD, and the data gathered on a randomly selected sample of universities show that a Department could also not announce any position for years. See the Appendix for more information.

²⁹ The consequence was that the first contest after many years of absence registered around 80 applicants for just one position (SSD P/01, University of Pavia, year 2010). Notice that, sometimes, some professors use the expression “there were no open contests”, to indicate that there were contests but the winners were already identified before starting the hiring procedure. The absence of open contests is usually assimilated to the absence of contests at all.

while waiting so that, the young PhD graduated, most of the times, feel to have no possibility. The consequence is that since the first year of the PhD course, students start to look for a way for creating their academic future through increasing the academic network and differentiating their “academic portfolio”, instead of focusing on their research work.

Moreover, on the level of local management of the hiring procedure, it can be noted a great **waste of time** between the publication date of the announcement on Gazzetta Ufficiale and the appointment date – a period time that is around one year but it can reach also a time of two years (data from MIUR)³⁰.



Figure 1. Example of time period between the date of publication of the announcement and the date of appointment

Since the Law do not establish of maximum lasting time per contest, there are no limits apparently. On the International Market for Economists the duration of a hiring procedure is around 8 months: from September to the end of November positions are announced and applications received; on January there are the first interviews; during the months of February/March a first short list of candidates are invited for holding a seminar; during the months of April/May a short-list of 3 candidates is invited for spending an entire day at the University site and before the end of June the winner is appointed. It is a long process, with a lot of steps for studying at best the candidates, but they are able to end it in less than one year. The Italian procedure, on the contrary, should ensure a quite rapid hiring process, since there it is made up by only two steps of evaluation. Real facts show the contrary, with a minimum of duration of 6 months. Probably due to administrative

³⁰ Ibidem.

time and long bureaucracy, or to the people assigned to the recruitment. I did not go in depth in the investigation of the causes of these extra-durations of contests in this dissertation, since the project focuses more on the functioning of the process. However, the lasting timings of contests will be at the center of further studies as outlined in the section of Conclusions. In fact, they add an interesting degree of uncertainty and of pressure on applicants who have no idea on when the winner will be declared, and could front the hard choice of accepting or not other offers while waiting.

As the lasting timings, also the problem of **thickness** is peculiar of each single contest. The number of applications is able to vary from 1 to 100, even for the same SSD. In opinion of some past candidates, if a competition is perceived as more accessible – i.e. there is not a favorite internal candidate – the number of candidates is very high (e.g. contest in 2010 at Università di Pavia – SECS/P01, number of candidates: 83) and is able to guarantee the right dimension of the market that allows to effectively select the most deserving applicant. Otherwise, when a contest is perceived to be “closed”, i.e. reserved to the ones already well known by the University, as referred by the subjects during the survey, the problems of thickness is manifest: there is a certain number of competitive exams with just one or two candidates. The problems related to the number of candidates for announcement are testified by the data found on the web-page of the MIUR³¹. It is not considered that a contest could end without the hiring of a candidate³², so the first consequence of a low number of candidates for contest is that low-qualified subjects can win, because they are the sole participant or because they compete with few individuals even less qualified. The general perception for single-candidate contests, is that it was a closed competition and so no other applicants showed up³³. Since this feeling cannot be empirically demonstrated through the data obtained for this dissertation, we will focus on what the data can tell about the

³¹ <http://reclutamento.mur.st.it/vincitori.html> .

³² Due to the costs in terms of time, organization and money linked to the public competition, it is very hard that a University conclude the procedure without hiring someone. Instead, if they did not receive a satisfying number of candidatures, the deadline for receiving the applications can be shifted to the month after.

³³ Out of the comments received through the survey, this feeling cannot be sustained with data since it is not possible to assess why other potential candidate did not send an application or refuse to have an interview.

possible causes. Having an adequate number of candidates ensures not only the

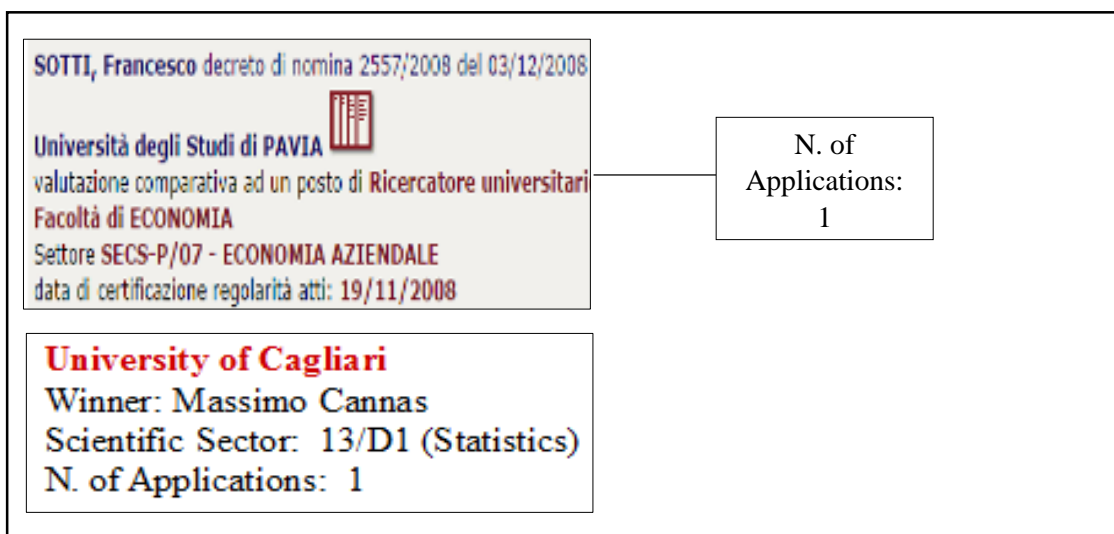


Figure 2. Examples of contests with one application

well-functioning of a matching market, but also the chance for the Institution to choose the best among many, and do not to settle to the one who applied. However, the causes of a low number of applications can depend on a large number of factors as: the research status of the University, its position into the national rank, its connections with foreign Universities, its past history of funds attraction, and so on. The survey revealed that the most important factors that have a positive incidence when selecting the Universities for sending applications are Geographic Position (69%) and Previous Experiences at the same institution (63%)³⁴. On the other hand, the most important factors that have a negative incidence when selecting the contests are the Presence of Internal Candidates (33%) and the feeling of a Non-Meritocratic Evaluation (27%) by the local Committee. Since the process is expensive for candidates, it is quite reasonable to assume that they will avoid the ones offered by universities located into disliked geographic areas, and the ones perceived as already reserved to someone. As a vicious cycle, this applicants' choice brings back us to the problem of sole-candidate contests. Moreover, the survey answers demonstrate that the attendees prefer universities where they had previous experiences. The data gathered on winners of the contest over the period 2004-2011 of the Dep. of Economics of a random sample of Universities, highlights that on average more the 50% of the

³⁴ All data about the positive factors of incidence on the selection of universities are in the Appendix.

winners has a previous connection with that institution.³⁵ The next step was then to understand if this big proportion was due to 1.) A candidates' preference (since up to the survey they prefer to apply where they had previous experiences); 2.) an institutions' preference; or, 3.) a sort of market self-selection such that when a candidate who had a previous experience at that university apply, all the others renounce because of her presence. In fact, it is interesting to notice that in the case of the Dep. of Economics of the University of Bergamo, any time that the winner was someone with a previous affiliation, the number of applicants is low (under 5), while in all the other cases the applications are above 10³⁶. Unfortunately, it is not possible to check if when an "external candidate" won, there were or not "internal candidates" because the complete list of applicants is not always available. This would have been an important piece of the puzzle. Whatever the cause is among the ones listed, any of them highlights a problem of the system, or of how it is perceived. The data gathered through the survey clear that there is a great **loss of confidence** about the system. The large part of the comments describes it as inadequate and inefficient, the "general efficiency" of the recruitment procedure obtained the mean value of 2.76 on a likert scale from 1 (extremely negative) to 7 (extremely positive)³⁷. The 41% indicated as preference a centralized system, while the others a localized one. The motivations on both sides are quite various, but most of respondents do not consider an Italian central system able to manage the recruitment procedure³⁸. On the other side, who chose the national mechanism, made recalls to equal treatments by the committees and the need of having rules that must be respected. Moreover, Prof. Checchi and his collaborators noted that the candidates of the contests for associate and full professors positions, exerted less effort in their research works after the switch from a national contest to the local ones (Checchi et al. 2014). As explanation of this phenomenon, the authors claimed that candidates perceived the national competition as more based on bibliometric dimensions with regard to the local ones. Since the hiring

³⁵ See the Appendix for detailed data.

³⁶ The comparison is more interesting for contests of the same SSD. More details into the Appendix.

³⁷ More details in the Appendix.

³⁸ The loss of confidence seems to be about not only the recruitment system, but about the infrastructure and the MIUR that should manage the process.

procedure of assistant professors has been never experiences a national contest, it is not possible to extend the research work also to this academic category. However, the results of this study could be considered as a warning sign of a decentralized structure's effects.

2.3 A theoretical model as a Tool of Efficiency Analysis

The analysis of the procedure and the empirical research carried out on the recruitment system, identify some problematic features. They constitute the basis for building the answer to the very first question of this project: **Is the Italian Recruitment System for Assistant Professors Inefficient?** The problems of thickness, congestion and safety brought to light in this paragraph already let us sustain that from a Market Design point of view, this is a failing matching market. It is sufficient for justifying a redesigning project, but it does not establish the path to follow in this work, neither what kind of intervention the market needs. In fact, the real cases of Market Design – seen in Section 1 – taught us the benefits of a centralized system, and the success it can have. However, there is not a single solution for all the matching markets, and this is not only about peculiarities that has to be added to a tailored centralized system. First of all, it would be necessary to demonstrate that the current decentralized system is not efficient. But the data gathered and partially presented, do not allow to robustly sustain the inefficiency of the market from a matching point of view, i.e. the failure of the assignment system that is at the core of the entire recruitment process. It is not possible to proceed on the redesign project without knowing if the matching mechanism is efficient or not, and in the negative case, what is affecting its functioning. For these reasons, it has been fundamental to develop a theoretical model for carrying on an analysis on the mechanism itself. The results obtained will then be used as a tool for evaluating the real-world matching procedure, and assessing, on a robust basis, its in/efficiency.

The theoretical model built up for analyzing the Italian recruitment procedure is presented in the following paper. The model explained in the paper is not the

perfect reproduction of the current system in formal terms, but it is a tool of analysis. Since the literature did not provide a model that was able to be implemented as a tool of analysis on this assignment procedure, it was necessary to build a new one that allowed to proceed on the study of the mechanism efficiency.

The Decentralized Multiple-Rounds Mechanism

E. Quintili

Abstract

The paper presents a dynamic non-revelation hiring procedure made up by a finite number of rounds that matches Institutions and Candidates. Any round has the same structure, and each time is played by a different sub-set of Institutions that commit to the temporary matching obtained at the end of the round. On the contrary, Candidates are able to play all the rounds of the game, and to break the temporary assignment. I demonstrate that in a setting of complete information the mechanism implements the Candidate-Optimal matching in Subgame Perfect Equilibrium. Moreover, the stability of the outcome is ensured for all agents truthfully revealing their preferences. Finally, I used the model as an analysis tool for the current hiring procedure for Assistant Professors in Italy. I succeed in proving that the real market is inefficient and which are the causes of its failure thanks to the implementation of the model, showing its theoretical and practical relevance.

Keywords: *decentralized mechanism, matching, non-revelation game, multi-rounds mechanism, dynamic game, truncation strategies, hiring procedure, stability*

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1. Introduction

A two-sided matching market is an economic space where the agents of one side of the market, are coupled with the ones of the other side, through a matching mechanism based on their preferences. There are different kind of inefficiencies that can affect this kind of market – such as problems of thickness, congestion and safety (Roth 2007) – and that can undermine the stability of the assignments. A matching is said to be stable if it is individually rational – i.e. any agents is coupled with someone they like more than being alone – and if it is not blocked by any pair – i.e. there are not two agents that would prefer to be matched together rather than their current mates. Since D. Gale and L. Shapley (1962) elaborated the Deferred-Acceptance Algorithm that guarantees stable outcomes, the entry-level labor markets have been the focus of both theoretical and practical studies (Roth 1991a). The first project of analysis and redesign of a matching market was conducted by A. E. Roth on the assignment of American physicians to hospitals (Roth 1984a; Roth and Peranson 2002), and he paved the way to a brand new literature. Most of the studies focused on centralized matching structure, where agents submit their lists of preferences to a central clearinghouse that runs a matching algorithm for obtaining the final couples. A great number of real-world markets benefited from the research on centralized mechanism, however, there are still many entry-level labor markets organized on a decentralized structure. First formalized and analyzed by A. E. Roth (Roth and Vate 1990, 1991), on the contrary of centralized systems, agents do not submit lists but reveal their preferences through their actions. Agents of different sides directly interact in a sequential game of offers and acceptances/rejections. This scheme is particularly common in the private job market, where Firms address Workers one by one proposing the job position. Most of the studies carried out so far, present models based on this scheme. Few exemptions are represented by the students-proposing mechanism described by Alcalde and A. Romero-Medina (2000), the college admission system described by A. Romero-Medina and M. Triossi

(2010)³⁹, and the Academic hiring mechanism formalized by M. Triossi (2009)⁴⁰. The latter, particularly, is based on the real-world procedure for hiring PhD candidates, where candidates apply to a finite number of potential employers and when receive the offers, select the ones they like the most. Triossi is the first to introduce the Academic job market into the literature of decentralized mechanism, and it is actually the closest model to the ones presented in this paper. However, the model relies on a clear and regular timing structure, such as all firms open the positions at the same time. The timing structure is a key feature of decentralized matching markets since define the functioning of the mechanism, and shape the organization of the market itself (Roth and Xing 1994, 1997).

I introduce a mechanism that take inspiration from the entry-level academic job market in Italy where Public Institutions (Universities) hire Assistant professor through public competitions and do not open their positions all at the same time. The hiring procedure is not carried on in a specific period of the year, but over multiple periods since each institution announces the job position at a different time respect to the others. The entire job offer of the year is not completed till the last university with an available position announces it on the market. This entails a mechanism build up by a finite number of rounds – called Decentralized Multiple-Round or DMR mechanism – for any round being structured at the same play and being played by a different sub-set of institutions each time. In Triossi (2009) during the first step of the mechanism all candidates simultaneously apply to any university, instead in the DMR mechanism, at any round candidates can send an application only to the institutions that belong to the sub-set that is opening the job position at that moment⁴¹. Moreover, I assume the applications to be costless and free of any restrictions per round and per the whole game, and I demonstrate that these are two of the key assumptions that guarantee the stability of the

³⁹ Even if both models are presented into the context of College Admissions, they can be easily translated into a Firm-Worker context.

⁴⁰ This paper actually presented itself as an extension of the Workers-Propose-and-Firms-Choose Mechanism presented in (Alcalde and Romero-Medina 2000).

⁴¹ Actually, the mechanism presented by Triossi (2009) is assimilable to the structure of a single round of the DMR mechanism.

assignments⁴². At the end of each round temporary matchings (Blum et al. 1997) are realized for any institution that announced a job position. I set a condition of one-sided commitment – the third key assumption – firstly introduced by Diamantoudi et al. (2015), such that only the side of Institutions is called to commit to these temporary assignments, while candidates are able to break them at some following round of the game. The multiple-rounds structure set in a one-sided commitment condition, makes the functioning of this decentralized mechanism much closer to the one of the D-A algorithm than any other. Even if far from real situations, the model has been formalized in a setting of complete information with the aim of offering a clear presentation of the mechanism. The results about equilibrium strategies, partially retrace the ones already reported by other authors. It seems quite usual for the Firms' side to have as weakly dominant strategy to truthfully reveal the real preferences (Blum et al. 1997; Haeringer and Wooders 2011; Sotomayor 2003; Triossi et al. 2010). I demonstrated that the same holds for the Institutions' side in the DMR mechanism, but only once the condition of "Institutional Acceptability" is introduced. This is one of the "contests' features" introduced in the mechanism whose aim is also to formalize the hiring procedures through public competitions that differs from the ones by "direct calls" – that have been the focus of the literature so far. On the other, there is always at least one candidate who can benefit from deviating from a truthfully revealing strategy⁴³, such that for the Candidates' side it is a best response to reveal only a truncation of their real preferences (Blum et al. 1997; Ehlers 2008; Roth and Rothblum 1999; Roth and Vate 1991). The mechanism implements the stable set of matching in Subgame Perfect Equilibrium, specifically the matching realized is the Candidate-Optimal assignment. However, a truncation strategy requires a lot of information and is risk-free only as long as the number of job positions equals the number of candidates. Aware that this last condition is quite unusual in real-world market, I tested the model under the assumption that all candidates truthfully reveal their real preferences over the game. As in the Institution-proposing D-A algorithm, the

⁴² On the contrary, in Triossi (2009) costly applications maintain the stability of the matching.

⁴³ As it has been demonstrated for the D-A algorithm when Institution propose first (Roth and Sotomayor 1992).

final matching is stable and Institutional-Optimal⁴⁴. The results are very close to the ones presented by Blum et al. in 1997 even if the mechanisms are different. The model presented by the authors – called Decentralized Deferred Acceptance – starts from an initial matching, as actually happens in the DMR after the first round⁴⁵. In both cases, the workers who are still unmatched, can benefit of misreporting their true preferences into the game. On the contrary, once they are matched they are no longer able to profit of a deviation from a truthfully revealing strategy. In the case of DMR, these results heavily rely on the assumption made on the mechanism. One of them is the one-sided commitment condition. On the contrary of the results obtained by Diamantoudi et al. (2015) about their firm-proposing decentralized model⁴⁶, the one-sided commitment is a necessary – even if not sufficient – condition for guaranteeing the stability of the assignments under the DMR mechanism. Finally, I offer a demonstration of a practical application of the model, using the theoretical results as an efficient tool of analysis of the efficiency of the hiring procedure for Assistant Professors in Italy. This particular real case-study has never been treated into the matching literature, and presents different interesting starting points for further studies on decentralized mechanism features.

The paper is organized as follows: Section 2 presents the basic model and the key assumptions on the mechanism; Section 3 detailed the hiring procedure; Section 4 describes the strategies of the agents into the game; Section 5 introduces the main results; Section 6 shows the application of the model on the real Italian case and Section 7 concludes. All the proofs are in the Appendix.⁴⁷

⁴⁴ (Gale and Shapley 1962).

⁴⁵ Then in both cases it has been done a differentiation between currently matched and unmatched workers.

⁴⁶ They state that under one-sided commitment unstable matching can arise, for a model where firms offer directly to one worker at a time, and commit as soon as matched.

⁴⁷ In the next sections I will use the subject “we” for avoiding confusions with the name of the Institutions’ set.

2. The Two-Sided Matching Model

Let $I = \{i_1, \dots, i_n\}$ be the non-empty and finite set of Institutions, and let $C = \{c_1, \dots, c_n\}$ be the non-empty and finite set of Candidates. Each agent has complete, transitive and strict **preferences** over the individuals on the other side that will be reported in their preferences' orders. Let $P(i)$ denote the preferences of any university $i \in I$ over $C \cup \{i\}$ ordered in a decreasing list $P(i) = \{c_1, c_2, \dots, c_n, i\}$, and $P(c)$ denote the preferences of any candidate $c \in C$ over $I \cup \{c\}$, $P(c) = \{i_1, i_2, \dots, i_n, c\}$. The order represents a strong preference relation such that $c_1 \succ_i c_2$, i.e. c_1 is strictly preferred to c_2 by i . A candidate $c \in C$ is acceptable for $i \in I$ if $c \succ_i i$; a university $i \in I$ is **acceptable** for $c \in C$ if $i \succ_c c$, i.e. it is considered acceptable any agent who is preferred to herself otherwise it is called **unacceptable**. Let denote by $P = \{P(i_1), \dots, P(i_n), P(c_1), \dots, P(c_n)\}$ the profile of preferences that gather the preferences of all the players into the matching procedure. The **non-revelation** setting implies that the order of preferences will never be directly reported into the markets, such that the agents have not to submit the order list before the starting of the assignment procedure⁴⁸. The players, instead, will reveal who is acceptable and who is not, and some preference relations, through their actions into the game. We can define the matching market as a triple of the form (I, C, P) and the outcome obtained at the end of the procedure is called matching. A matching μ for (I, C, P) is a one-to-one correspondence of $I \cup C$ into itself of order two, i.e. $\mu: I \cup C \rightarrow I \cup C$, such that $\mu(i)=c \Leftrightarrow \mu(c)=i$, and so $\mu(i) \in C$ and $\mu(c) \in I$. If $\mu(x)=x$ the agent is matched to herself, and so she is considered as **unmatched** since she is not coupled with an agent of the other side of the market. The problem is set as a one-to-one matching, so the quota of offered positions is fixed to one, for any $i \in I$, $q_i=1$ by assumption⁴⁹. We say that a matching μ is **individually rational** if each agent is coupled with an acceptable mate, for any $c \in C$ and for any $i \in I$, $\mu(c)=i \Leftrightarrow i \succ_c c$ and $\mu(i)=c \Leftrightarrow c \succ_i i$. If there is a pair (i, c) that prefers each other at the agent they are matched to, such that $i \succ_c \mu(c)$ and $c \succ_i \mu(i)$, we say that the couple (i, c) is a blocking pair. Anytime that

⁴⁸ As it happens in centralized procedures.

⁴⁹ It is possible to allow an institution to offer more positions and simultaneously respecting the constraint of $q = 1$ by creating a number of copies of the same institution equal to the number of job positions it desires to open (Gale and Sotomayor 1985a).

a blocking pair is detected into the outcome of the assignment procedure, it is said that the matching is **blocked**. A matching μ is defined **stable** if it is individually rational and it is not blocked by any pair.

2.1 Key Assumptions on the Model

We assume that applying for a job position is costless for any candidate and that there is not a maximum number of applications allowed per round or per game, such that they do not have any kind of constraint during the hiring procedure. Then, it is rational to believe that they will apply as much as they can, relatively to their preferences and strategies. Moreover, we set a “**one-sided commitment**”⁵⁰ condition into the game, according to only one side of the market will be committed to any matching during the procedure, i.e. no agent of the selected side can terminate the assignment, for any reason. On the contrary, the agents on the other side will be able to break the temporary pair for creating a new one. Once the procedure ends and the matchings are officially finalized, nobody will be still able to change the couples. In this model, the Institutions are the committed side, while candidates are not officially bounded to any matching until the end of the game, when no more rounds are available. We will use the term of “**temporary matching**”⁵¹ for denoting the non-official assignments that will be formed during the game and it will be formally represented by μ^t and $\mu^t(i)$. The agents involved will be addressed as “temporary matched”, to underline their condition at some point into the game and to differentiate them from the ones who are still unassigned. Since candidates do not commit to their temporary mates, they are able to change the assignments during the game by applying, and obtaining, a new job position at another Institution. On the other hand, Institutions commit to the temporary match and so they cannot re-offer the job position on the market, or simply offer it to someone else, as long as the position is fulfilled. This means that the temporary matching is binding for the Institutions and not for the Candidates. This is a fundamental assumption for obtaining stable outcomes in setting under

⁵⁰ (Diamantoudi et al. 2015)

⁵¹ (Blum et al. 1997).

incomplete information and uncertainty. In fact, it will be demonstrated that under a stronger assumption of two-sided commitment, it is impossible to achieve stable results in a contest different from the one of complete information. We set also a constraint on the acceptability conditions of the Institutions' side by aligning it more to real-world situations, by introducing the new concept of "*institutionally acceptable*".

Definition 1. An agent is defined **institutionally acceptable** if she meets all the requirements imposed by the job position announcement, and she can never be treated as unacceptable by the Institution.⁵²

This means that an institutionally acceptable agent can never be rejected: (a) if she is the only applicant then she is automatically temporarily matched with the institution; (b) if someone better than her is chosen, then she stays in the waiting list. From the Institutions' point of view, it means that they are always obliged to pick a candidate from the group who applied, as long as there is at least one of them who is institutionally acceptable. Consequently, we consider to be unacceptable mates for Institutions, only the agents who do not comply with the mandatory requirements. For convenience of the model presentation, in the paper we will consider all the candidates who send an application as institutionally acceptable. Note that, the same condition can be easily detected (and applied) in a more general job market when the hiring procedure is costly in terms of money and time for firms. In this case, they would prefer to hire someone who is good enough (acceptable) among the ones who applied as soon as they can, instead of rejecting everyone and keep looking for the best candidate. The final effect is the same, there is at least one candidate who is chosen and the "reject-all" effect is avoided. This real-life constraint has a huge impact on the Institutions' strategies, as it will be showed in the specific paragraph.

⁵² This is a real-life condition imposed in hiring procedures through public contest. Candidates have to meet requirements for participating, and at the same time, they can be discarded directly from the procedure only if they do not meet all the requirements, and the winner is the ones who satisfies the requirements better than all the other candidates.

3. The Decentralized Multiple Rounds Mechanism (DMR)

Let $R = \{r, r+1, \dots, r+n\}$ be the finite set of **rounds** that compose the whole game. Each round is actively played by sub-sets of I and C identified by $I^r \subset I$ and $C^r \subseteq C$, for any $r \in R$. While C^r could even totally correspond to C , this is never possible for I^r by assumption. The aim is to reproduce a hiring procedure of different institutions that is distributed over a long time period, and not concentrated in a sole well-defined short term. This is a typical real characteristic of the job market, where companies open job positions according to their needs, at any time of the year, independently by any time-constraint⁵³. For representing this **unregulated timing-structure** of the hiring procedure, Institutions do not offer the positions all at the same time but over the different rounds. This is the reason why it is never possible for a round sub-set I^r to be equal to the entire original set I by assumption. Let γ be the **permutation** of the set I over the game defined as $\gamma = \{(i, i')_r, (i'')_{r+1}, \dots, (i^n)_{r+n}\}$ where the sequential order identifies the distribution of the set I over the rounds of the game for $i_r = \gamma I$ for all $r = 1, 2, \dots, n$ and the brackets denoting the institutions that play simultaneously during the same round.

2.1 The Round Structure and Mechanism

The proposal – acceptance/rejection matching mechanism of the game is carried out per round, and each round has the same following structure:

- Step_0: Some Institutions simultaneously announce a job position.

At any round, the institutions that play identified by the set I^r are the ones with a free position to offer⁵⁴. The universities who announce the position belongs now to I^r , and they are the only ones that will make active moves during this round. Formally, any $i \in I$ sends an “activity message” at each round, denoted by $a^r(i)$ in the form $a^r(i) = \{c \in C : c \succ_i i\}$ if it offers a position, or $a^r(i) = \{i\}$ if it stays inactive.

⁵³ Examples of time-constrained hiring procedure, are the cases of matching medical interns to hospitals or students to schools and colleges, where the starting and finishing point are well-defined and have to be respected by all the Institutions.

⁵⁴ In the real world, the opening of a job position could be subject to some requirements or constraints; in the model the order of announcement is taken as randomly given.

Then, the round sub-set I^r is defined as $I^r = \{i \in I \mid c \in a^r(i)\}$. Since γ is randomly given by assumption, the composition of all the round sub-sets is already known in advance. The job is publicly announced, and the offer is addressed to all the ones who owns the mandatory requirements inserted into the announcement.

- Step_1: Once the positions are offered, candidates simultaneously send the applications to the job positions they are interested in.

The candidates who apply for the position define the set of active players C^r , and they are the only ones that will make moves during r . Formally we say that each $c \in C$ sends a message denoted by s^r in the form $s^r = \{i \in I^r : i \succ_c c\}$ if she applies for the job, or $s^r = \{c\}$ if she stays inactive. So, during Step 1 the general round sub-set $C^r = \{c \in C \mid c \notin s^r\}$ is defined, together with the proposing sub-sets per institution $C^r_{i'} = \{c \in C^r \mid i' \in s^r\}$, for $i, i' \in I^r$. While the first sub-set identifies all the active candidates who are playing during the round, the second gathers specifically the candidates per institutions they applied to. Note that, candidates can propose to all the active universities of the round. In fact, remember that candidates have no limited number of applications per round and per game, and that sending a proposal is costless by assumption.

- Step_2: Any institution simultaneously picks up one candidate and orders the others on the waiting list according to its preferences.

Any $i \in I^r$ chooses one candidate within the set of the ones who defined it an acceptable mate – i.e. C^r_i – and sends a message $s^r(i)=\{c\}$. If two different institutions $i, i' \in I^r$, for $i \neq i'$, select the same candidate, such as $s^r(i) = s^r(i') \equiv c$, then the decision is given to the candidate. This “**back-and-forth**” phase allows the mechanism to work perfectly even if more institutions select the same applicant. Since the final decision is remitted to the candidate, and it is not arbitrary defined by a sequential-choice-mechanism of the Institutions, the result is never influenced by how many institutions per round choose the same applicant. Moreover, since candidates have not constraints on applications and do not

commit to temporary matchings, the mechanism is not influenced by how many and which institutions play during the round.

Proposition 1. *The functioning of the mechanism induced by the game G^Y_R is **never affected** by the permutation γ of the set I over the rounds and the number of institutions that simultaneously pick the same candidate during the same round.*

Proof in the Appendix. For a clearer presentation I^r will be posed equal to 1 for the entire paper, i.e. $I^r = \{i\}$ and $\gamma = \{i_r, \dots, i_{r+k}, \dots, i_{r+n}\}$.

The candidates who have not been chosen as “winners of the contest” are not definitely rejected, and stay in the set of the **waiting list** $WL(i) = \{C^r_i \setminus \mu^t_r(i) | P(i)\}$, composed by all the candidates who applied less the one who has been picked up during the active round.

- Step_3: Temporary matchings are defined and the round ends.

Each round ends with a temporary matching for any $i \in I^r$ in the form $\mu^t_r(i) \equiv x$ for $x \in C \cup \{i\}$, and the set of temporary matching is defined by $M^t_r = \{\mu^t_r(i), \dots, \mu^t_r(i^n)\}$ that gathers all the temporary assignments realized up to round r .

2.1 From the single Round to the whole Game

As we consider the whole game, we need to characterized some features of the mechanism. The agents of the round sub-sets I^r and C^r are identified as **active players** of the round, since they are the only ones who have the chance to make moves. Instead, the other players have the passive or the inactive status. Agents stay in an **inactive status** when they are still waiting for their active round (institutions) or if they have not sent an application yet (candidates). On the contrary, an agent has a **passive status** if she is being already active for at least one round of the game. For an institution, being passive means being temporary matched, such that $i \in I^P \Leftrightarrow i \in M^t_r$ for any $r \in R$. They are not allowed to re-open the job position or to receive applications, but they can send job offers to the candidates who are on their waiting list as long as their temporary matching has been broken. In fact, Institutions commit to temporary assignments but candidates

can break the temporary matching at any following round of the game. Consequently, if during Step 2 of a following round, a temporary matched candidate receives a new offer from an active institution, in Step 3 she is automatically assigned to the new job position⁵⁵. As an applicant leaves her current assignment, the left institution has to fulfill again the job position by scrolling down the $WL(i)$ – as long as $WL(i) \neq \{\emptyset\}$ – and offering the position to the first candidate into the waiting list, or announcing it again – if $WL(i) = \{\emptyset\}$ – after that all the other Institutions have played one active round. When candidates on the waiting list receive the job offer – from an institution which the candidate sent an application to during a previous round – the assignment is not automatic, and the decision is up to the candidate. The institution keeps sending offers to the components of its $WL(i)$ until the position is fulfilled or the $WL(i)$ goes empty – and so $\mu^t(i)=i$. The institution that is matched to itself stays in a passive status until all other institutions has actively played at least one round. As the round $r+k$ where the last institution offers its available job position ends, all the ones temporary matched to themselves are now automatically considered as unmatched. They announce again the job positions respecting the timing sequentially-round order given by γ for the rounds from r to $r+k$ ⁵⁶. Since candidates play different rounds, they can simultaneously have an active and a passive status. For an applicant, being passive means having already sent a candidacy to some institution or simply being on some waiting list, such that $c \in C^P \Leftrightarrow c \in WL_i$ for some $i \in I^P$. When a candidate holds a passive status, it means that she is able to receive an offer from the institution whose waiting list she is part of. The **waiting list** introduce a typical contest-feature that represents a constraint for the Candidates' side: if $c \notin C_i$ then $c \notin WL(i)$ and so she cannot be chosen by i during the game, unless $\mu^t(i)=i$ at some point of the

⁵⁵ Since the candidates' preferences are revealed into the game by applying to institutions who offers job positions, it is rational to assume that when a candidate is picked as winner of the contest – during the same round of the application – she would not reject the offer. Given this assumption, for fluidity of the model it has been removed the “acceptance step” where the selected candidates officially accept the job, and they are automatically assigned. It is different for candidates who are in the waiting list and receive an offer in some following round, since in the meanwhile they could have been temporary assigned to some institution they prefer more.

⁵⁶ As long as round $r+k$ ends, the institutions receive a call for going on the job market according to the sequential order given by the permutation γ , and if $\mu^{t_{r+k}}(i)=i$ then the institution offers the available position o.w. the institution maintain the temporary assignment and the procedure goes on till the last one.

game⁵⁷ and so it will re-open the position, giving the chance to candidate c to apply. If a candidate does not apply to one institution while it is actively playing, then she can lose the opportunity of being chosen for the rest of the game (if the waiting list is long enough). The aim is to reflect into the model a real-life feature of both contests' setting and, more generally, of the hiring procedures on the job market.

At the end of each round the set M^t_r is updated by adding the new temporary matching of $i \in I^r$ realized during the new round, and the new assignments that occurred for $I^{\bar{P}}(I^r, \gamma)$, i.e. the set of all institutions that played before I^r under the permutation γ . As soon as no more job positions are available – or the ones left are considered as unacceptable by all the candidates – the game ends. At this point, the temporary matchings of the last round M^t_{r+n} become official, $M^t_{r+n} \equiv M$.

Example

$$I = \{i, i'\}; C = \{c, c'\} - \gamma = \{i, i'\}$$

$$P(i) = P(i') = \{c, c'\}; P_I = \{i', i\}, P(c') = \{i, i'\}$$

Round 1.

- $I^1 = \{i\}; s^1 I = s^1(c') = \{i\}$ such that $C^1_i = \{c, c'\}$

$$s^1(i) = \{c\} \rightarrow \mu^{t_1}(i) = c; WL^1_i = \{c'\};$$

- $M^t_1 = \{\mu^{t_1}(i, c)\}$

Round 2.

- $I^2 = \{i'\}; s^2 I = s^2(c') = \{i'\}$ such that $C^2_{i'} = \{c, c'\}$

$$s^2(i') = \{c\} \rightarrow \mu^{t_2}(i') = c; WL^2_{i'} = \{c'\}$$

- $I^{\bar{P}} = \{I\}; c$ broke $\mu^{t_1}(i, c)$

so i scrolls the WL $\rightarrow s^2(i) = \{c'\} \rightarrow \mu^{t_2}(i) = c'; WL^2_i = \{\emptyset\}$

- $M^t_2 = \{\mu^{t_2}(i', c), \mu^{t_2}(i, c')\}$

⁵⁷ Because $WL(i)$ goes empty.

End: $M^t_2 \equiv M$

4. Strategies of the Players

The strategies of the players are analyzed in a setting of complete information. The following features of the model are common knowledge:

- The composition of the sets I and C ;
- Any $i \in I$ will offer a job position;
- The permutation of the set I over the rounds of the game γ – taken as randomly given – and so the composition of any I^r for any $r \in R$;
- The preference profile P ;
- All the assumptions on the game are known

The sole private information is the message that candidates send at the starting of each round, s^r_I . Since they all act simultaneously, there is no possibility for one candidate to know in advance what strategies the others selected. However, the information is acquired at the end of the round when the round-actions of all agents become common knowledge. Then, at the end of the round players acquire the following information:

- s^r_I of any $c \in C$, and so the composition of C^r_i for any $i \in I^r$ (candidates' actions)
- $s^r(i)$ of any $i \in I^r$, and so the composition of $M^t_r = \{\mu^t(i), \dots, \mu^t(i^n)\}$ (institutions' actions)

2.1 Institutions' Dominant Strategy

The “institutionally acceptable” constraint is a fundamental component of the mechanism that reduces the set of possible strategies for institutions. In fact, they have not the opportunity to reject all the candidates who applied to the job in order to hold the position for someone they prefer more. This entails a restriction of the strategies' space.

Proposition 2. *The **institutional acceptability** condition causes a restriction of the strategy profile space of the agents on the Institutions' Side.*

Proof in the Appendix. Formally, any $i \in I^r$ make a choice onto the set C_i^r , such as its strategy is a function $Ch_i: P(i) \rightarrow C_i^r$ that defines its message profile $s^r(i) = \{c\}$. As long as the Institutions do not have the possibility of rejecting all the candidates, given the assignment mechanism induced by the decentralized multiple-round game G^r , it is a dominant strategy for any $i \in I$ to pick the candidate it prefers the most, from the sub-set of candidates who applied to the offered job position. Formally, for any $i \in I$, given $P(i)$ and $C_i^r = \{c \in C : i \in s^r(i)\}$, and for $s^r(i) \neq \{\emptyset\}$ as $C_i^r \neq \{\emptyset\}$, then $s^r(i) = \arg \max_{P(i)} \text{ on } \{c \mid c \in C_i^r\}$ or $s^r(i) = \arg \max_{Ch_i} := \{c \mid c \in P(i) \wedge \forall (c' \in C_i^r) \in P(i) : Ch_i(c') \leq Ch_i(c)\}$ is always a dominant strategy, for any γ and any $r \in R$.

Theorem 1. *Given the constraint of institutional acceptability, such as $s^r(i) \neq \{\emptyset\}$ as long as $C_i^r \neq \{\emptyset\}$, the strategy $s^r(i) = \arg \max_{Ch_i}$ is a **weakly dominant strategy for each Institution**, into the decentralized multiple-rounds mechanism induced by the game G^r for any $r \in R$, and for any γ .*

Proof in the Appendix. The strategy followed by the Institutions' side can then be assimilated to the dominant strategy followed by the proposing-side into a Deferred-Acceptance mechanism. Under the D-A allocative mechanism, the proposing agents have the dominant strategy of truthfully submitting their preferences' orders (Dubins and Freedman 1981; Roth 1982b), such that they start proposing to their most preferred choice. In a similar way, the players on the Institutions side offer the job position to the candidate they like the most, identified into the set of the ones who sent the applications. Actually, this is their reference set since they cannot send direct offers to any $c \in C$, such that C_i^r represents the only set institution i can interact with, for any $i \in I$. Assume, then, that institutions update their preferences' lists given the "institutions' candidates set" C_i^r , i.e. they remove from the lists all the ones who did not apply for the position. As long as the institution follows its preference list for announcing the winner of the competition, then it is revealing its real preference profile given the constraint of the reduced

candidates' set. Then the following Lemma is stated on the basis of this assessment.

Lemma 1. *The weakly dominant strategy $s^r(i) = \arg \max C^r_i$ is a **truthful reporting strategy** with respect to the reference set C^r_i for any $i \in I$.*

Theorem 2. *In the decentralized multiple-rounds mechanism induced by the game G^Y_R , it is a **weakly dominant strategy for each Institution to truthfully reveal its preferences**, for any $r \in R$, and for any γ .*

The proof of the theorem comes directly from Theorem 1 and Lemma 1.

The results obtained from the Institutions' side retrace the ones reported by Blum et al. (1997), Sotomayor (2003), and Haeringer and Woodens (2011) for decentralized mechanisms where employers sequentially and directly propose the job position to one candidate per time.

2.1 Candidates' Strategies

The strategy of a candidate is determined by the choice of applying or not to the institutions that are offering the job position. Candidates are able to play multiple rounds and their strategies can be defined per round as $s^r_i \in S^r_i$, for $S^r_i = \{s^{r1}_i, \dots, s^{rn}_i\}$ be the strategy profile of candidate c at round r , i.e. the set of all possible strategies at round r for any $c \in C$. On the other hand, the strategy can be considered on a more general level, and can be defined for the entire game as $s_i \in S_i$, for $S_i = \{s_i^1, \dots, s_i^n\}$ be the strategy profile of candidate c for the game G^Y_R . Moreover, given the temporary allocations defined at the end of each round, it is also possible to differentiate the analysis for **candidates who are temporary matched – identified by μ -candidates – and** candidates who are still **unmatched**⁵⁸. Once candidates are temporary assigned, they have their current mate as point of reference for the next actions into the game. As outlined also in the work of Blum et al. (1997), it is natural for who is already holding an offer or a job, to compare the new proposal to that one, and then to follow the kind of

⁵⁸ (Blum et al. 1997)

strategies denoted by the authors as “preference strategies”. These strategies make the μ -candidates decide whether apply or not to a new job position while already holding another one. Note that, once the agent applies and is selected by the institutions, she cannot reject, since the action of sending the application is considered the expression of the candidate’s willingness of accepting the job⁵⁹. Candidates have also non-preference strategies, as apply to any new position even after been matched to one of them, or hold the first job proposal and stop to apply to any other contest. However, the non-preference strategies are always dominated – and sometimes non-rational – strategies for all the rounds of the game. The strictly dominated strategies are excluded from the study, such as only weakly dominated and non-dominated strategies will be taken into account for the analysis of candidates’ strategies. We focused then on the truthfully reporting strategy and on the truncation strategies⁶⁰, i.e. revealing into the game only the top part of the real preference list.

2.1.2 Truthfully Revealing Strategy

By **truthfully revealing strategy (TRS)** traditionally we denote the act of sending an application to all institutions that belong to the preference list of the candidate, i.e. $s^i = \{i \in I^r \mid i \in P^i\}$ for any $r \in R$. However, as soon as a candidate is temporary assigned, her TRS entails to send applications to all the institutions that belong to her preferences’ lists and that she likes more than the current mate. Then, we redefined the TRS strategy such that it could be valid for all $c \in C$ independently if they have been already assigned or not. In a DMR mechanism, a candidate displays a truthfully revealing strategy when, given her preference profile P^i , she sends an application to all the institutions she prefers more than her current matching.

⁵⁹ The situation is different for the candidates who result to be temporary matched and receive an offer from an institution they applied before because of the scrolling down of the Waiting List. In this case the agent can reject the offer, or can simply choose to remove herself from the WL once obtained a better job.

⁶⁰ (Roth 1984b; Roth and Rothblum 1999)

Proposition 3. *In a DMR mechanism, for all $c \in C$ it is a **Truthfully Revealing Strategy (TRS)** to send the message $s^r I = \{i \in I^r : i \succ_c \mu^t I\}$, given PI , for $\mu^t I = x$ and $x \in IU\{c\}$, for any $r \in R$.*

The TRS is considered to be an **implementing preference strategy** for the temporary matched candidates. The term implementing preference strategy denotes the strategy that tells to the μ -candidate to apply only to job positions that she prefers more than the current one. Since the only application does not cause the loss of the temporary assignment, the candidate can try to obtain a better job without damage. It would be irrational to apply to all institutions even where already matched, or to not apply anymore just because temporary matched as not being consistent with any preference order. In the first case, agents would risk to reject an old assignment that was better than the new one; in the second case, they would renounce to the chance of getting a job more preferred than the current one. Remember that the applications are assumed to be costless, such that candidate should be incentivized to participate at contests for positions she likes more. The implementing preference strategy defines an action profile for the entire game as μ -candidate, defined as $s^*(\mu\text{-candidate})$, that aims to improve the current temporary condition $\mu^t I$ using the preference profile PI as a guide of this escalation path. Formally we obtain exactly, for $c \in M_{r-1}^t$ and $\mu^t I = i$, given PI , for $i, i' \in I$ and $i \neq i'$, $s^r(\mu\text{-candidate}) = \{i' \in I^r \mid i' \succ_c \mu^t I\}$, i.e. the TRS just defined. But the implementing strategy can be also considered as an update process of the preference profile of the candidate that occurs at the end of any round. All the institutions the candidate likes less than $\mu^t I$, are now treated as unacceptable since it would be irrational to leave the current position for a worse offer. The update process consist of removing such agents from the preference list, i.e. as $c \in M_{r-1}^t$ and $\mu^t I = i$, then PI is adjusted as $P^* I = \{i', \dots, \mu^t I\}$. The implementing preference strategy can be defined also as $s^r(\mu\text{-candidate}) = \{i' \in I^r \mid i' \in P^* I\}$, where $P^* I$ is

actually a truncation of the preference list P_i where the last element is the current assignment⁶¹.

Proposition 4. *Into the decentralized multiple-round mechanism induced by the game G^{γ_R} , for all the agents truthfully revealing their preferences, it is a **best response** for any temporary matched candidate to follow an implementing preference strategy for any $r \in R$ and for any γ .*

Proof in the Appendix.

On the contrary, for the still unmatched candidates revealing their real preferences is not a best response.

Theorem 3. *In a DMR mechanism induced by the game G^{γ_R} , if the players of both sides truthfully reveal their preferences into the game, there is always at least one candidate who can benefit from deviating, for any $r \in R$ and for any γ .*

Proofs in the Appendix. Unless the game has a singleton core, there is always at least one candidate who has an incentive to deviate from a truthfully revealing strategy, given the dominant strategy played by any $i \in I$.

2.1.2 Truncation Strategy

The alternative to a truthfully revealing strategy for a candidate is a **truncation strategy**. By truncating their preferences' lists, candidates reveal only part of their true preferences into the game. Since candidates' preferences are reported into the game by applying or not to the institutions, truncating their lists means that they do not apply to all acceptable institutions but only to the ones that are on the top part of their preferences' orders. Formally, candidate c truncates the preference list P_i after k -elements, and $P_i = \{i, \dots, i^k, \dots, i^n\}$ is reported into the game as being $P_i^* = \{i, \dots, i^k\}$, i.e. candidate c does not send an application to any institution less preferred than i^k . The truncation strategy profile per round is defined as $\mathbf{s}^* = \{s_i \in I^r$

⁶¹ Note that in this case the truncation of the preference list is not a misrepresentation, on the contrary, it helps the candidate to correctly reveal the preferences' order into the game by removing institutions less preferred than the current matching.

$\{i \in P^*I\}$, for $P^*I \subseteq PI$ and for any $r \in R$. Given the dominant strategy played by the Institutions' side, there is always at least one candidate⁶² who is better off by reporting a truncation of PI over the game, i.e. $s^*I = \{i \in I \mid i \in P^*I\}$ for $P^*I \subseteq PI$.

Proposition 5. *In a DMR mechanism, for all the other agents truthfully revealing their preferences, there is always at least one candidate $c \in C$ who is better off by revealing a truncation P^*I of the real preference list PI .*⁶³

The Proof of Proposition 5 comes directly from the Proof of Theorem 3.

Theorem 4. *For a candidate in a DMR mechanism, given the dominant strategy played by any $i \in I$, reporting a truncation of her real preferences dominates any non-truncation strategy, for all the other candidates playing a weakly dominated or non-dominated strategy.*⁶⁴

Formally, for $c \in C$ and for P^*I being the truncation of the real preferences' list PI , then for the preference profile P_{-c} of the other players, **DMR $[P^*I, P_{-c}] \succ_c$ DMR $[PI, P_{-c}]$.**

Proofs in the Appendix. By not applying to some acceptable institutions, and mostly by avoiding the institution that would be her worst assignment⁶⁵, the candidate triggers a domino-effect over the entire game (Roth 1984b). Then, it could be sufficient the deviation of a single player for improving the welfare of all the Candidates-side. However, applicants do not know what kind of strategy will follow the others, since they only know the preferences' profiles but not the messages sent at the beginning of the round. Since revealing only the top-part of the preferences' order dominates all the other strategies, then we can state the following corollary to Theorem 4.

⁶² Any candidate whose assignment is different under an Institutional-Optimal or Candidates-Optimal outcome, is better off by playing a truncation strategy. As long as the game has not a singleton core, then there is at least one candidate who prefers any matching different from the one obtained under an I-Optimal outcome.

⁶³ The result is not so different from what stated for the D-A mechanism (Roth and Sotomayor 1992).

⁶⁴ The result is close to the one reported by Roth and Rothblum (1999) for the D-A mechanism.

⁶⁵ The institution she would be match with under an Institutional-Optimal result, denoted as $\mu^I(c)$. Remember that this is the assignment obtained under a truthfully revealing strategy.

Corollary 1. *In a DMR mechanism, given the dominant strategy played by all $i \in I$, for any candidate who reveals a truncation of her true preferences there are no incentives to deviate, given all the other candidates playing a non-dominated or a weakly dominated strategy.*

The proof of Corollary 1 comes directly from the proof of Theorem 4. We can affirm that reporting only a truncation of the real preferences is a best response for any candidate, for the given strategies of the other players.

As long as a candidate is assigned to a job position – after have played a truncation strategy – she is still incentivized to send an application to all the institutions that belongs to P^* and that she likes more than μ^i . Then, an implementing preference strategy is still the best response strategy for a μ -candidate.

Theorem 5. *Into the decentralized multiple-round mechanism induced by the game $G^{\gamma, R}$, given the dominant strategy played by all $i \in I$, it is a **best response** for any temporary matched candidate to follow an implementing preference strategy for all the other candidates playing a weakly dominated or a non-dominated strategy, for any $r \in R$ and for any γ .*

Proof in the Appendix. Then, as long as a temporary assignment occurs, a candidate is better off by implementing a preference strategy based on her updated preference profile. This result is independent of the strategy they followed as unmatched candidates and of the strategy that the other applicants are displaying.

5. Equilibrium Strategies and Stability Properties

Given the dynamic nature of the game identified by the DMR mechanism, we will refer to Subgame Perfect Equilibrium. A strategy profile S is a Subgame Perfect Equilibrium if for each player $x \in I \cup C$, given a preference profile P , there is not an S' such that the outcome obtained under S' is preferred by some x to the outcome

obtained under S . Then, S is an SPE if there is not an agent x who would be better off under S' with respect to the outcome obtained under S , and so who would have an incentive to deviate from S .

2.1 Equilibrium Strategies

The analysis of players' strategies outlined a weakly dominant strategy for the Institutions' side and a Best Response for the Candidates' side. We demonstrated then that for the game induced by the DMR mechanism there exists a strategy profile under which no agent is incentivized to deviate.

Theorem 6. *Let S be such that $s(i) = \arg \max C_{hI}$ for all $i \in I$, and $s^*I = \{i \in I \mid i \in P^*I\}$ for all $c \in C$ and $P^*I = \{i, \dots, \mu^cI\}$, then S identifies a Subgame Perfect Equilibrium of the decentralized multiple-rounds mechanism induced by the game G^{γ_R} , for any $r \in R$ and for any γ .⁶⁶*

The proof of the Theorem comes partly from Theorem 1 – dominant strategy of Institutions – and Theorem 4. The truncation of the preferences' list is posed after element μ^cI , since it identifies the optimal strategy that allows candidate to reach a Candidate-Optimal outcome – i.e. the best possible stable outcome they could ever end up with.⁶⁷ Note that for all candidates revealing $P^*I = \{i, \dots, \mu^cI\}$ into the game, it is not necessary to redefine the implementing strategy for temporary matched candidates. In fact, the last element of the list of preferences that will be revealed into the game is the best possible outcome of each candidate, denoted as μ^cI . This means that any $i \in P^*$, for $i \neq \mu^cI$, such that $i \succ_c \mu^cI$, is the best assignment of some other candidate c' , for $c \neq c'$ and $c, c' \in C$. Then, the implementing preference strategy of a μ -candidate would be equal to the one identified by s^*I for $P^*I = \{i, \dots, \mu^cI\}$. As long as no other assignments could occur for candidate c revealing P^* but μ^cI , then $P^*I = \{i, \dots, \mu^cI\}$ correspond to $s^r(\mu\text{-candidate}) = \{i \in I^r \mid i \succ_c \mu^cI\}$.

⁶⁶ By $\mu^C(c)$ we denote the best possible stable outcome for candidate c into the DMR mechanism, given P .

⁶⁷ The Proof is in the Proof of Theorem 7.

Theorem 7. *For S being an SPE of the decentralized multiple-rounds mechanism induced by the game G^R , the outcome of the mechanism is stable and Candidates-Optimal.*

Proof in the Appendix. These results hold under a two-sided commitment assumption as long as $s^I = \{\mu^C\}$ ⁶⁸.

Some considerations are necessary about the implementation of a truncation strategy by the candidates. In a setting where almost all information are available – mostly the preferences' profiles of all agents – the applicants have sufficient tools for revealing into the market the optimal number of preferences. Questions may rise about what would happen if candidates are not able to truncate their lists exactly as s^I , especially if we think in practical applications of the model. As long as the number of job positions equals the number of candidates into the market:

- if a candidate reveals at first a too short truncation of her preferences, such as $\mu^C \notin P^I$, as long as all the other candidates c' play $s^*(c')$, the job position at the institution $i \equiv \mu^C$ after its first active round, would still be available. As the institution i re-announces the position on the market, candidate c – already discarded by all others institutions i' such that $i' \succ_c \mu^C$ – will send the application. So, at the end of the game candidate c actually revealed into the market exactly P^I .
- assume candidate c follows a TRS, such that reveals her real preferences. As soon as she is assigned to some institution, then she will keep following an implementing preference strategy, by applying to any $i \succ_c \mu^I$ for $\mu^I \neq \mu^C$. As long as all the other candidates c' play $s^*(c')$, then the final outcome is M^C .

Different are the consequences if we assume that the **number of candidates** into the market is **larger than the number of available positions** – as actually happens in real markets. Let consider the DMR procedure with m Candidates and n Institutions, for $m > n$. It is clear that at the end of the mechanism, some

⁶⁸ Under the assumption of two-sided commitment, the results on the mechanism are quite close to the ones presented by Sotomayor (2003).

candidates will be matched to herself. We already know that for any stable outcome, the agents who are matched are always the same, such as for any stable outcome the sub-set of candidates who will end up without a job is unchanged⁶⁹. We analyzed how the number of candidates in the market can influence the strategy decision of the agents. Consider that for any $c \in C_{un}$, for $C_{un} \subset C$ and $C_{un} = \{c \in C : c \notin M\}$ ⁷⁰, the final outcome is $\mu^C|_{\mu^I} \equiv c$. Then, their non-dominated strategy entails to apply to any institution that is on their preference list. As long as the players follow the equilibrium strategies, the increase in the number of candidates does not have any effect onto the game. Assume now that candidate c' , for $c' \in C$ and $c' \notin C_{un}$, reveals at first a too short preference list – as in the case above-mentioned – i.e. c' does not apply at $\mu^C(c')$ during its active round. When the number of candidates was the same of the available job positions, there were no consequences. However, now there is some $c \in C_{un}$ who will send an application to $\mu^C(c')$ and that will not break the assignment – since she will not be accepted by any other institution⁷¹. Since candidate c does not leave the temporary matching, candidate c' will not have the chance to apply to $\mu^C(c')$ in any following round. Then, candidate c' will be matched to herself and the final matching will be no longer stable – instability arises for unassignments. Though, the numerical asymmetry of the market does not affect the theoretical model in anyway, and the implications are more related to real-world cases. In fact, even if on a theoretical basis the mechanism has been proven to have an SPE and to be stable, it has also been demonstrated in other studies that most of the time real agents are not able or do not choose to display a misrepresenting strategy.⁷²

Note that, in the situation where all agents are truthfully revealing their preferences, the numerical asymmetry between the two sides of the market has

⁶⁹ (McVitie and Wilson 1970)

⁷⁰ For M being the set of stable matchings.

⁷¹ For all the other candidates who keeps playing the equilibrium strategy $s^*(c)$, and then who apply to their $\mu^C(c)$.

⁷² The laboratory experiment carried on by Featherstone and Niederle (2009) tested the capacity of students of identifying and implementing the best misrepresenting strategy and most of them failed. Moreover, it has been demonstrated that also in a mechanism managed through a D-A mechanism, candidates would be better off by misreporting their preferences – when the procedure is Employer-Proposing. Even though, an analysis on the American Residents market highlighted that the majority of candidates report the real list of preferences as cited by Ehlers and Massò (2007).

no influence at all. The TRS appears to be a less risky strategy, mostly into procedure where the numerical asymmetry is particularly strong. As concluded also by Roth and Rothblum (1999) about the D-A mechanism, the truncation strategy is possible and is the best response strategy for a candidate, but it is also the riskier one. Remember also that the analysis of this procedure was carried out in a complete information setting – that is far from reality – where candidates are facilitated in recognizing the value of a truncated reporting. It would be reasonable to consider that in real-world markets, some information – as the preference profile P – are not available, and that candidates could be oriented more to a TRS. Indeed, a wrong truncation could cause the loss of any assignment, while revealing true preferences always guarantees a final matching – even if not the most preferred.

Proposition 6. *In a DMR mechanism induced by the game G^{γ_R} , as long as a candidate cannot implement a truncation strategy, the best response is to reveal her true preferences, for all the other agents playing a weakly dominated or a non-dominated strategy, for all $r \in R$ and for all γ .⁷³*

2.1 Stability Properties

At this point, it acquires more value an analysis of the stability condition of the mechanism for all agents truthfully revealing their preferences into the market. We can assume a sort of myopia according to which all candidates only report real preferences into the game. Given the dominant strategy played by any $i \in I$, for all $c \in C$ playing a TRS identified by s^I at any $r \in R$, the outcome of the DMR mechanism is stable and Institutions-Optimal, denoted by $M^I = \{\mu^I(i), \dots, \mu^I(i^n)\}$. This means that given the preference profile P , as long as any agent truthfully reveals her preference, under the DMR mechanism, any institution obtains the best

⁷³ (Ehlers 2008; Roth and Rothblum 1999)

possible mate. This outcome corresponds to the worst possible stable assignment for candidates⁷⁴.

Theorem 8. *In a DMR mechanism induced by the game G^{γ_R} , for all institutions having the dominant strategy of truthfully reporting, if all candidates truthfully reveal their real preferences into the game, the outcome is stable and Institutions-Optimal, for any $r \in R$ and for any γ .⁷⁵*

Proof in the Appendix. The stability result heavily depends on the key assumption of one-sided commitment.

Proposition 7. *In a DMR mechanism induced by the game G^{γ_R} , for any $x \in IUC$ truthfully revealing her real preferences, the one-sided commitment is a necessary but not sufficient condition for ensuring the stability of the outcome.*

As long as also candidates are called to commit to the assignment realized at the end of the round, this result does not hold anymore⁷⁶. However, the one-sided commitment is not a sufficient condition for the stability of the outcome. Indeed, if restrictions are applied on applications – as costs or limit number of applications per round – the final matching is unstable due to blocking pairs.

Proposition 8. *In a DMR mechanism induced by the game G^{γ_R} , for any $x \in IUC$ truthfully revealing her real preferences, no costs and no restrictions on application are a necessary but not sufficient condition for ensuring the stability of the outcome.*

⁷⁴ The matching is considered Optimal for the Institution' side in the sense that there is no other stable matching v that associate to any institution i an assignment $v(i)$ that it would prefer to $\mu^I(i)$. Specifically, “when preferences are strict the set of stable matching is a lattice with respect to the partial order $>_M$ ” [from the Lattice Theorem of Conway and Knuth (Knuth 1976)], given the lattice structure of the set of the stable matchings (Blair 1988), $\mu^I(i)$ represents the maximum for any $i \in I$ and $\mu^C(i)$ is the minimum (and vice-versa for candidates).

⁷⁵ It is quite easy to see that most of the results obtained so far are close to the ones presented for the D-A algorithm with the Institutions proposing first (Roth 2008).

⁷⁶ Assume $I = \{i, i'\}$ and $C = \{c, c'\}$ and let $P(i)=P(i')=\{c, c'\}$, $P(c)=\{i, i'\}$ and $P(c')=\{i, i'\}$ for $\gamma=\{i_r, i_{r+1}\}$. All candidates follow a TRS strategy and all institutions play their dominant strategy. At the first round both c and c' send an application to institution i whose message will be $s^c(i)=\{c\}$, and $\mu(i)=c$. Under a two-sided commitment assumption, as the matching is realized both agents are removed from the market. Then at round $r+1$ the only possible assignment is $\mu(i')=\{c'\}$. It is easy to see that the outcome is blocked by the pair (i', c) .

It follows that the key assumptions on the model, are also the key features of the mechanism that allows the outcome to be stable when all the agents truthfully reveal their preferences.

Theorem 9. *In a DMR mechanism induced by the game G^Y_R for any $x \in IUC$ truthfully revealing her real preferences, the assumptions of one-sided commitment and of “no-costs and no-restrictions” on applications are necessary, and sufficient when applied simultaneously, conditions for ensuring the stability of the outcome.*

Proofs in the Appendix. As long as all the agents into the game follow a weakly dominated or a non-dominated strategy, the stability of the matching depends exclusively on the characteristics of the market. This result has been highlighted mostly in relation to the practical real case study presented in the next section.

6. The Hiring Procedure for Assistant Professors in Italy

The hiring procedure for Assistant Professors at public universities in Italy is carried out on a decentralized systems of public competitions over the entire year. The Ministry of Education, University and Research (MIUR) draw the mandatory guidelines for the hiring procedure which all universities have to be aligned to, into the Law 240/2010⁷⁷. Out of the respect of the Law requirements, the universities have their autonomy in managing the entire process. As they receive a formal approval by MIUR⁷⁸, each university publicly announces the job position at some point of the year, starts to collect the applications and builds up the committee. The contest is made up by two parts, at first an evaluation of the academic titles of the candidates and then an oral examination – held at the university site – where only part of the applicants is accepted. Then the committee expresses a judgment about all of them and publicly announces the winner of the contest. The market

⁷⁷ The Law 240/2010 also known as Gelmini reform is a recent revision of the old rules about the hiring procedure.

⁷⁸ Since the opening of a new position is linked to the disposability of financial resources, MIUR has to officially approve it.

has been reformed different times over the years, but it seems to still suffer some sort of **inefficiencies** that have not been understood or addressed by the restyling policies. We implemented the theoretical model of the Decentralized Multiple-Round mechanism for study the causes of failures – if any – of this hiring procedure. Even if the model has been formalized in a complete information setting, it is robust enough to demonstrate the bad functioning also of the real hiring procedure where information is scarce. However, we need some consideration about how the theoretical DMR mechanism is assimilated to the real process. The public competition of any university has a duration that can vary from a couple of months till 2 years in some extreme cases⁷⁹, but we assume that the contest starts and unofficially – since the matching is just temporary – ends during the same round of the position announcement. As above-mentioned, the hiring procedure has not a bounded timing, and announcement can be published at any time over the year. This unregulated time structure can be easily assimilated to the round structure defined by the DMR. Since the job position can be obtained only through a public contest, universities cannot make “direct calls” to candidates, and vice-versa, candidates cannot apply out of the procedure – as in the DMR. Then let consider the real hiring process as a DMR mechanism with numerical asymmetry where universities play the dominant strategy⁸⁰ – picking the candidate they like the most – and candidates truthfully reveal their preferences by assumption⁸¹. We learned from the theoretical analysis of the model that the outcome of such procedure can be stable only under certain key assumptions⁸². First of all, we assume that sending applications for candidates is costless so they are incentivized to participate in any contest they like. Unfortunately, in the Italian case the application has some costs – added to timing costs – due to travel expenses sustained for reaching the universities’ sites. This costs can trigger the

⁷⁹ Data available on the MIUR website.

⁸⁰ Remember that the constraint of “institutional acceptability” is a feature taken from reality and specifically from the real structure of public contests.

⁸¹ As already outlined, the displaying of a truncation strategy depends on the amount of information available and also on the risk profile of an agent. For the clearness and simplicity of the presentation, it is more convenient to assume candidates to be sincere.

⁸² They are presented in Theorem 9 and demonstrated into the Appendix.

candidates to not reveal their preference for some universities because of their geographical distance⁸³, applying more to close institutions, even if they are less liked than the other ones. Second, in the model we assume that candidates have no restriction on applications, such as they can apply to any institution at any round. However, restrictions apply in the real market due to overlapping of oral examinations timings at different universities' site⁸⁴. Since the candidate have to be physically present at the oral examination, and the date cannot be changed once publicly announced, she has to renounce to one of the two job interviews – i.e. withdraw herself from one of the two contests⁸⁵. Last, the key assumption of the DMR model is the one-sided commitment, that leaves candidate free to reject a previous matching and to accept a new one. In the real hiring procedure, the job offers received at the end of the contest cannot be held for a long time and once accepted, it is very hard that someone will leave it another proposal: first because the contract may have some constraints; second for reputational reasons – a candidate that accepts and then withdraws for a better offer would be retained unreliable. Since all the competitions end at different timing – and that it is not possible to know in advance when a contest will end – candidates have no chances to evaluate all the potential offers they could receive from all the universities they applied to. Of course the model does not reproduce the real timing structure where the matchings are not made official all at the same time – i.e. after the last round of the game – but as soon as they end⁸⁶. Assume then, that the real procedure is adjusted such that the contests are finalized all at the same

⁸³ Data available on the MIUR Website shows that for some competitions sometimes there are only three or two candidates – and sometimes even one candidate – while for others the number of candidates is close to 100. Due to the travel expenses, it is not possible to affirm that one university is more preferred than the other, since the applications number can rely more on a cost condition rather than on a preference profile.

⁸⁴ In the model we assume the order of the announcements – and so the hypothetical date of examinations – as randomly given. In the reality, it is not excluded that University can use the timing decisions as a strategy for slimming down the list of candidates or for addressing a specific class of candidates.

⁸⁵ Demonstration in the Appendix at the proof of Theorem 9.

⁸⁶ Imagine that a contest starts during round r and ends at round $r+k$ such that during the rounds between r and $r+k$ the candidate has applied to other institutions and maybe has received some proposal. Even a two-sided commitment will not be suitable for representing this timing structure as long as we assume that the contest lasts only the duration of one round and that the matching occurs at its end. In fact, the strategy of candidates is not more only “apply or not apply” to some institution, but also “accept or reject” a job offer, before knowing if a better propose will arrive.

time⁸⁷, as long as applications are costly and can suffer restrictions due to timing overlapping, the system still shows inefficiencies and unstable outcomes⁸⁸. On the reverse, as demonstrated by Theorem 9, it would be useless to make a costless-applications system where examinations date are all different, without allowing candidates to have the chance of evaluating all the offers.⁸⁹ The analysis of the Italian hiring procedure based on the theoretical model of the DMR mechanism, allow us at first to assert that the market is inefficient on a solid basis, second, to identify the causes of failures and consequently to give advice on future restyling policies. For maintaining the decentralized structure, it would be necessary to define a clear timing for the timing procedure, such as all university can open the job position at the moment they prefer, but they would have to be able to announce the winners all during the same fixed period of time. After all job offers have been sent, candidates will have a specific amount of time for accepting one of them and reject all the others (and so on). Moreover, universities could agree on organizing the job interviews all during a national conference on the Economics field⁹⁰, solving simultaneously the problem of application costs and dates overlapping. The procedure would be more controlled by the central body represented by MIUR, but at the same time universities would keep their degree of autonomy on the hiring procedure, and the evaluation structure would be unchanged.

7. Conclusions

The paper presented the formalization and the analysis of a new decentralized mechanism that works per round. It recalls the real-world functioning of public competitions, but also of hiring procedures in general, and matches Institutions – that open job positions – to Candidates – who apply to job positions. Each round of the mechanism has the same structure – some institutions announce a job

⁸⁷ That is, assume that candidates can evaluate all the offers and one-sided commitment is implemented in the fixed time period of offers.

⁸⁸ See the demonstrations of Theorem 9 in the Appendix.

⁸⁹ Note that, as we are assuming that all candidates are sincere, the final outcome would be Institutional-Optimal.

⁹⁰ Following the example of the International Academic job market for the Economics field. It would be cheaper than reimburse travel expenses to all candidates called for the oral examination.

position, candidates apply, institutions select one candidate and order the others into a waiting list⁹¹ – and ends with temporary assignments that can be broken at some following round by the Candidates side, while Institutions commit to their temporary mates. Moreover, the applications are assumed to be costless and free of any possible restrictions, such as candidates should be incentivized to apply to job they like, as much as they can. On the other hand, Institutions are not allowed to reject all the candidates as long as there is at least one of them who satisfy the mandatory requirements of the job announcement. This restriction of the strategy space of any institution is formalized as “institutional acceptability” – or as the cost of the hiring procedure for Employers – and it is introduced by this paper for the first time. The game is never affected by the order according to which institutions open the job positions, or the amount of institutions that play simultaneously during the same round, and neither by the number of institutions that simultaneously select the same candidate⁹². As in other decentralized one-to-one matching mechanisms (Blum et al. 1997; Haeringer and Wooders 2011; Sotomayor 2003), for the Institutions’ side it is a weakly dominant strategy to truthfully reveal their preferences into the game by picking the candidate they like the most among the ones who sent an application for the job position. In a setting of complete information, for all candidates revealing a truncated list of preferences in the form $P^i = \{j, \dots, \mu^i\}$, the mechanism implements the stable C-Optimal matching in SPE. However, a truncation strategy of this kind requires a lot of information and it could be not feasible or highly risky in real-world applications – especially when the number of candidates is greater than the number of jobs available. On the basis of this considerations, we assumed candidate to be sincere – i.e. truthfully reveal their preferences – and tested the mechanism whose outcome happened to be stable and I-Optimal. In both cases, the results heavily rely on the key assumptions of the game, such as they do not hold anymore under the condition of two-sided commitment, or when costs and restrictions apply on candidatures⁹³. The real

⁹¹ All the steps are played simultaneously by the agents.

⁹² Thanks to a back-and-forth phase where the final decision is remitted to the candidate.

⁹³ It is sufficient that just one of these three assumptions is not respected for having an unstable outcome at the end of the game.

case-study of the hiring procedure for Assistant Professors in Italy, fails all the three assumptions. The model, then, helps to assert the inefficiency of the real-world market, to rapidly identify the causes and to propose the necessary improvements. The paper enriches the literature on decentralized mechanism, and on matching theories and market design in general, and offers a new useful tool for the analysis of real-world markets. However, the formalization of the model suffers of three limitations: first, in real cases the contests or hiring procedures do not end all at the same time, and candidates cannot hold offers for long periods; second, the entrance of Institutions into the market has been taken as randomly given, while it could be a strategy itself; third, strategies are analyzed in a setting of complete information that is far from reality. Then, the aim of future studies is to stress the model in settings of incomplete information that are as similar to reality as possible, and to add the timing features to the model. In fact, at this point it is not possible to determine the role of the information into the mechanism – and so if and how it influences the strategies of players – as well as the effects of timing characteristics – as the timing of job announcement, the duration of single competitions, the differences in end-timing of the contests. This paper establishes the basis for a large number of interesting developments whose results will be a precious enrichment for the whole matching literature.

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Appendix

Structure of the Game

Then, the general decentralized multiple-rounds game G^Y_R is characterized by:

- Two finite sets of agents $I = \{i, \dots, i_n\}$ and $C = \{c, \dots, c_n\}$;
- A preference profile $P = \{P(i_1), \dots, P(i_n), P(c_1), \dots, P(c_n)\}$;
- A message profile $S^R = \{S(i_1), \dots, S(i_n), \dots, S(c_1), \dots, S(c_n)\}$;
- A finite set of rounds $R = \{r, \dots, r+n\}$, each round with the same characteristics;
- A permutation γ of the set I over the finite number of rounds that identifies I^r for each $r \in R$;
- A decentralized proposal – acceptance/rejection matching mechanism of I and C , carried out per round called DMR;
- An outcome i.e. the set of final matchings $M = \{\mu(i), \dots, \mu(i_n)\}$.

Each round is characterized by:

- A set of activity messages $A^r = \{a(i_1), \dots, a(i_n)\}$ pre-defined by γ for each round;
- A sub-set of Institutions that offer a job position $I^r = \{i \in I : a^r(i) = (c \in C \mid c \succ_i i)\}$ for $I^r \subset I$;
- A sub-set of Candidates who apply to the offers $C^r = \{c \in C : s^r = (i \in I^r \mid i \succ_c c)\}$ for $C^r \subseteq C$;
- A set of messages $S^r = \{s^r(i_1), \dots, s^r(i_n), s^r(c_1), \dots, s^r(c_n)\}$;
- A sub-set of Institutions that have a passive status $I^P \subseteq I$;
- A finite number of Institutions that have an inactive status, $i \in I \wedge i \notin I^r \wedge i \notin I^P$;
- A decentralized proposal – acceptance/rejection mechanism carried out by I^r and C^r ;
- An outcome as a set of temporary matchings $M^t_r = \{\mu^t(i), \dots, \mu^t(i_n)\}$ for any $i \in I^r$.⁹⁴

⁹⁴ If no one candidate sends a proposal to some $i \in I^r$, then it is considered temporary assigned to itself until the round when it will have the chance of re-offer the job position.

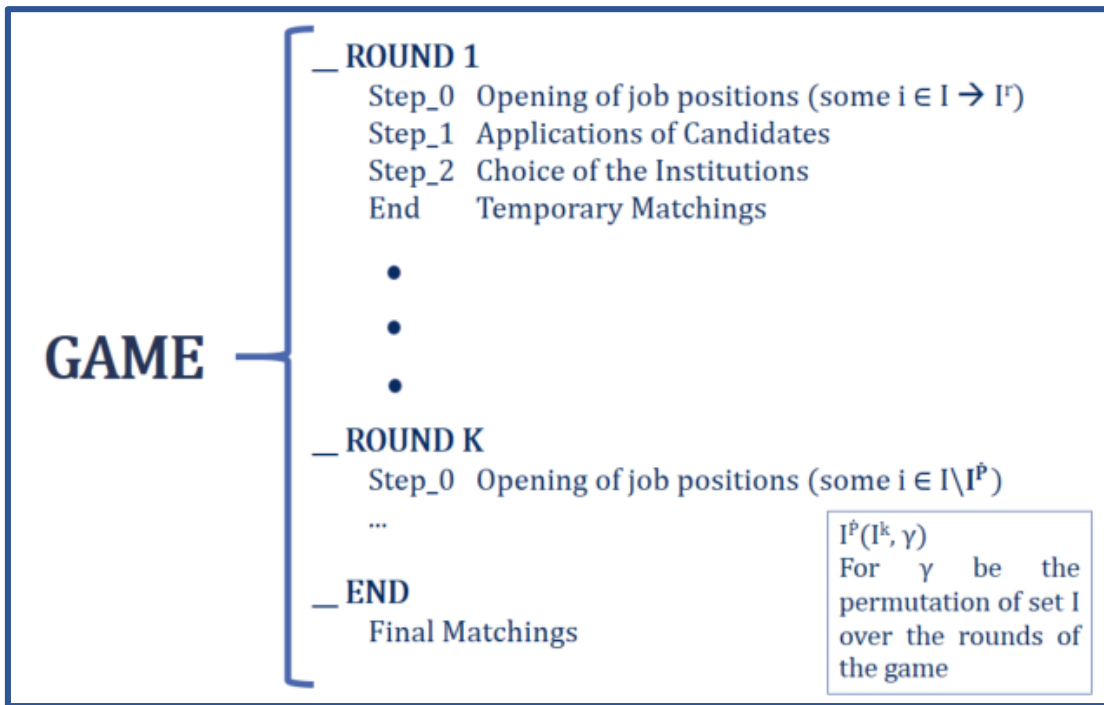


Figure 3. Representation of the DMR mechanism

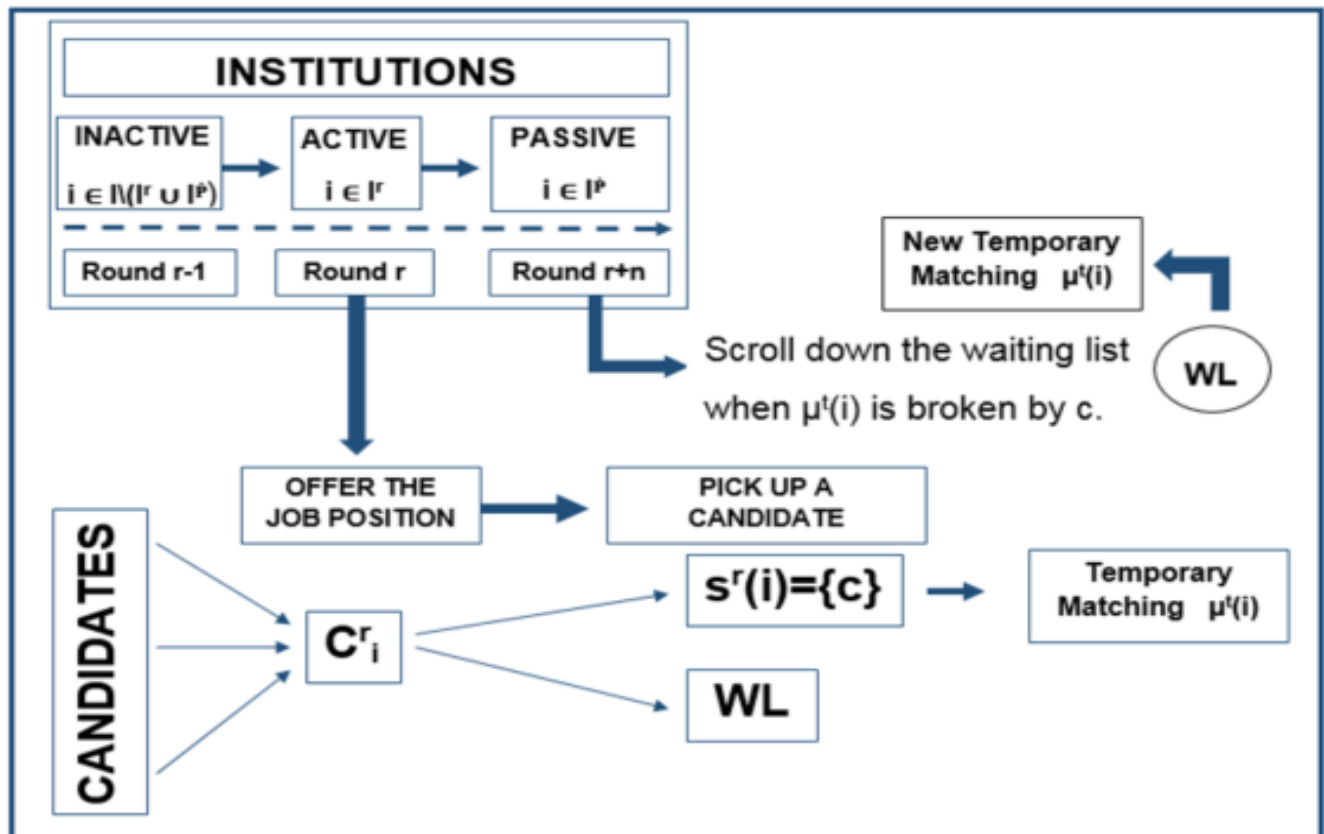


Figure 4. Scheme of the DMR mechanism

Proof 1 – Proposition 1

Let be $I=\{i, i'\}$ the set of institutions and $C=\{c, c'\}$ the set of candidates. Let P be the preference profile, and let be respectively $P(i)=\{c, c'\}$, $P(i')=\{c', c\}$, $PI=\{i', i\}$ and $P(c')=\{i, i'\}$ the list of preferences of each agent. Since there are no limits on applications, if $\gamma=\{i, i'\}$ or $\gamma'=\{i', i\}$ there is actually no difference for candidates. For γ , at round 1 institution i receives the applications of candidates c and c' , and $\mu^t_1=(i, c)$. At round 2, institution i' receives the applications of candidates c and c' and the final matching is $M=\{(i, c'), (i', c)\}$. For γ' , institution i' receives the applications of c and c' , and $\mu^t_1(i')=c$. At round 2, institution i receives the application of c' , and the final matching is $M=\{(i, c'), (i', c)\}$. Then, the final outcome of the game never depends on the permutation γ of the set I over the entire game.

- Back and Forth Phase

Assume that $i \in I^r$ and $i' \in I^{r+1}$, that $PI = \{i, i'\}$ for $c \in C$ and $P(i) = P(i') = \{c, c'\}$; at round r , $c \in C^r_i$ and $\mu^t_r(i)=c$; at round $r+1$, since $i \succ_c i'$ then $c \notin C^{r+1}_{i'}$ and so, $\mu(i')=c'$ and $\mu(i)=c$. Assume that $i' \in I^r$ and $i \in I^{r+1}$, that $PI = \{i, i'\}$ and $P(i) = P(i') = \{c, c'\}$; at round r , $c \in C^r_{i'}$ and $\mu^t_r(i')=c$; at round $r+1$, since $i \succ_c i'$ then $c \in C^{r+1}_i$ and so, $\mu(i)=c$ while i' scroll the waiting list and then $\mu(i')=c'$. Now, let $i, i' \in I^r$ and let be $PI = \{i, i'\}$ and $P(i) = P(i') = \{c, c'\}$. At round r , $c \in C^r_i$ and $c \in C^r_{i'}$ such that $s^r(i) = s^r(i') = c$; at this step, since $i \succ_c i'$ then $\mu^t_r=i$ and so $\mu(i')=c'$.

Proof 2 – Proposition 2

Assume $s^r(i)=\{\emptyset\}$ is acceptable⁹⁵. Let be $I=\{i, i'\}$ the set of institutions and $C=\{c, c'\}$ the set of candidates. Let P be the preference profile, and let be respectively $P(i)=\{c, c'\}$, $P(i')=\{c', c\}$, $PI=\{i', i\}$ and $P(c')=\{i, i'\}$ the list of preferences of each agent. At the first round r , let i receive the applications by $C^r_i = \{c'\}$ and let be $S^r(i)=\{(s^r(i)^1=\{\emptyset\}), (s^r(i)^2=\{c'\})\}$ the set of all possible strategy profiles of institution i at round r . Let i follow the strategy $s^r(i)^1=\{\emptyset\}$ such that $\mu^t_r(i)=i$. At the next round, let

⁹⁵ Assume that the institutional acceptability constraint has been removed.

i' receive the applications by $C^{r+1}_{i'} = \{c\}$ and let be $S^{r+1}(i') = \{(s^{r+1}(i')^1 = \{\emptyset\}), (s^{r+1}(i')^2 = \{c\})\}$ the set of all possible strategy profiles of institution i' at round $r+1$. Let i' send the message $s^{r+1}(i') = \{\emptyset\}$ and let $\mu^{t_{r+1}}(i') = i'$ be its temporary matching. At the following round $r+2$, both institutions re-offer their positions, and due to the rejections received let c apply to i , and c' apply to i' . Since both the institutions received their preferred candidates, they both accept them and the final matchings are $\mu_{r+2}(i) = c$ and $\mu_{r+2}(i') = c'$. Note that the result is stable and Institutions-Optimal. Now consider $s^r(i) = \{\emptyset\}$ unacceptable as imposed by the institutional acceptability condition. At the first round r , the set of possible strategies is now $S^r(i) = \{(s^r(i) = \{c'\})\}$, such that the institution i has only one strategy instead of two. The same happens for i' at the following round $r+1$.

Proof 3 – Theorem 1.

Let M be the set of stable matchings. If $s^*(i)$ is not a dominant strategy for i , then there is at least another $s'(i)$, for $s'(i) \neq s^*(i)$ and $s^*(i), s'(i) \in S(i)$, for which i is better off, for some strategies followed by all the other players. It means that $(s'(i), s_{-i}) \geq (s^*(i), s_{-i})$. Let i receive the applications by $C^r_i = \{c, c'\}$ for $c \succ_i c'$, and let i follow the strategy $s'(i) = \{c'\}$ such that $\mu^t_r(i) = c'$. At the next round, let i' receive the applications by $C^{r+1}_{i'} = \{c, c'\}$ and let be $s^{r+1}(i') = \{c\}$ its strategy, for $c \succ_{i'} c'$, and $\mu^{t_{r+1}}(i') = c$ its temporary matching. Assume that $i \succ_c i'$, such that (i', c) would form a blocking pair, then the matching would no longer be stable since i have no chances to re-offer its job position to c . This is a contradiction of the statement that the final matching M is stable. If i would had followed $s^*(i) = c$ no blocking pair would have been formed and the matching would be stable. Moreover, following $s^*(i)$ the institution i can obtain its first preference, while the strategy $s'(i)$ causes to be paired with its second choice. Assume, instead, that $i' \succ_c i$ and so (i', c) is not blocking pair. At this point the final matching is represented by the couples (i, c') and (i', c) , and the result would be the same even under $s^*(i)$. In fact, at the second round c would have broken the previous matching with i for accepting the offer by i' , and i would have come up with c' by scrolling the waiting list. Let i' follow the strategy $s'(i') = \{c'\}$,

and let $\mu_{r+1}^t(i')=c'$ be its temporary matching. For i following $s'(i)$ then the temporary matching $\mu_r^t(i)=c'$ is now broken, and i scrolls down the WL and forms the new matching $\mu_{r+1}^t(i)=c$ with its first preference. If i would have followed the strategy $s^*(i)$, then i would have obtained the same result directly at the first round, saving time. This means that $s^*(i)$ respect to $s'(i)$ offers to i more chances to obtain its most preferred candidate that will be lost if and only if she will be accepted by some other institutions she likes more. Then, $(s^*(i), s_{-i}) \geq (s'(i), s_{-i})$ such that $s^*(i)$ is a weakly dominant strategy for any $i \in I$, at any $r \in R$ and for any γ , into the assignment mechanism induced by the game G^Y_R .

Proof 4 – Proposition 4.

Let M be the set of stable matchings. Let $I=\{i, i'\}$ and $C=\{c, c'\}$. Let candidate c be temporary matched after round r , such as $\mu_r^t(i)=i$, and let $PI=\{i', i\}$ be her preference list. Assume that $s^*(\mu\text{-candidates})$ is not a best response for c , such that there is a strategy $s'_{r+1}l$ such that $(s'_{r+1}l, s_{-c}) \geq (s^*(\mu\text{-cand}), s_{-c})$, given s_{-c} . At round $r+1$, let the candidate play $s'_{r+1}l=\{c\}$ such that $\mu_r^t(i)=i$ even if $i' \succ_c i$. Assume that $P(i')=\{c, c'\}$ and since $c \notin C^{r+1}_{i'}$ the final couples are (i, c) and (i', c') , but then (i', c) is a blocking pair and this contradicts the initial assessment that the final matching is stable. For candidate c playing $s^*(\mu\text{-cand})$ instead, the result would be (i, c') and (i', c) . It is stable, and moreover, c is better off respect to the outcome obtained under $s'_{r+1}l$. Assume that $P(i')=\{c', c\}$, then the outcome would be the same under both strategies. Let $PI=\{i, i'\}$ and let $\mu_r^t(i)=i$ be the temporary matching of c . At the next round $r+1$, let the candidate play $s'_{r+1}l=\{i'\}$ such that $\mu_{r+1}^t(i)=i'$ for $P(i')=\{c, c'\}$. Since $i \succ_c i'$ the candidate c is worse off under the strategy $s'_{r+1}l$. On the contrary, under the strategy $s^*(\mu\text{-cand})$ the candidate would have avoided to send an application to the institution i' at round $r+1$, holding the better job $\mu_r^t(i)=i$. Assume that $P(i')=\{c', c\}$, then the outcome would be the same under both strategies. Then the strategy $s^*(\mu\text{-cand})$ weakly dominates the strategy $s'_{r+1}l$ for any other strategy played by the other agents, i.e. $(s^*(\mu\text{-cand}), s_{-c}) \geq (s'_{r+1}l, s_{-c})$.

Proof 5 - Theorem 3

Let M be the set of stable matchings. If S^{RI} is not a best response for c , then there is at least another S^I , for $S^I \neq S^{RI}$ for which c is better off, such that $(S^I, s_c) \geq (S^{RI}, s_c)$, for S^{RI} and S^I being the strategy of c for the entire game. Assume that $I = \{i, i'\}$ and $C = \{c, c'\}$ and let $P(i) = \{c, c'\}$, $P(i') = \{c', c\}$, $P_I = \{i', i\}$ and $P(c')$ be their preferences' profiles. Let all $i \in I$ play their dominant strategy and let all $c \in C$ play $S^{RI} = \{i \in P_I\}$. Then,

1. $I^r = \{i\}$ $C^r_i = \{c, c'\}$ and $\mu^t(i) = c$
2. $I^{r+1} = \{i'\}$ $C^{r+1}_{i'} = \{c, c'\}$ and $\mu^t(i') = c'$
3. Final matching $M = \{(i, c); (i', c')\}$ i.e. I -Optimal

Let all $i \in I$ play their dominant strategy and let c deviates from $S^{RI} = \{i \in P_I\}$ in the form $S^I = \{i \in P^I\}$ for $P^I = \{i'\}$.

1. $I^r = \{i\}$ $C^r_i = \{c'\}$ and $\mu^t(i) = c'$
2. $I^{r+1} = \{i'\}$ $C^{r+1}_{i'} = \{c\}$ and $\mu^t(i') = c$
3. Final matching $M = \{(i, c'); (i', c)\}$ i.e. C -Optimal

Candidate c is better off from not applying to i even if it is considered acceptable, such that $(S^I, s_c) \geq (S^{RI}, s_c)$, then she has an incentive to deviate from $S^{RI} = \{i \in P_I\}$ for all $i \in I$ playing their dominant strategy, and for $c' \in C$ playing $S^R(c') = \{i \in P(c')\}$. Note that c' is also better off thanks to c deviation.

Proof 6 – Theorem 4

Let M be the set of stable matchings. If Theorem 4 is not true, then there is at least one non-truncation strategy $S_I = \{i \in I \mid i \in P_I\}$ that dominates a truncation strategy $S^*I = \{i \in I \mid i \in P^*I\}$, for $P^*I \subseteq P_I$, such that $(S_I, S_c) \geq (S^*I, S_c)$, for any $i \in I$ playing its dominant strategy.

Assume that $I = \{i, i'\}$ and $C = \{c, c'\}$, and let $P(i) = \{c, c'\}$, $P(i') = \{c', c\}$, $P_I = \{i', i\}$, $P(c') = \{i, i'\}$ be their preferences' profiles. Let all Institutions play their dominant strategy and let all any $c \in C$ play S^*I , such that $P^*I = \{i'\}$ and $P^*(c') = \{i\}$, then:

Round 1. $I^r = \{ i \}$ and $C^r_i = \{ c' \}$ and $\mu^t(i) = c'$

Round 2. $I^{r+1} = \{ i' \}$ and $C^r_{i'} = \{ c \}$ and $\mu^t(i') = c$

This outcome is $\mathbf{M} = \{(i, c'), (i', c)\}$ and it is C-optimal, i.e. any candidate obtained the best outcome she could ever obtained into this game. Now assume that c' keeps playing S^*I while c plays SI :

Round 1. $I^r = \{ i \}$ and $C^r_i = \{ c, c' \}$ and $\mu^t(i)_1 = c$ and $WL(i) = \{ c' \}$

Round 2. $I^{r+1} = \{ i' \}$ and $C^r_{i'} = \{ c \}$ and $\mu^t(i')_2 = c$ then $\mu^t(i)_2 = c'$

The outcome is $\mathbf{M} = \{(i, c'), (i', c)\}$. Even if the candidate c is truthfully revealing her preferences, candidate c' is still better off by reporting only a truncation of her list. This means that given the institutions that play their dominant strategy, any $c \in C$ who plays S^*I has no incentives to deviate to another strategy SI , for the other applicants following the same strategy or a weakly dominated strategy. In fact, she is not worse off respect to any other SI , and she is good as well as under any other strategy that makes her obtaining the best outcome possible, such that $(S^*I, S_{-c}) \geq (SI, S_{-c})$, given the institutions that play their dominant strategy.

Proof 7 – Theorem 5

Let M be the set of stable matchings. Let $I = \{i, i'\}$ and $C = \{c, c'\}$. Let candidate c be temporary matched after round r , such as $\mu^t_r(i) = c$, and let $PI = \{i', i\}$ be her preference list. Let candidate c' be temporary matched after round r , such as $\mu^t_r(i') = c'$, and let $PI = \{i', i\}$ be her preference list. Assume that $s^*(\mu\text{-candidates})$ is not a best response strategy for any $c' \in C$ revealing a truncation of true preferences, such that there is a strategy $s'_{r+1}|I$ for $(s'_{r+1}|I, s_{-c}) \geq (s^*(\mu\text{-cand}), s_{-c})$, given s_{-c} . Let then be $P^*(c') = \{i\}$, and $P(i) = \{c, c'\}$, such that after round r $\mu^t_r(i) = c$ for c revealing true preferences. At round $r+1$, let the candidate play $s'_{r+1}|I = \{c\}$ such that $\mu^t_{r+1}(i) = c$ even if $i' \succ_c i$. Assume

that $P(i')=\{c, c'\}$ and since $c \notin C^{r+1}_{i'}$ the final couples are (i, c) and (i', c') , but then (i', c) is a blocking pair and this contradicts the initial assessment that the final matching is stable. For candidate c playing $s^*(\mu\text{-cand})$ instead, the result would be (i, c') and (i', c) . It is stable, and moreover, c is better off respect to the outcome obtained under s'_{r+1} . Assume that $P_i=P(c')=\{i, i'\}$ for $\gamma=\{i', i\}$, $P(i)=\{c, c'\}$ and $P(i')=\{c', c\}$. Assume then that any candidate plays a truncation strategy $s^*I=\{i, \dots, \mu^C I\}$ such that $P^*I=\{i\}$ and $P^*(c')=\{i, i'\}$. After round r candidate c' is temporary matched $\mu^t_r(c')=i'$; at round $r+1$ $s^{r+1}(c')=\{i\}$, even though the final outcome is $M=\{(i, c), (i', c')\}$. But assume that for some reason candidate c not apply to institution i , then candidate c' would have obtained her first preference. Then, under an implementing strategy for all the other candidates playing a weakly dominated – demonstrated in proof 4 – or a non-dominated strategy, a temporary matching is not worse off respect to other strategies and she is as well as under a strategy that allow her to obtain the same result.

Proof 8 – Theorem 7 – Stability

For all the institutions playing their dominant strategy and for all candidates playing their best response, the final outcome is stable. All matchings are individually rational since any $i \in I$ accepts only candidates who respect the condition of institutional acceptability – i.e. they all belongs to $P(i)$; any $c \in C$ sends applications to any $i \in P^*I$ for $P^*I=\{i, \dots, \mu^C I\}$. The matching is not blocked by pairs, in fact if institution i did not receive the application by candidate c then $i \notin P^*I$; on the other hand, if c applies to i and $s^r(i) \neq \{c\}$ then there is another $c' \neq c$ for $c' \in C^r_i$ such that $c' \succ_i c$, for $c, c' \in P(i)$.

- **Candidates-Optimal**

As long as all candidates reveal only a truncation of their preferences, the choice of the institutions is reduced to that ones who likes it the most – instead of all the one who consider it acceptable. Mostly, P^*I excludes all the potential stable mates of c but the optimal one represented by $\mu^C I$. Then, since the outcome of the mechanism has been proof to be stable, the only outcome possible for each candidate c is $\mu^C I$, and the final matching is Candidates-Optimal. Assume that $I=\{i, i'\}$ and $C=\{c, c'\}$, and let be $P(i)=\{c, c'\}$, $P(i')=\{c', c\}$, $PI=\{i', i\}$, $P(c')=\{i, i'\}$ their preferences' profiles.

Proof 9 – Theorem 8 – Stability

All matchings are individually rational since any $i \in I$ accepts only candidates who respect the condition of institutional acceptability – i.e. they all belongs to $P(i)$; any $c \in C$ sends applications to any $i \in PI$. The matching is not blocked by pairs, in fact if institution i did not receive the application by candidate c then $i \notin PI$; on the other hand, if $s^I=\{i\}$ but $s^I(i) \neq \{c\}$ then there is another $c' \neq c$ for $c' \in C^I_i$ such that $c' \succ_i c$, for $c, c' \in P(i)$.

– Institutions-Optimal

As long as all candidates truthfully reveal their preferences, then Institutions can choose among all the candidates it is acceptable for, and by playing its dominant strategy it will pick up the most preferred. So, it is automatically selecting its optimal matching. Any Institution has the same conditions, such as each of them will be coupled with the mate they can have in a stable matching.

Proof 10 – Prop. 8/Prop. 9/Theorem 9

- Two-Sided vs. One-Sided Commitment

Let be $I=\{i, i'\}$ the set of institutions and $C=\{c, c'\}$ the set of candidates. Let P be the preference profile, and let be respectively $P(i)=\{c, c'\}$, $P(i')=\{c, c'\}$, $PI=\{i', i\}$ and $P(c')=\{i, i'\}$ the list of preferences of each agent. Assume that all candidates follow a TRS strategy and all institutions play their dominant strategy. At the first round

both c and c' send an application to institution i whose message will be $s^r(i)=\{c\}$, and $\mu(i)=c$. Under a two-sided commitment assumption, as the matching is realized both agents are removed from the market. Then at round $r+1$ the only possible assignment is $\mu(i')=\{c'\}$. It is easy to see that the outcome is blocked by the pair (i', c) .

- **Costs Restrictions on Applications**

Let be $I=\{i, i'\}$ the set of institutions and $C=\{c, c'\}$ the set of candidates. Let P be the preference profile, and let be respectively $P(i)=\{c, c'\}$, $P(i')=\{c', c\}$, $P_I=\{i', i\}$ and $P(c')=\{i, i'\}$ the list of preferences of each agent.

Assume that applications are costly, such as any candidate has the resources for apply only a limited number of times. In this demonstration, let set the maximum number of applications per candidate to one. Since we are in the setting of “truthfully revealing strategy” for all agents, suppose that candidates will apply to the first announced job they like. At the first round both c and c' send an application to institution i whose message will be $s^r(i)=\{c\}$, and $\mu(i)=c$. Since both candidates applied at round r , no one of them have more resources to apply at round $r+1$. The outcome is unstable due to unassignment and blocking pairs. Then, the “no-costs” on applications condition is necessary for guaranteeing the stability, and the one-sided commitment condition alone is not sufficient.

Assume that applications are costly, such as some candidate has the resources for apply only a limited number of times. In this demonstration, let suppose that candidate c can apply two times and c' one time. Since we are in the setting of “truthfully revealing strategy” for all agents, suppose that candidates will apply to the first announced job they like.

- ❖ Under Two-Sided Commitment: At the first round both c and c' send an application to institution i whose message will be $s^r(i)=\{c\}$, and $\mu(i)=c$. The couple (i, c) is removed from the market and c' has no more resources for a second round. The outcome is unstable due to unassignment and blocking pairs.

If we assume the reverse, such that candidate c has resources only for one round but candidate c' for two rounds, the final matching is made up by the couples (i, c) and (i', c') and is blocked by the pair (i', c) .

- ❖ Under One-Sided Commitment: At the first round both c and c' send an application to institution i whose message will be $s^r(i)=\{c\}$, and $\mu(i)=c$. At round $r+1$ candidate c apply to institution i' and get the job; institution i scrolls the WL and choose c' . The final outcome is stable.

If we assume the reverse, such that candidate c has resources only for one round but candidate c' for two rounds, the final matching is made up by the couples (i, c) and (i', c') and is blocked by the pair (i', c) .

Assume applications are costless:

- ❖ Under Two-Sided Commitment: At the first round both c and c' send an application to institution i whose message will be $s^r(i)=\{c\}$, and $\mu(i)=c$. The couple (i, c) is removed from the market and c' applies to i' at round $r+1$. The final matching is made up by the couples (i, c) and (i', c') and is blocked by the pair (i', c) .

Then, the “no-costs” on applications condition is not sufficient for guaranteeing the stability.

- ❖ Under One-Sided Commitment: At the first round both c and c' send an application to institution i whose message will be $s^r(i)=\{c\}$, and $\mu(i)=c$. At round $r+1$ candidate c apply to institution i' and get the job; institution i scrolls the WL and choose c' . The final outcome is stable.

- **Per Round Restrictions on Applications**

Assume that candidates can apply only to one institution per round for $I^r > 1$ at any $r \in R$.

Let be $I=\{i, i', i''\}$ the set of institutions and $C=\{c, c', c''\}$ the set of candidates. Let P be the preference profile, and let be respectively $P(i)=\{c, c', c''\}$, $P(i')=\{c', c'', c\}$, $P(i'')=\{c'', c, c'\}$, $P_i=\{i', i, i''\}$, $P(c')=\{i', i, i''\}$ and $P(c'')=\{i, i', i''\}$ the list of preferences of each agent. Suppose that all candidates truthfully reveal their preferences, such as under a constraint on the number of applications per round, they reveal the preference profile by applying to the institution they like the most. Let be $\gamma=\{(i, i'), i''\}$. At round r , $I^r = \{i, i'\}$ and $s^r I = s^r(c') \equiv \{i'\}$ while $s^r(c'') = \{i\}$, then the temporary couples are (i, c'') and (i', c') . At round $r+1$, $I^{r+1}=\{i''\}$ and only candidate c applies such that the set of final matchings is $M=\{(i, c''), (i', c'), (i'', c)\}$. The outcome is blocked by the pair (i, c) . At round r since c' has not the chance to apply to both institutions, she selected the one she liked the most accordingly to her preference list. Unfortunately, in the meanwhile her second choice fulfilled the position, even though this institution would have preferred her to its temporary assignment.

The “no-restrictions” on applications is a necessary condition to guarantee the stability of the outcome.

Now, assume that there are no per round restrictions on the applications but that also candidates are called to commit to temporary matchings.

Let be $I=\{i, i', i''\}$ the set of institutions and $C=\{c, c', c''\}$ the set of candidates. Let P be the preference profile, and let be respectively $P(i)=\{c, c', c''\}$, $P(i')=\{c'', c, c'\}$, $P(i'')=\{c'', c, c'\}$, $P_i=\{i', i, i''\}$, $P(c')=\{i', i, i''\}$ and $P(c'')=\{i, i'', i'\}$ the list of preferences of each agent. Suppose that all candidates truthfully reveal their preferences. Let be $\gamma=\{(i, i'), i''\}$. At round r , $I^r = \{i, i'\}$ and $C_i^r = C_{i'}^r \equiv \{c, c', c''\}$, then $s^r(i)=\{c\}$ and $s^r(i')=\{c''\}$, such that (i, c) and (i', c'') are the couples realized and removed from the market. At round $r+1$, $I^{r+1}=\{i''\}$ and only c' can apply such that the set of final matchings is $M=\{(i, c), (i', c''), (i'', c')\}$, and it is blocked by the pair (i'', c'') ⁹⁶.

⁹⁶ It is clear that the outcome would be stable if candidate c'' would be “patient enough” (Diamantoudi et al. 2015) to wait for her best mate, but it would actually mean that candidate c'' plays a truncation strategy (since she does not apply to the acceptable institutions at the first round) while we assumed a truth-telling behavior for all the candidates.

The “no-restrictions” on applications is not a sufficient condition to guarantee the stability of the outcome.

Note that, when the three conditions are not respected simultaneously, the mechanism is highly sensible to the permutation of the set I over the game – i.e. γ – and to the structure of the Preferences’ Profiles of the agents.

2.4 The Decentralized Matching Mechanism: Real System and Theoretical Model

The paper offered a theoretical formalization of the real characteristics of the market presented in the previous paragraphs. However, it could be difficult to recognize some real features into the model, or to understand the reason why the

final version of the model has that structure. In fact, the paper could not report all the steps of the research path. It only shows the starter point – i.e. the information on the current recruitment system – and the last step – i.e. the final version of the model. Even though, there were a lot of adjustments to the theoretic formalization before arriving at that last step. This paragraph will reveal how the inter-steps worked, in order to explain 1) how real-world features have been translated into modelled features; and 2) why the last version of the model is not the exact reproduction of the market, but the exact tool of analysis.

2.4.1 From the Real-World Market to the First Version of the Theoretical Model

The aim of this first sub-paragraph is to illustrate how the theoretical model has been built up starting from the information gathered on the real market.

At first, the general information about the system have been used for determining the kind of matching market in terms of structure (de/centralized), of model (one/two-sided matching) and assignment (one-to-one/many).

The recruitment procedure is also characterized by features never modelled before for other applications, such that they have been formalized in this model for the first time. It is important to underline that they are specific of the recruitment procedure – a public contest system – and not only of the Italian market. This means that these new introductions – as the “Institutional Acceptability” property – are not peculiar of the Italian case, but of the public contests system. So, their utility is not confined and this sole model but they are able to be implemented in other projects for markets that relies on a similar procedure.

Real-World Procedure (general)	Theoretic Model
It involves Institutions and Candidates	Two-sided matching market, two finite set of agents $I = \{i^1, \dots, i^n\}$ and $C = \{c^1, \dots, c^n\}$
Candidates apply to Institutions; Institutions select Candidates	Any agent has strict, complete, transitive preferences over the agents on the other side, $P = \{P_{i^1}, \dots, P_{i^n}, P_{c^1}, \dots, P_{c^n}\}$
Each Institution selects only one winner per contest, and no candidate can accept two job positions	One-to-one Assignment, $\mu = (i, c)$
Each Institution carries on the hiring procedure autonomously (aligned to Law 240/2010)	Decentralized Mechanism \rightarrow Non-revelation dynamic or sequential game
Any job announcement contains mandatory and non-mandatory requirements	<i>Institutional Acceptability</i>
Institutions open one job positions per scientific sector per year	Institutions play only one time during the game

The second step was to report the characteristics of the recruitment system into the model, that, at this point was just modelled from a structural point of view. This new information shapes the theoretical model making it tailored for a more specific kind of market. Indeed, the result is a game whose main features are the series of rounds that constitutes that entire game, and the permutation of the set of Institutions over these rounds – and together combined they represent an innovation for the literature. The first draft of the model was a very close reproduction of the real market. This entails that the restrictions on candidates' applications as costs and the impossibility of participating to interviews held the same day, were integrated into the model. Moreover, the first draft was tested also under the original condition of two-sided commitment, for reproducing the real conditions of the market – that actually do not allow to candidate to compare different offers and choose the one they like the most, neither they can withdraw from a previous job offer. To not complicate furtherly the model, the contests are assumed to select a winner at the same round of job opening. This structure do not represent the overcrossing of opening/closing of competitions or the waiting times that have been reported during the real-market analysis. However, it does not affect the validity of the model whose aim is actually to test the efficiency of the matching mechanism – i.e. the recruitment procedure – and not of the system built around it.

Real-World Procedure (general)	Theoretic Model
Candidates can send multiple applications	Candidates play multiple times / Candidates express their preferences through the applications
Institutions open job positions at different timings during the year, and there is an application deadline	The game is build up by <i>Multiple Rounds</i> , at each round some institution plays and any institution plays only one round.
The oral examination of Candidates is held exclusively at the Institution site	Expensive for candidates → could limit the applications → manipulation of preferences
Sometimes the oral examination dates at different institutions overlap	Candidates can apply to one institution per round → restriction of choices for candidates
Hiring procedures end at different timing by Institution	No chance of comparing potential job proposals → two-sided commitment

Once determined the game functioning, the hiring procedure carried on by each Institution was formulated. It actually represents the round functioning and it is the core of the model, since it defines the specific features of the assignment mechanism.

Real-World Procedure (specific)	Theoretic Model
The Institution open a job position	The institution $i \in I$ enter the market at round r , such that $i \in I^r$
Candidate choose if sending an application or not The group of applicants is defined for the institution	Any $c \in C$ send a message $s(c) = \{i\}$ or $s(c) = \{c\}$ $C_i^r = \{c \in C : i \in s(c)\}$
The institution picks a candidate and send a job proposal, and order the others into a waiting list	Any $i \in I^r$ send a message $s(i) = \{c c \in C_i^r\}$ $WL_i = \{C_i^r \setminus c\} P_i$
The matching is realized	$\mu = (i, c)$

Once the first draft was completed, the strategies' analysis and the stability investigation started.

2.4.2 From the first draft to the final version of the Model

The tests were focused on the stability properties of the matching mechanism, under different strategies of the players.

The first focus point was the two-sided commitment of agents. This feature makes the model more similar to other matching mechanisms, and the results were quite close to the ones presented by M. Sotomayor (2003). In a complete information setting, candidates are able to know their best mate under a stable result and their best response was to apply only to this specific institution⁹⁷. In this case, it is sufficient the deviation of one applicant for ending with an unstable result. The stability results were heavily based on the information setting. Indeed, under the assumption that candidates play truthfully by revealing their real preferences⁹⁸ – instead of applying only to a determined institution – the result was unstable due to the presence of blocking pairs. Beside the general candidates' strategy of addressing just the optimal institution, it was possible to define a characterization of different strategies by kind of applicants. For example, the existence of blocking pairs was influenced by the permutation of Institutions over the game – i.e. the order by which Institutions open job positions. The finding was not a novelty in the literature (Pais 2008), and the following (adapted)⁹⁹ proposition formulated by Sotomayor (Sotomayor 2003) perfectly worked also on the model: “Let $c \in C$. Let γ be such that Institutions are ordered in accordance to c 's preference. Let PI be the set of Institutions that are truly acceptable to c . Then $SI = \{PI\}$ is a dominant strategy in the game induced by G_R^γ .” Moreover, a key role was played by the agents involved in “priority matchings”, i.e. agents who are the each other first preference¹⁰⁰. In some very specific cases, it is sufficient that these couples are not blocked for ensuring the stability of the result. In these cases, for any candidate part of a priority matching playing her dominant strategy – i.e. to apply only to her first preference – even if all the others play a truth-telling strategy the final result is

⁹⁷ A very close result was presented by Sotomayor (2003).

⁹⁸ As if they do not know their best stable mate, or if they decide to not play the game for other market reasons.

⁹⁹ The original proposition formulated for the model presented by Sotomayor (2003) involves men and women.

¹⁰⁰ E.g. $P_c = \{i, \dots, i^n\}$ and $P_i = \{c, \dots, c^n\}$, then (i, c) is a priority matching.

able to be stable. However, it happens for a narrow class of cases, and candidates can never achieve their best outcome unless the core of the game is a singleton. Moreover, it is important to highlight that the two-sided commitment change also the analysis of the game. Indeed, in the paper we have seen that the strategies of players are considered simultaneously at round and game level¹⁰¹. For the two-sided condition, a deeply analysis on strategies at round and game level was carried out. In this case, it was interesting to make a differentiation, since candidates play rounds only till the couple is formed, and pairs are built up at different rounds but, at the end, they are evaluated together for checking the stability of the game outcome. The result of the analysis showed that the stability of each round was a necessary but not sufficient condition for obtaining the stability of the entire game. Once concluded the study of the model under this condition, we tested the functioning the mechanism in the case of one-sided commitment. Thanks to the stability properties achieved under this new condition – that made the model easier to be studied and mostly to be analyzed also in different information settings – we choose to include it in the model by assumption¹⁰².

Once determined the one-sided commitment condition, we tested the model for the other critical features: the costs of applications and the restrictions on the number of contest candidates can participate to due to the overlapping of interviews' appointments. This last element was introduced by imposing to applicants the chance of responding to only one vacancy per round¹⁰³. The restriction was able to cause the formation of blocking pairs when candidates revealed their true preferences¹⁰⁴. Then, it nullified the goal obtained by the introduction of the one-sided commitment. For this reason, it was removed and

¹⁰¹ Under the one-sided commitment condition, candidates are able to play multiple rounds, such that they can adjust strategies over the game. Most of all, the outcome of the game is stable either if candidates follow the equilibrium strategies or if they reveal their true preferences.

¹⁰² The aim of this paragraph is to make clear that we perfectly know how the model works under the original conditions that reflect the real market, and that we choose to remove it for specific purposes.

¹⁰³ In this case, the strategy of each candidate is to apply for the Institution they like the most among the ones that are opening a job position, for the Institutions that belong to P_c or P_c^* - on the basis of the strategy they are following.

¹⁰⁴ Notice that the restriction has no consequences when candidates play the best response (truncation) strategy, but since this kind of strategy is hard to be displayed in the real-world, the attention was focused on reaching a stable outcome under a truth-telling strategy.

substituted by the assumption of having no restrictions on applications. The same procedure was followed for the costs on applications.

Then, the three key assumptions have been identified with a sort of backward induction system. The first model realized was as close as possible to the real hiring procedure. Then, I identified and removed step by step the conditions that impeded the system to achieve a stable outcome. This is how the final version of the model was obtained, and the reason why it does not exactly reflect the current system, but it is the perfect tool for checking the functioning of its matching mechanism. Moreover, the model acquired a more general form that makes it implementable on a variety of markets instead of being useful just for this particular project. The generalizability of a model represents its strength out the boundaries of the specific aim it was built for, and so the final formalization was highly more useful and valuable than all the previous drafts. But the aim of this paragraph was to highlight also the importance of the research path that brought the first draft to acquire its final version, since it includes the information fundamental for deeply understanding the functioning of the mechanism as presented into the paper¹⁰⁵.

2.5 The Importance of Information Availability into the Procedure

The first paper presented the model of the decentralized multiple-rounds matching mechanism analyzed into a complete information setting. The findings allow to sustain the efficiency of the mechanism but the inefficiency of some aspects of the Italian recruitment procedure. Unfortunately, the complete information setting is an unreal condition able to work only in theoretical studies. The model where agents have all the information is then considered a *benchmark model*, to be compared with the same model treated under different information settings. This is the aim and the core of the next paper. The theoretical model developed in the first paper is transported into more realistic scenarios where the information is incomplete or almost absent. The primary interest in this project was to understand how the model works under real-world conditions, that usually includes a degree of

¹⁰⁵ It is mostly important in case of new implementation of the model or the introduction of new features.

uncertainty and complexity reduced in theoretical analysis. Once achieved the results of the benchmark analysis, we stressed the model for the degree of uncertainty – while the degree of complexity is the focus of future studies – by reducing the amount of information available for each agent into the game. The information is one of the key element of any game. The aim was to discover if and how the lack of information affects the functioning of the assignment, but also to obtain a model as close as possible to reality. This desire made us go over the literature boundaries, by introducing a strong incomplete information setting for the first time. At the project level, the findings confirmed the need of elaborating the *stress-tests*, and of evaluating the model under similar real-world conditions. At the literature level, the paper shows the consequences of a change into the information setting and clearly highlight the importance of this element in the strategy choice of the agents.

Reaching Stable Outcomes through the Decentralized Multiple-Rounds Mechanism under Uncertainty

E. Quintilii

Abstract

The paper presents a dynamic non-revelation hiring procedure made up by a finite number of rounds that matches Institutions and Candidates. At any round a different sub-set of Institutions open a job positions and candidates apply. Temporary matching are obtained at the end of the round, to which Institutions commit while Candidates do not. I demonstrate that in a setting of incomplete information the mechanism implements the Candidate-Optimal matching in Bayesian Nash Equilibrium. Moreover, the stability of the outcome is ensured for all agents truthfully revealing their preferences. Finally, I stressed the model into a strong incomplete information setting – i.e. almost all information are removed, and candidates do not have a prepared preferences' lists but just an utility function. I succeed in proving the existence of Bayesian Nash Equilibrium for all the agents truthfully revealing their preferences. The lack of information – huge or not – does not affect the functioning of this decentralized mechanism and it drives the agent toward truth-telling strategies.

Keywords: *decentralized mechanism, matching, non-revelation game, multi-rounds mechanism, dynamic game, incomplete information, uncertainty, stability*

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2. Introduction

A two-sided matching market is an economic space where the agents of one side of the market, are coupled with the ones of the other side, through a matching mechanism based on their preferences. There are different kind of inefficiencies that can affect this kind of market – such as problems of thickness, congestion and safety (Roth 2007) – and that can undermine the stability of the assignments. A matching is said to be stable if it is individually rational – i.e. any agents is coupled with someone they like more than being alone – and if it is not blocked by any pair – i.e. there are not two agents that would prefer to be matched together rather than their current mates. Most of the studies focused on centralized matching structure, where agents submit their lists of preferences to a central clearinghouse that runs a matching algorithm for obtaining the final couples. A great number of real-world markets benefited from the research on centralized mechanism, however, there are still many entry-level labor markets organized on a decentralized structure. First formalized and analyzed by A. E. Roth (Roth and Vate 1990, 1991), on the contrary of centralized systems, agents do not submit lists but reveal their preferences through their actions. Agents of different sides directly interact in a sequential game of offers and acceptances/rejections. This scheme is particularly common in the private job market, where Firms address Workers one by one proposing the job position (Alcalde et al. 1998; Blum et al. 1997; Haeringer and Wooders 2011), and in the Academic Market (Triossi 2009).

In this paper, I present the analysis of a decentralized mechanism that works per rounds in settings of incomplete information. Taking inspiration from the real information conditions of the entry-level academic job market in Italy, I stressed the model over the traditional setting of incomplete information, and I introduce – for the first time – the strong incomplete information condition. The peculiarity of the procedure analyzed, is that the entire job offer is sub-divided over a finite number of rounds. At each round, some Institutions open a job position; at this point candidates can send applications, and then, they wait for knowing who among them has been selected for the job – while the others are ordered into waiting lists. The end of the round is officialized by the formation of temporary matchings, that can be broken by candidates but not by institutions – due to an

assumption of one-sided commitment (Diamantoudi et al. 2015). Each round is played by a different sub-set of institutions and the game ends when no one of them has still a vacant position to be fulfilled. The model has been formalized in a setting of complete information (E. Quintilii 2016)¹⁰⁶. The mechanism implements the Candidate-Optimal assignment in Subgame Perfect Equilibrium when institutions follow their dominant strategy – picking the candidate they like the most among the ones who applied – and candidates reveal a truncation of their real preference lists – and the last element of the truncated lists is the optimal stable mate. This truncation strategy requires a lot of information and is risk-free only as long as the number of job positions equals the number of candidates. However, the mechanism resulted to achieve stable results also for all the agents truthfully revealing their preferences. These findings, added to the awareness that a complete information setting is unusual in real-world market, drove me to test the model under the incomplete information and strong incomplete information – i.e. the private and unknown information is more than just the preferences' profiles of the other agents. In 2009, Niederle and Yariv noticed that, in a setting of aligned preferences, a decentralized mechanism could generate unstable outcomes under uncertainty – and in presence of market frictions (Niederle and Yariv 2009). In order to drive outcomes to stability then it is necessary to add strong assumptions on the richness of the economy. Previously – in 2008 – Pais considered the **lack of information** into a decentralized mechanism considering ordinal preferences and implementing the notion of Ordinal Bayesian Nash Equilibrium (Pais 2008). The same notion was introduced in 2007 by Ehlers and Massò for exploring the Ordinal Bayesian Nash Equilibrium into a centralized structure that runs the D-A. The authors demonstrated that the strategy profile of the OBNE in the central mechanism that works with a D-A algorithm, is a truth-telling strategy profile¹⁰⁷. However, the study of how partial information can affect a – centralized – mechanism has been firstly introduced by Roth and Rothblum in 1999 (Roth and Rothblum 1999). The two authors introduced a symmetric

¹⁰⁶ The working paper is contained into the Ph.D. Dissertation “The role of Information into Matching Markets”, 2017.

¹⁰⁷ The theoretical results give an explanation to the real data gathered for the American Physicians market (Ehlers and Massó 2007).

information model, implemented for exploring the stochastic dominance of the agents' strategies. I recall and implement this model for analyzing the strategies of the players in the setting of incomplete information – where the preferences' profiles are not common knowledge. Even if the I focus on a mechanism with a different structure – centralized vs decentralized – the findings are the same: for the workers (candidates) the stochastically dominant strategy is to reveal a truncation of the preferences' lists. The game implements the set of the stable matchings in a Bayesian Nash Equilibrium strategy profile that identifies multiple equilibria. However, the risk linked to a truncation strategy is high and it increases as the number of candidates surpasses the number of vacancies. Interesting are the laboratory experiments carried out by Niederle and Featherstone (2009) that demonstrated how difficult is for the agents to identify the optimal strategy. Students has been found to try to strategize the game with terrible outcomes that at the end are worse than revealing true preferences (Featherstone and Niederle 2011). Their results give a foundation for sustaining that even if theoretically the mechanism perfectly works, in the real markets the situation can be highly different. This is the reason why it is necessary to involve into theoretical models always more details from real-world case studies. With this purpose in mind, I worked specially on the exploration of what happens if the information conditions of the model reflect the ones of a real market. The experiments carried out by Pais et al. (2012) demonstrated that the lack of information did not affect the functioning of the decentralized mechanism implemented in the laboratory (Pais et al. 2012). However, since the model is different from the DMR, it was not possible just to generalize the result on the DMR and a specific analysis was required. It is quite clear that the decentralized mechanism literature – in general – and the analysis of dynamic games into incomplete information setting – specifically – is still developing. The aim of this paper is to enrich and to foster further studies on decentralized mechanism in different information conditions; firstly, for testing the functioning of real-world model that influence our everyday life, and secondly, for having at disposal models that are as close as possible to real situations. The introduction of the strong incomplete information setting goes toward this direction,

by stressing the model in a context where traditional known information are now removed. Its closeness to the real condition of the hiring procedure for Assistant Professors in Italy, allows me to strongly assess that – opposite to first hypotheses – the huge lack of information does not affect its functioning, and it is surely not a cause of its failure. On the contrary, the incompleteness of information drives the agents to truthfully reveal their preference – and this strategy profile implements the Institutional-Optimal matching in Bayesian Nash Equilibrium.

The paper is organized as follows: Section 2 presents the basic model and the key assumptions on the mechanism; Section 3 detailed the hiring procedure and the strategies of the agents into a complete information setting; Section 5 introduces the incomplete information setting and the main results; Section 6 presents the mechanism stressed till a strong incomplete information condition and the main results; Section 7 concludes by resuming the paper and illustrating the practical consequences of the findings.¹⁰⁸

2. The Model

Let $I = \{i_1, \dots, i_n\}$ be the non-empty and finite set of Institutions, and let $C = \{c_1, \dots, c_n\}$ be the non-empty and finite set of Candidates. Each agent has complete, transitive and strict **preferences** over the individuals on the other side that will be reported in their preferences' orders. Let $P(i)$ denote the preferences of any university $i \in I$ over $C \cup \{i\}$ ordered in a decreasing list $P(i) = \{c_1, c_2, \dots, c_n, i\}$, and P_i denote the preferences of any candidate $c \in C$ over $I \cup \{c\}$, $P_i = \{i_1, i_2, \dots, i_n, c\}$. The order represents a strong preference relation such that $c_1 \succ_i c_2$, i.e. c_1 is strictly preferred to c_2 by i . A candidate $c \in C$ is acceptable for $i \in I$ if $c \succ_i i$; a university $i \in I$ is **acceptable** for $c \in C$ if $i \succ_c c$, i.e. it is considered acceptable any agent who is preferred to herself otherwise it is called **98nacceptable**. Let denote by $P = \{P(i_1), \dots, P(i_n), P(c_1), \dots, P(c_n)\}$ the profile of preferences that gather the preferences of all the players into the matching procedure. The **non-revelation** setting implies that the order of preferences will never be directly reported into the markets, such that

¹⁰⁸ In the next sections I will use the subject “we” for avoiding confusions with the name of the Institutions' set.

the agents have not to submit the order list before the starting of the assignment procedure¹⁰⁹. The players, instead, will reveal who is acceptable and who is not, and some preference relations, through their actions into the game. We can define the matching market as a triple of the form (I, C, P) and the outcome obtained at the end of the procedure is called matching. A matching μ for (I, C, P) is a one-to-one correspondence of $I \cup C$ into itself of order two, i.e. $\mu: I \cup C \rightarrow I \cup C$, such that $\mu(i)=c \Leftrightarrow \mu(c)=i$, and so $\mu(i) \in C$ and $\mu(c) \in I$. If $\mu(x)=x$ the agent is matched to herself, and so she is considered as **unmatched** since she is not coupled with an agent of the other side of the market. The problem is set as a one-to-one matching, so the quota of offered positions is fixed to one, for any $i \in I$, $q_i=1$ by assumption¹¹⁰. We say that a matching μ is **individually rational** if each agent is coupled with an acceptable mate, for any $c \in C$ and for any $i \in I$, $\mu(c)=i \Leftrightarrow i \succ_c c$ and $\mu(i)=c \Leftrightarrow c \succ_i i$. If there is a pair (i, c) that prefers each other at the agent they are matched to, such that $i \succ_c \mu(c)$ and $c \succ_i \mu(i)$, we say that the couple (i, c) is a blocking pair. Anytime that a blocking pair is detected into the outcome of the assignment procedure, it is said that the matching is **blocked**. A matching μ is defined **stable** if it is individually rational and it is not blocked by any pair.

2.1 Key Assumptions on the Model

We assume that applying for a job position is costless for any candidate and that there is not a maximum number of applications allowed per round or per game, such that they do not have any kind of constraint during the hiring procedure. Moreover, we set a “**one-sided commitment**”¹¹¹ condition into the game; In this model, the Institutions are the committed side, while candidates are not officially bounded to any matching until the end of the game, when no more rounds are available. We will use the term of “**temporary matching**”¹¹² for denoting the non-official assignments that will be formed during the game and it will be formally

¹⁰⁹ As it happens in centralized procedures.

¹¹⁰ It is possible to allow an institution to offer more positions and simultaneously respecting the constraint of $q = 1$ by creating a number of copies of the same institution equal to the number of job positions it desires to open (Gale and Sotomayor 1985a).

¹¹¹ (Diamantoudi et al. 2015)

¹¹² (Blum et al. 1997).

represented by μ^I and $\mu^t(i)$. The agents involved will be addressed as “temporary matched”, to underline their condition at some point into the game and to differentiate them from the ones who are still unassigned. We set also the constraint on the acceptability conditions of the Institutions’ side called **institutional acceptability**¹¹³. A Candidate is considered institutionally acceptable if she meets all the requirements imposed by the job position announcement. An institutionally acceptable agent can never be treated as unacceptable by the Institution, i.e. she cannot be rejected without reason. Consequently, we consider to be unacceptable mates for Institutions, only the agents who do not comply with the mandatory requirements. For convenience of the model presentation, in the paper we will consider all the candidates who send an application as institutionally acceptable.

3. The Decentralized Multiple Rounds Mechanism (DMR)

3.1 The Hiring Procedure

Let $R = \{r, r+1, \dots, r+n\}$ be the finite set of **rounds** that compose the whole game. Each round is actively played by sub-sets of I and C identified by $I^r \subset I$ and $C^r \subseteq C$, for any $r \in R$. Let γ be the **permutation** of the set I over the game defined as $\gamma = \{(i, i^1)_r, (i^2)_{r+1}, \dots, (i^n)_{r+n}\}$ where the sequential order identifies the distribution of the set I over the rounds of the game for $i_r = \gamma(r)$ for all $r = 1, 2, \dots, n$ and the brackets denoting the institutions that play simultaneously during the same round. The proposal – acceptance/rejection matching mechanism of the game is carried out per round, and each round has the same following structure:

- **Step_0:** Some Institutions *simultaneously* announce a job position.

Formally, any $i \in I$ sends an “activity message” at each round, denoted by $a^r(i)$ in the form $a^r(i) = \{c \in C : c \succ_i i\}$ if it offers a position, or $a^r(i) = \{i\}$ if it stays inactive.

¹¹³ (E. Quintilii 2016).

- **Step_1:** Once the positions are offered, candidates simultaneously send the applications to the job positions they are interested in.

Formally we say that each $c \in C$ sends a message denoted by $s^r(c)$ in the form $s^r(c) = \{i \in I^r : i \succ_c c\}$ if she applies for the job, or $s^r(c) = \{c\}$ if she stays inactive. So, during Step 1 the general round sub-set $C^r = \{c \in C \mid c \notin s^r(c)\}$ is defined, together with the proposing sub-sets per institution $C_{i'}^r = \{c \in C^r \mid i' \in s^r(c)\}$, for $i, i' \in I^r$.

- **Step_2:** Any institution simultaneously picks up one candidate and orders the others on the waiting list according to its preferences.

Any $i \in I^r$ chooses one candidate within the set of the ones who defined it an acceptable mate – i.e. $C_{i'}^r$ – and sends a message $s^r(i)=\{c\}$. If two different institutions $i, i' \in I^r$, for $i \neq i'$, select the same candidate, such as $s^r(i) = s^r(i') \equiv c$, then the decision is given to the candidate. This “back-and-forth” phase allows the mechanism to work perfectly even if more institutions select the same applicant. For a clearer presentation I^r will be posed equal to 1 for the entire paper, i.e. $I^r = \{i\}$ and $\gamma = \{i_r, \dots, i_{r+k}, \dots, i_{r+n}\}$.

The candidates who have not been chosen as “winners of the contest” are ordered into the set of the waiting list $WL(i) = \{C_{i'}^r \setminus \mu_r^t(i) \mid P(i)\}$.

- **Step_3:** Temporary matchings are defined and the round ends.

Each round ends with a temporary matching for any $i \in I^r$ in the form $\mu_r^t(i) \equiv x$ for $x \in C \cup \{i\}$, and the set of temporary matching is defined by $M_r^t = \{\mu_r^t(i), \dots, \mu_r^t(i^n)\}$ that gathers all the temporary assignments realized up to round r .

At the end of each round the set M_r^t is updated by adding the new temporary matching of $i \in I^r$ realized during the new round, and the new assignments that occurred for $I^P(I^r, \gamma)$, i.e. the set of all institutions that played before I^r under the permutation γ . As soon as no more job positions are available – or the ones left are considered as unacceptable by all the candidates – the game ends. At this point, the temporary matchings of the last round M_{r+n}^t become official, $M_{r+n}^t \equiv M$.

3.2 Strategies into Complete Information Setting

The strategy profiles of agents will be analyzed in two different settings of information, and for both we will illustrate the equilibrium strategies. However, some previous considerations on the strategies profiles of the agents are useful. The following statements are derived from the analysis of the DMR into a complete information setting, whose results are taken into account as point of reference.

3.2.1 Institutions' Dominant Strategy

The “institutionally acceptable” constraint is a fundamental component of the mechanism that reduces the set of possible strategies for institutions. In fact, they have not the opportunity to reject all the candidates who applied to the job in order to hold the position for someone they prefer more. This entails a restriction of the strategies' space, such that given the constraint of institutional acceptability – i.e. $s^r(i) \neq \{\emptyset\}$ as long as $C_i^r \neq \{\emptyset\}$ – the strategy $\mathbf{s}^r(\mathbf{i}) = \arg \max \mathbf{Ch}_i(\mathbf{c})$ is a **weakly dominant strategy for each Institution**, for any $r \in R$, and for any γ . It is important to notice that *the weakly dominant strategy $s^r(i) = \arg \max Ch_i(c)$ is a **truthful reporting strategy** with respect to the reference set C_i^r for any $i \in I$, that is actually the only set the Institution can directly address. We can affirm that in the decentralized multiple-rounds mechanism induced by the game G^Y_R , it is a **weakly dominant strategy for each Institution to truthfully reveal its preferences**, for any $r \in R$, and for any γ .¹¹⁴ Given the properties of a dominant strategy, there is no need to demonstrate that the result is valid also in this information setting. Moreover, it is easy to note that the dominant strategy of the Institutions' side does not depend on the knowing the preference profiles of other agents, such as it is not affected at all by the new conditions.*

3.2.2 Candidates' Strategies

¹¹⁴ These statements correspond to Theorem 1, Lemma 1 and Theorem 2 of the working paper « Decentralized Multiple-Rounds Mechanism' by E. Quintilii 2016, part of the Ph.D. dissertation “The Role of Information into Matching Markets” (2017).

The strategy of a candidate is determined by the choice of applying or not to the institutions that are offering the job position. The analysis of the mechanisms in the setting of complete information revealed a weakly dominated strategy – the truthfully revealing strategy – and a non-dominated strategy – so-called truncation strategy¹¹⁵, i.e. revealing into the game only the top part of the real preference list. By **truthfully revealing strategy (TRS)** traditionally we denote the act of sending an application to all institutions that belong to the preference list of the candidate, i.e. $s^r(c) = \{i \in I^r \mid i \in P(c)\}$ for any $r \in R$. In the DMR, there is always a still unmatched candidate who can benefit by deviating from a TRS strategy. The alternative to a truthfully revealing strategy for a candidate is a **truncation strategy**. Formally, candidate c truncates the preference list $P(c)$ after k -elements, and $P(c)=\{i, \dots, i^k, \dots, i^n\}$ is reported into the game as being $P^*(c)=\{i, \dots, i^k\}$, i.e. candidate c does not send an application to any institution less preferred than i^k . The truncation strategy profile per round is defined as $\mathbf{s}^*(c)=\{i \in I^r \mid i \in P^*(c)\}$, for $P^*(c) \subseteq P(c)$ and for any $r \in R$. Given the dominant strategy played by the Institutions' side, there is always at least one candidate¹¹⁶ who is better off by reporting a truncation of $P(c)$ over the game, i.e. $s^*(c) = \{i \in I \mid i \in P^*(c)\}$ for $P^*(c) \subseteq P(c)$.¹¹⁷ Revealing just the top-part of the preferences' list dominates any non-truncation strategy, in particular it weakly dominates the TRS and strongly dominates any other strategy.¹¹⁸ Formally, for $c \in C$ and for $P^*(c)$ being the truncation of the real preferences' list $P(c)$, then for the preference profile P_{-c} of the other players, $\mathbf{DMR} [P^*(c), P_{-c}](c) \succ_c \mathbf{DMR} [P(c), P_{-c}](c)$. Then; for any candidate who reveals a truncation of her true preferences there are no incentives to deviate, given the Institutions playing their dominant strategy. For the temporary matched candidates it was identified an implementing preference strategy denoted by $\mathbf{s}^r(c) = \{i \in I^r : i \succ_c \mu^t(c)\}$, given $P(c)$, for $\mu^t(c)=x$ and $x \in IU\{c\}$, for any $r \in R$. This strategy is a best response for any temporary matched candidate for

¹¹⁵ (Roth 1984b; Roth and Rothblum 1999)

¹¹⁶ Any candidate whose assignment is different under an Institutional-Optimal or Candidates-Optimal outcome, is better off by playing a truncation strategy. As long as the game has not a singleton core, then there is at least one candidate who prefers any matching different from the one obtained under an I-Optimal outcome.

¹¹⁷ The result is not so different from what stated for the D-A mechanism (Roth and Sotomayor 1992).

¹¹⁸ The result is close to the one reported by Roth and Rothblum (1999) for the D-A mechanism.

all the other candidates playing a weakly dominated or a non-dominated strategy, given the dominant strategy of Institutions.

3.3 Equilibrium Strategies and Stability Properties

In the setting of Complete Information (E. Quintilii 2016) it has been identified a unique Subgame Perfect Equilibrium whose outcome corresponds to the Candidate-Optimal matching.

Theorem A. *Let S be such that $s(i) = \arg \max C_h(c)$ for all $i \in I$, and $s^*(c) = \{j \in I \mid i \in P^*(c)\}$ for all $c \in C$ and $P^*(c) = \{i, \dots, \mu^c(c)\}$, then S identifies a Subgame Perfect Equilibrium of the decentralized multiple-rounds mechanism induced by the game G^{γ_R} , for any $r \in R$ and for any γ .¹¹⁹*

Theorem B. *For S being an SPE of the decentralized multiple-rounds mechanism induced by the game G^{γ_R} , the outcome of the mechanism is stable and Candidate-Optimal.*

However, the mechanism showed interesting stability properties, and the ability of reaching stable outcomes not only at the SPE, but also for all the agents truthfully revealing their preferences.

Theorem C. *In a DMR mechanism induced by the game G^{γ_R} , for all institutions having the dominant strategy of truthfully reporting, if all candidates truthfully reveal their real preferences into the game, the outcome is stable and Institutions-Optimal, for any $r \in R$ and for any γ .¹²⁰*

However, it cannot be forgotten that the assumptions of one-sided commitment and of “no-costs and no-restrictions” on applications are necessary – and sufficient when applied simultaneously – conditions for ensuring the stability of this outcome.

¹¹⁹ By $\mu^c(c)$ we denote the best possible stable outcome for candidate c into the DMR mechanism, given P .

¹²⁰ It is quite easy to see that most of the results obtained so far are close to the ones presented for the D-A algorithm with the Institutions proposing first (Roth 2008).

The findings of the analysis into a complete information setting allow to sustain also for the DMR mechanism, a statement firstly achieved also by Roth and Rothblum (1999) for a D-A mechanism:

Proposition 1. *In a DMR mechanism induced by the game G^{γ_R} , as long as a candidate cannot implement a truncation strategy, the best response is to reveal her true preferences, for all the other agents playing a weakly dominated or a non-dominated strategy, for all $r \in R$ and for all γ .¹²¹*

The content of this proposition would be a key point of the further exploration of the mechanism into incomplete information settings.

4. Incomplete Information Setting

The strategies of the players are analyzed in a setting of incomplete information. The following features of the model are common knowledge:

- The composition of the sets I and C ;
- Any $i \in I$ will offer a job position;
- The permutation of the set I over the rounds of the game γ – taken as randomly given – and so the composition of any I^r for any $r \in R$;
- All the assumptions on the game are known

Information acquired at the end of the round:

- $s^r(c)$ of any $c \in C$, and so the composition of C^r_i for any $i \in I^r$ (candidates' actions)
- $s^r(i)$ of any $i \in I^r$, and so the composition of $M^t_r = \{\mu^t(i), \dots, \mu^t(i^n)\}$ (institutions' actions)

The set of private information is formed by:

- Preferences' List

¹²¹ (Ehlers 2008; Roth and Rothblum 1999)

Set of Unknown Information:

- Preference Profile P

Given that the Preference Profile P is not part of the information set, we consider a common belief \hat{P} over P – i.e. a probability distribution over the preference profile – and we assume each agent $x \in IUC$ to have a common belief \hat{P}_x over the preferences' of all the other agents into the game. As the focus of the analysis is on the Candidates' side, we will generally refer to a candidate c behaviors, for candidate c having a preference list $P(c)$ and a belief over the preferences of all the other agents \hat{P}_{-c} . For exploring the candidates' actions into this setting, we decided to follow the **model of symmetric information** introduced by Roth and Rothblum (1999)¹²². According to this model, the lack of information of candidate c about the preferences' of Institutions and of the others candidates, can be formulated as follows. Given two institutions i and i', for $i \neq i'$ and $i, i' \in I$, we define the common belief \hat{P}_{-c} as $\{i, i'\}$ -symmetric if the distributions of \hat{P}_{-c} and $(\hat{P}_{-c})^{i \leftrightarrow i'}$ coincide. This means that for any realization of P_{-c} the probability $\Pr\{\hat{P}_{-c} = P_{-c}\}$ corresponds to the probability $\Pr\{\hat{P}_{-c}^{i \leftrightarrow i'} = P_{-c}\}$ ¹²³. Less formally, the symmetric belief entails a certain degree of knowledge about the preferences of the Institutions. The candidate is able to identify the kind of applicants the two institutions likes more, but cannot know precisely how candidates are ordered up to their preferences. For example, if the announcement of a job position requires applicants to have specific characteristics, she knows that the ones who have all of them and satisfy them better than the others would be on the top-list, but she cannot identify who will be first, second, and so on. Since she does not have any information about preferences of other applicants, she cannot know which of the two institutions they like most. If the Candidate has beliefs that are $\{I\}$ -symmetric that it would mean that she cannot differentiate any Institution from the other, i.e. she has the same amount of information for each of them. However, candidates

¹²² Other models developed for analyzing behaviors into incomplete information settings involved ordinal preferences for defining Ordinal Bayesian Nash Equilibrium (Ehlers and Massó 2007; Pais 2008).

¹²³ This is also equivalent to $\Pr\{\hat{P}_{-c} = P_{-c}\} = \Pr\{\hat{P}_{-c} = P_{-c}^{i \leftrightarrow i'}\}$.

can acquire information additional to the ones reported into the announcement of the job position, for example by knowing someone who works in that institution, or because she already worked there, and so on. The diverse knowledge an applicant has on the Institutions, let us identify partitions of I in the form $\{i^1, \dots, i^k\}$ for which she has the same amount of information, i.e. her beliefs \hat{P}_{-c} are I_v -symmetric for $v = 1, \dots, k$. For the institutions that belong to same partition, the candidate is not able to have a specific knowledge about the preferences they have over the candidates' set and from whom they are desired. Then, the available information into the market allows the applicants to distinguish institutions in classes whose differences are reflected onto the general expectations about preferences, but they are not sufficient for having determined expectations on the lists of single institutions. This entails that a candidate can identify the group of institutions for which she would be one of the most preferred, but she cannot assess how much she is more preferred than the others.

4.1 Stochastically Dominant Strategy

Given the randomness linked to this information setting, it is necessary to introduce the concept of **random matching**¹²⁴. A random matching denoted by $\hat{\mu}$ is the random outcome obtained from a matching mechanism that works with agents' random preferences¹²⁵. At each random matching, any player of the game receives a random assignment denoted by $\hat{\mu}(x)$ for $x \in IUC$ whose range is given by the union of x and the set of the agents on the opposite side of the market. For any $\hat{\mu}$, it is possible to define the expected utility function of each agent with respect to $\hat{\mu}(x)$ – on the defined range – as $E_{\hat{\mu}(x)}(u_x)$, for u_x being the utility function of agent x , for $x \in IUC$.

Given the incompleteness of the information set, we will refer to a stochastically dominance and Bayesian Nash Equilibrium. Given two random matchings $\hat{\mu}$ and

¹²⁴ (Ehlers and Massó 2007; Roth and Rothblum 1999).

¹²⁵ Formally, it is a random variable whose outcome is one of the assignment that belongs to the matching set of the game.

$\hat{\mu}'$, for $c \in C$ and for her $P(c)$, $\hat{\mu}(c)$ is said to **stochastically dominates** $\hat{\mu}'(c)$, denoted by $\hat{\mu}(c) \succcurlyeq_{P_c} \hat{\mu}'(c)$, if $Pr\{\hat{\mu}(c) \geq_{P_c} x\} \geq Pr\{\hat{\mu}'(c) \geq_{P_c} x\}$, for every $x \in I \cup \{c\}$. Moreover, any utility function $u_c : I \cup \{c\} \rightarrow R$ is defined as P_c -monotone if it is monotone with respect to the preference list $P(c)$. Roth and Rothblum (1999) recognized any P_c -monotone function as the expected utility function of some candidate whose ordinal preferences are expressed by $P(c)$. Then, a random matching stochastically dominates another random matching up to the ordinal preferences of candidate $c - \hat{\mu}(c) \succcurlyeq_{P_c} \hat{\mu}'(c) -$ if and only if for any P_c -monotone utility function u_c the expected utility obtained under the random matching $\hat{\mu}$ is as preferred as the one obtained under $\hat{\mu}'$, i.e. $E_{\hat{\mu}}(u_c) \geq E_{\hat{\mu}'}(u_c)$.

The main result that stemmed from the symmetric information model is the following:

Theorem (Roth and Rothblum 1999). *For a candidate with $\{I\}$ -symmetric information, any non-truncation strategy $s_c(P_c)$ is stochastically dominated by a truncation of the true preferences $s_c(P_c^*)$, for P_c^* being a truncation of P_c , for every random preference \hat{P}_{-c} that is $\{I\}$ -symmetric of all the other players.*

The authors concentrated in demonstrating the stochastically dominance of some strategies over others, and they did not consider expected utility functions further¹²⁶. In this paper we will consider them both. Specifically, we started from Theorem 4 for testing their result on the DMR¹²⁷ and identifying the non-dominated strategy in the setting of incomplete information. Then, we reflexed these results on the expected utility functions and implemented them for the definition of the Bayesian Nash Equilibrium of the game.

As in the complete information setting, this study did not involve strictly dominated strategies (so only truthful and truncation revealing strategy have been taken into account). For the demonstration that the result of Roth and Rothblum holds in the

¹²⁶ The aim of the paper (Roth and Rothblum 1999) was to address a question of a dean about which advice should students receive when participating into the mechanism for Intern positions at Hospitals. Then, they were just interested in finding the stochastically dominant strategies for students.

¹²⁷ Theorem 4 refers to a Deferred Acceptance algorithm context.

setting of a DMR mechanism, we will refer to the findings about the outcome of the mechanism and we will implement a proof inspired by the above-mentioned authors (Roth and Rothblum 1999).

From the analysis carried out by E. Quintilii (2016) we learned that for all the agents truthfully revealing their preferences, the outcome of a DMR mechanism is Institutional-Optimal. This matching represents the worst possible stable assignment for any candidate and it is denoted by $\mu^I(c)$ for any $c \in C$. Formally, it is expressed by $DMR[P_c, P_{-c}](c) = \mu^I(c)$ that in a contest of incomplete information, for a common belief \hat{P}_{-c} over P_{-c} , is expressed by $DMR[P_c, \hat{P}_{-c}](c) = \mu^I(c)$ for $\Pr\{\hat{P}_{-c} = P_{-c}\} > 0$. The truncation strategy identified in the SPE under complete information, it is not implementable with a lack information – since candidates cannot know in advance which of the institutions is their optimal stable assignment. For this reason, we will generally address the truncation strategy, that refers to report a truncation P_c^* of the entire list P_c that respects the same restriction to I. For all the candidates reporting a truncation of their preferences, given the dominant strategy played by Institutions, the generated random matching has its range in the set of stable matching M, whose minimum is determined by $\mu^I(c)$ and the maximum by $\mu^C(c)$.¹²⁸ Then, by displaying a truncation strategy any candidate has more probability of obtaining a better outcome than revealing her true preferences. This statement is valid both for all other candidates reporting the entire list or its truncation, given the Institutions' side playing its dominant strategy.

Lemma 1. *Let i and i' be two institutions such that $i \neq i'$ and $i' \equiv \mu^I(c)$, and let P_c be the true preference list of $c \in C$, such that $i \succ_{P_c} i'$, and let P_c^* be the truncation of P_c , then for some $\partial \geq 0$*

$$\Pr\{DMR[P_c^*, \hat{P}_{-c}](c) = x\} - \Pr\{DMR[P_c, \hat{P}_{-c}](c) = x\} = \begin{cases} \partial & \text{if } x = i \\ -\partial & \text{if } x = i' \end{cases}$$

¹²⁸ Due to the lack of information we cannot assume that candidates are able to eliminate from the reported truncation the institutions that corresponds to $\mu^I(c)$.

for \hat{P}_{-c} be the $\{i, i'\}$ -symmetric preference profile of all the other players, and for $i, i' \in M$, i.e. the set of final stable matchings.

The Proof of Lemma 1 comes directly from the findings under the complete information setting. If a candidate truthfully reveals her preferences, for all the other agents being sincere, there is a unique stable outcome given by $\mu^I(c)$. This means that the probability of achieving any other $i \in I$ such that $i \succ_{P_c} \mu^I(c)$, for $\Pr\{\hat{P}_{-c} = P_{-c}\} > 0$ is close to zero, i.e. $\Pr\{DMR[P_c, \hat{P}_{-c}](c) = i\} \approx 0$. However, under the same conditions, if she reports a truncation of her list the probability is close to one, i.e. $\Pr\{DMR[P_c^*, \hat{P}_{-c}](c) = i\} \approx 1$ for any $i \in I$ such that $i \succ_{P_c} \mu^I(c)$.

Theorem 1. In a DMR mechanism induced by the game G^{γ_R} , for any candidate $c \in C$ having $\{I\}$ -symmetric information, revealing a truncation P_c^* of her true preference list P_c stochastically dominates the truthfully reporting, such that

$$DMR[P_c^*, \hat{P}_{-c}](c) \succcurlyeq_{P_c} DMR[P_c, \hat{P}_{-c}](c)$$

given the \hat{P}_{-c} $\{I\}$ -symmetric preference profile of all the other players, for all $r \in R$ and for any γ .

The proof of Theorem 1 comes directly from Lemma 1. Revealing only part of the preference list gives to any candidate a greater probability of obtaining a stable assignment that she prefers more than the Institution-Optimal mate – that is the only possible stable outcome for all agents truthfully reporting.

Corollary 1. Let $\hat{\mu}$ and $\hat{\mu}'$ be two random matchings. Assume that for any $c \in C$ whose preference list is denoted by P_c , given the \hat{P}_{-c} $\{I\}$ -symmetric preference profile of all the other players, $\hat{\mu} \succcurlyeq_{P_c} \hat{\mu}'$, i.e. $E_{\hat{\mu}}(u_c) \geq E_{\hat{\mu}'}(u_c)$ for u_c being the P_c -monotone utility function of c . Then, for some $\partial \geq 0$

$$\begin{aligned} & \Pr\{DMR[P_c^*, \hat{P}_{-c}](c) = x\} - \Pr\{DMR[P_c, \hat{P}_{-c}](c) = x\} \\ &= \begin{cases} \partial & \text{if } x = \hat{\mu} \\ -\partial & \text{if } x = \hat{\mu}' \end{cases} \end{aligned}$$

such that

$$E_{\hat{\mu}}[u_c(\mathbf{P}_c^*, \hat{\mathbf{P}}_{-c})] \geq E_{\hat{\mu}'}[u_c(\mathbf{P}_c, \hat{\mathbf{P}}_{-c})]$$

4.2 Bayesian Nash Equilibrium

In the previous section we identified the strategy that stochastically dominates all the others. In a game with incomplete information a strategy of a player x , for $x \in \text{IUC}$; is defined as a function $s_x: P_x \rightarrow A_x$, for P_x being the preference profile x ¹²⁹ and A_x being the set of all possible actions of x . A strategy s_x is a **Bayesian Nash Equilibrium** strategy, if for each player $x \in \text{IUC}$, given the expectation over the preferences' profiles of the other agents (\hat{P}_{-x}), it is a best response to $s_{-x}(\hat{P}_{-x})$. Then, a strategy profile S is an BNE if there is not an agent x who would be better off under some S' with respect to the outcome obtained under S , and so who would have an incentive to deviate from S , for her type defined by P_x and given the expectation over the preferences' profiles of the other agents (\hat{P}_{-x}). Formally,

$$E_{\hat{P}_{-x}}[u_x(s_x(P_x), s_{-x}(\hat{P}_{-x})|P_x)] \geq E_{\hat{P}_{-x}}[u_x(s'_x, s_{-x}(\hat{P}_{-x})|P_x)]$$

for $E_{\hat{P}_{-x}}$ being the expectation over the preferences' profiles of all the other agents on which the agent x has a common belief \hat{P}_{-x} .¹³⁰

Given the dominant strategy played by the Institutions, we want to demonstrate that for any candidate it is a best response to report only a truncation of her preferences' list. This result comes directly from Theorem 1 that proves the stochastic domination of a truncation strategy over a truthfully revealing one. The P_c -stochastic domination implies that the random outcome obtained by candidate $c \in C$ for reporting the top-part of the list has a larger probability of being preferred by c to the random outcome obtained for a truth-telling strategy. Then, the P_c -monotone utility function of candidate c defined by u_c ¹³¹ is expected to have a higher value for her revealing a truncation than the entire preferences' list.

¹²⁹ The preference profile defines the candidate type.

¹³⁰ (Nisan et al. 2007).

¹³¹ It has been firstly defined as $u_c: I \cup \{c\} \rightarrow R$ but it can be also defined by $u_c: P_c \times A_c \rightarrow R$, for A_c being the set of actions.

Lemma 2. In the DMR mechanism under incomplete information, given an expectation over the preferences' profiles of all the other agents, for any $c \in C$ and for P_c being her preference list whose truncation is defined by P_c^* :

$$E_{\hat{P}_{-c}}[u_c(s_c(P_c^*), s_{-c}(\hat{P}_{-c})|P_c)] \geq E_{\hat{P}_{-c}}[u_c(s_c(P_c), s_{-c}(\hat{P}_{-c})|P_c)]$$

for u_c being a P_c -monotone function.

The proof is given by Theorem 1 and Corollary 1. If $s_c(P_c^*)$ stochastically dominates $s_c(P_c)$ up to the real preferences P_c of any candidate $c \in C$, then the expected utility value assigned by c to a random outcome $\hat{\mu}$ that could be obtained by displaying P_c^* is greater than the one assigned by c to a random outcome $\hat{\mu}'$ that could be obtained by displaying P_c . It follows the next Theorem.

Theorem 2. Let \hat{P} be a common belief over the preference profile P and let any preference profile be $\{I\}$ -symmetric. Then, the strategy profile S , for Institutions playing their dominant strategy and any candidate revealing a truncation of her preferences' list, is a Bayesian Nash Equilibrium for the DMR mechanism induced by game G^{γ_R} , for any $r \in R$ and for any γ .

The strategy profile of the BNE partially corresponds to the SPE identified in the setting of complete information. In a setting of complete information candidates are able to identify the optimal truncation strategy, such that there is a unique SPE that implements the Candidate-Optimal outcome. In the incomplete information setting candidates are able to recognize the benefit of truncating their preferences' lists but they have not enough information for knowing the exact point of the cut. Then, the strategy profile of the BNE identifies multiple equilibria. The stability of the outcome is ensured for all the equilibria¹³² as long as the numerical asymmetry between job positions and candidates is respected. However, as the numerical asymmetry assumption does not hold – as in real markets – unstable outcome

¹³² Mostly thanks to the one-sided commitment and to the round structure: candidate do not commit before the end of the game, and have the possibility of adjusting their strategies during the game. Since they do not submit a list, it is easy for them to update the strategy. For example, if a candidate is unmatched yet, they can decide to send an application to an institution that still have a job position, even if they excluded it at some previous round of the game.

can arise due to blocking pairs between institutions and unmatched candidates¹³³. This is the reason why truncation strategies – even if are able to guarantee a higher outcome – are the riskiest strategy for a candidate, and the risk-level increases with the decrease of information (and the increase of the number of candidates over the number of job positions).¹³⁴

5. Strong Incomplete Information Setting

The strategies of the players are analyzed in a setting of strong incomplete information with a high degree of uncertainty. The following features of the model are common knowledge:

- The composition of the set I ;
- Some $i \in I$ will offer a job position;
- Information on past games (average # of job offered per year)
- Candidates' set of actions $A = \{\text{"apply"}, \text{"not apply"}\}$
- All the assumptions on the game

Information acquired at the end of the round:

- $s^r(c)$ of any $c \in C$, and so the composition of C^r_i for any $i \in I^r$ (candidates' actions)
- $i \in I^r$

The set of private information is formed by:

- $s^r(c)$ and $s^*(i)$
- Agent type $\theta(x)$, for $x \in I \cup C$
- Utility function $u(x)$

¹³³ As the number of candidate increases the risk of being unmatched because of a too-short truncation strategy is extremely high. Taking the example of the above notation: if the (still unmatched) candidate did not apply to that institution as soon as it opened the position, probably she will never have again a chance of applying, because there could be some other candidate – who would be unmatched in a stable outcome – who applied and fulfilled the position such that the institution will not re-appear on the market during the same game.

¹³⁴ Roth and Rothblum (1999) expressed the same conclusions.

- $\mu^t(x)$ is a private info of x until the end of the game
- Preference profile $P(x)$

Set of Unknown Information:

- Preference Profile P
- The type θ of other agents
- γ is random (up to real-world rule);
- $I^R = \{i \in I : c \in S(i)\}$ for $I^R \subseteq I$, i.e. the set of institutions that will open a position.
- The composition of set C is discovered by rounds, such that any $i \in I^R$

knows only some of the candidates $c \in C$.

- The composition of the set temporary matchings of each round M_r^t is unknown.

Given that the Preference Profile P is not part of the information set, we consider a common belief \hat{P} over P – i.e. a probability distribution over the preference profile – and we assume each agent $x \in I \cup C$ to have a common belief \hat{P}_{-x} over the preferences' of all the other agents into the game. As the focus of the analysis is on the Candidates' side, we will generally refer to a candidate c behaviors, for candidate c having a preference list $P(c)$ and a belief over the preferences of all the other agents \hat{P}_{-c} . Since the composition of the set I^R and its permutation over the game is not known – such that also the finite number of rounds that compose the game is unknown – we consider a common belief \hat{W} over the possible states of the world W . At each round, the possible future states of the world are: 1. There will be another round, and the institutions that will offer the positions are liked by the candidate at least as well as the institutions of the current round (W_1); 2. There will be another round, and the institutions that will offer the positions are all considered as unacceptable by the candidate (W_2); 3. There will be not another round (W_3). Then, a common belief is the probability distribution over W , that is shaped by the probability that there will another round conditioned to the probability that the institutions that will play at that round are acceptable, i.e. $Pr[R > r] | Pr[i \in$

$I^{r+1}: i \succcurlyeq \mu^t(c)$], for r being the current round and R being the set of the total rounds of the game, and for $\mu^t(c)$ being the current situation of the candidate c – who can be temporary matched to some other $i \in I$ or to herself. For the analysis of the players strategies, we assume that $\Pr[\widehat{W} = W_3] > 0$, such that each agent is driven to play during the current round as it would be the last round of the game.

In this setting we assumed that the composition of the entire set C is not known by all the institutions. In the real world, it would cost a lot of time and effort for an institution to have a complete list of all possible suitable candidates for the job they are announcing – and still there could be someone missing. Then, the idea is that each institution knows only the sub-set of candidates who sent an application. Given that C is unknown, any $i \in I$, after receiving the candidatures, builds up a list of applicants that reflects its preference profile over the set C_i^r , for $C_i^r = \{c \in C : i \in s_c^r\}$. However, this feature has no consequences on the dominant strategy of Institutions, on the reverse, it highlights the truth-telling behavior behind their strategy.

Taking into account the preferences' profiles, there was two different ways of carrying on the analysis: 1. By assuming that each candidate has a prepared list of preferences over the set I (traditional setting); 2. By assuming that each candidate does not have a ready-to-use list, but that she knows her utility function u_c (assumption of naïve behavior). We selected the second naïve assumption because it is closer to reality, and because it allows candidates to be more flexible in a market where they have no knowledge about the job offer. Specifically: by having an *a priori* preference list over the entire set I , could trigger the candidate to do not apply for some institutions because too low in their lists compared to all the others. However, not all the institutions of that list are going to open a position, such that up to sub-set of institutions that will go on the market (I^R), that same institution could be the first of the list. The hypothesis behind the choice of a candidate naïve behavior, then, is that having a pre-prepared preference list, in a setting of strong uncertainty, could mislead the candidate's actions. The final findings partially sustain this hypothesis. Instead of having a preference profile,

candidates have a *utility profile*: based on the utility function u_c they identify when it is preferable to apply or not, by comparing the utility level of the current situation $\mu^t(c)$ to the one of the potential new mate.

Let $u_c(i)$ in the form $u_c(g_i, r_i)$ be the monotone utility function of c over any $i \in I^R$, for g_i being the geographic position of i and for r_i being the reputation level of i . This definition of the utility functions that relies over two features of institutions i has a demonstrative scope, and helps to clarify the functioning of the utility-preference profile. The two features have been selected on the basis of the results of a survey conducted in 2014 on the Italian researchers. They have been asked about which characteristics of institutions – in their personal situation – drove them to apply or not to some competitions. The geographic position and the reputational level have been the one most selected as reasons of their choices. This form of the utility function desires to outline that the preference profile of each candidate, is based upon an evaluation of the institutions under different point of views, and each of them constitutes part of the final utility value. Of course, it can involve more parameters according to the necessity of the market. Since it is only demonstrative, we are not going to analyze the utility function more deeply.

At each round, a candidate is called to decide whether to apply or not, and to which institution. By implementing the utility function – instead of the preference list – the choice is not direct and there is an evaluation process antecedent:

For $u_c(\mu^t(c))$ being the current utility of candidate c :

- If $u_c(g_i, r_i) < u_c(\mu^t(c))$ then the institution i is **unacceptable** → **not apply**
- If $u_c(g_i, r_i) > u_c(\mu^t(c))$ then the institution i is **acceptable** → **apply**

for $\mu^t(c)$ defining the current situation of the candidate (that could be $\mu^t(c) \equiv c$ or $\mu^t(c) \equiv i$ for $i \in I^R$). Note that if $\mu^t(c) \equiv i$ then $u_c(g_{i'}, r_{i'}) \leq u_c(g_i, r_i)$, and $u_c(g_{i'}, r_{i'}) > u_c(g_i, r_i)$ implies that $i' \succ_c i$ such that $P_c = \{i', i\}$. The utility function defines the agent type θ of any candidate, for $\theta = \{\theta^1, \dots, \theta^n\}$, such that θ_c represents the type of candidate c . The type of the candidate denotes if she is a potential applicant [for $u_c(g_i, r_i) > u_c(\mu^t(c))$] or not [for $u_c(g_i, r_i) < u_c(\mu^t(c))$]. Since each candidates

knows only her personal utility function, and there are no preferences profiles for candidates, then it is necessary to assume that each agent has a common belief $\hat{\theta}$ over Θ , for $\hat{\theta}$ being the probability distribution over Θ , and for $\hat{\theta}_{-c}$ being the belief of candidate c over the types of all the other agents.¹³⁵

Stochastically Dominant Strategy

A strategy is defined as a function $\mathbf{s}(\mathbf{c}): \Theta \rightarrow \mathbf{A}$, for $A = \{a^1, \dots, a^n\}$ the set of possible actions of each candidate. The strategy defined by the utility profile is an implementing truth-telling strategy up to the utility function u_c in the form $\mathbf{s}^*(\mathbf{c}) = \{\mathbf{i} \in \mathbf{I} : u_c(\mathbf{i}) > u_c(\mu^t(\mathbf{c}))\}$ that aims to better the current condition of the candidate at each round of the game. However, there is the chance that even if a potential assignment with one of the institutions that are opening a position, is better than the current condition, a candidate decides to not apply displaying a sort of truncation strategy. We want to demonstrate that any kind of truncation strategy is stochastically dominated by a strategy that reveal the true candidate type at each round.

The result heavily relies on the assumption that each agent plays as it would be the last round, such that the $E_{\hat{\theta}_{-c}}[u_c(r+1)(s_c, s_{-c}(\hat{\theta}_{-c}) | \theta_c, \hat{W})] \approx 0$, for any s_c , for $r+1 \in R$ and for \hat{W} be the common belief over W , for $\Pr[R > r] \approx 0$, for $R = \{r, r+1, \dots, r+n\}$. When deciding if implementing a truncation strategy – i.e. not applying to an acceptable institution up to u_c – a candidate is comparing the levels of the expected utility for not applying at round r and applying to a potential more preferred institutions at round $r+1$, and applying at round r – and to apply to a more preferred institution at the next round if there will be a next round. If the round $r+1$ is strongly believed to not exist, the expected utility related to this period of the game is near to zero, for any possible strategy of candidate c and of all the others players at round $r+1$. Moreover, if candidate c is not applying at round r , even the expected utility related to round r is close to zero, i.e.

¹³⁵ Note that also the Preferences of Institutions can be made up following the same process, given a utility function that implements as parameters the announcement's requirements for the job.

$E_{\hat{\theta}_{-c}}[u_c(r)(s_c(\theta_c^*), s_{-c}(\hat{\theta}_{-c}) | \theta_c, \hat{W})] \approx 0$, for θ_c^* being a misrepresentation of the true type θ_c of candidate c . It is possible to conclude then the expected utility value obtained by displaying a truncation strategy is close to zero.

Lemma 3. *The expected utility value of a candidate $c \in C$, in a setting of strong incomplete information, for the monotone utility function u_c that identifies a type θ_c and defines a preference profile P_c of candidate c , is close to zero for any strategy that misreports her true type $s_c(\theta_c^*)$, for θ_c^* being a misrepresentation of the true type θ_c of candidate c , that is*

$$E_{\hat{\theta}_{-c}}[u_c(s_c(\theta_c^*), s_{-c}(\hat{\theta}_{-c}) | \theta_c, \hat{W})] \approx 0$$

given a common belief $\hat{\theta}_{-c}$ over the other agents' types, for \hat{W} be the common belief over W such that $Pr[R > r] \approx 0$, for $R = \{r, r+1, \dots, r+n\}$.

Since by reporting the true type, the candidate has the potentiality of being assigned to some institutions she likes more than her current situation, we can assess that at each round of the game $E_{\hat{\theta}_{-c}}[u_c(s_c(\theta_c), s_{-c}(\hat{\theta}_{-c}) | \theta_c, \hat{W})] \geq 0$.

Lemma 4. *In a setting of strong incomplete information, for the monotone utility function u_c that identifies a type θ_c and defines a preference profile P_c of some candidate $c \in C$,*

$$E_{\hat{\theta}_{-c}}[u_c(s_c(\theta_c), s_{-c}(\hat{\theta}_{-c}) | \theta_c, \hat{W})] \geq E_{\hat{\theta}_{-c}}[u_c(s_c(\theta_c^*), s_{-c}(\hat{\theta}_{-c}) | \theta_c, \hat{W})]$$

for θ_c^* being a misrepresentation of the true type θ_c of candidate c , given a common belief $\hat{\theta}_{-c}$ over the other agents' types, for \hat{W} be the common belief over W such that $Pr[R > r] \approx 0$, for $R = \{r, r+1, \dots, r+n\}$.

Recalling the definition of stochastic dominances, given the findings of Lemma 1 and Lemma 2, it is possible to state the following Theorem.

Theorem 3. *In a DMR mechanism where information is strongly incomplete, for the monotone utility function u_c that identifies a type θ_c and defines a preference profile P_c of some candidate $c \in C$,*

$$DMR[\theta_c, \hat{\theta}_{-c} | \hat{W}](c) \succ_{P_c} DMR[\theta_c^*, \hat{\theta}_{-c} | \hat{W}](c)$$

for θ_c^ being a misrepresentation of the true type θ_c of candidate c , given a common belief $\hat{\theta}_{-c}$ over the other agents' types, for \hat{W} be the common belief over W such that $Pr[R > r] \approx 0$, for $R = \{r, r+1, \dots, r+n\}$.*

The proof of the Theorem comes directly from Lemma 3 and Lemma 4.

Moreover, we can demonstrate the validity of the next Corollary:

Corollary 2. *In a DMR mechanism where information is strongly incomplete, for the monotone utility function u_c that identifies a type θ_c and defines a preference profile P_c of some candidate $c \in C$, given two random outcomes $\hat{\mu}$ and $\hat{\mu}'$ such that $\hat{\mu} \succ_{P_c} \hat{\mu}'$, for some $\partial \geq 0$*

$$\begin{aligned} & Pr\{DMR[\theta_c, \hat{\theta}_{-c} | \hat{W}](c) = x\} - Pr\{DMR[\theta_c^*, \hat{\theta}_{-c} | \hat{W}](c) = x\} \\ &= \begin{cases} \partial & \text{if } x = \hat{\mu} \\ -\partial & \text{if } x = \hat{\mu}' \end{cases} \end{aligned}$$

for θ_c^ being a misrepresentation of the true type θ_c of candidate c , given a common belief $\hat{\theta}_{-c}$ over the other agents' types, for \hat{W} be the common belief over W such that $Pr[R > r] \approx 0$, for $R = \{r, r+1, \dots, r+n\}$.*

As long as the probability that the game will have another round is close to zero, displaying an action aligned to a truncation strategy at the current round, can cause the candidate a worse assignment than revealing her true type. This is because, differently for what strategized by the candidate, it could happen that there is not another round or even that all the following rounds are played by unacceptable institutions. Then, as the probability that the game will continue is close to zero, the probability that a misreporting strategy succeeds is close to zero too. The scenario is even more uncertain – and risky – if we take into account the numerical asymmetry between candidates and number of available job positions.

Theorem 4. *In a DMR mechanism where information is strongly incomplete, for the monotone utility function u_x that identifies a type θ_x and defines a preference profile P_x of some candidate $x \in IUC$, truthfully revealing the personal type θ_x is a **Bayesian Nash Equilibrium** for each round of the game G^R , given a common belief $\hat{\theta}_{-c}$ over the other agents' types, for \hat{W} be the common belief over W such that $Pr[R > r] \approx 0$, for $R = \{r, r+1, \dots, r+n\}$.*

The proof of the Theorem comes directly from what stated so far. The strong lack of information impedes any displaying of a truncation strategy, that represented an equilibrium profile under the setting of complete and incomplete information. However, there are no implications from a stability point of view, since it has already been demonstrated that for all agents revealing true preferences, the outcome is stable and Institution-Optimal. Then, despite the findings on the previous setting of information – that would have suggested to advice candidates in truncating their lists – a truthfully revealing strategy is the best strategy candidates can implement in a market that suffers of strong incomplete information.

Note that these results continue to heavily rely on the three assumptions of no costs and no restrictions on applications, and no commitment to temporary matchings for the Candidates' side.

6. Conclusions

The paper presented the stress test of a decentralized mechanism that works per round in two context of incomplete and strong incomplete information. Each round of the mechanism has the same structure – some institutions announce a job position, candidates apply, institutions select one candidate and order the others into a waiting list¹³⁶ – and ends with temporary assignments that can be broken at some following round by the Candidates side, while Institutions commit to their

¹³⁶ All the steps are played simultaneously by the agents.

temporary mates. Moreover, the applications are assumed to be costless and free of any possible restrictions, such as candidates should be incentivized to apply to job they like, as much as they can. The game resulted to be not particularly affected by the absence of knowledge on the preferences' profile of the other players. Implementing the symmetric information model (Roth and Rothblum 1999), I demonstrated that when the preferences of all agents are $\{I\}$ -symmetric then for any candidate, a truncation strategy stochastically dominates truthfully revealing the entire lists. Even if this strategy is risky, it is still implementable – even if candidate cannot identify the equilibrium optimal truncation strategy, possible only with complete information. The results are different for the context of strong incomplete information. In this setting, most of the information that previously were considered common knowledge, have now been removed, such that it is not possible to define which institutions is going to offer a job positions, when and how many of them will enter the market. Mostly due to the assumptions on the common perception that the round agents are playing would be the last, I demonstrated that there exists a Bayesian Nash Equilibrium that implements the set of stable matchings for all candidates revealing their true types. The paper enriches the literature on decentralized mechanism, and on matching theories and market design in general, and offers a new useful tool for the analysis of real-world markets. However, the formalization of the model suffers still of two limitations: first, in real cases the contests or hiring procedures do not end all at the same time, and candidates cannot hold offers for long periods; second, the entrance of Institutions into the market has been taken as randomly given, while it could be a strategy itself. Moreover, it would be interesting to conduct the same analysis removing the assumption on the state of the world, for checking the robustness of the results. In the meantime, this paper introduces a brand new information setting and drives the attention on the consequences of removing – and on the contrary, of adding – information into a matching system. It establishes the basis for a large number of interesting developments whose results will be a precious enrichment for the whole matching literature.

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2.6 From the Theoretic Findings to Policy Advices and Efficiency Improving Re-Organization Proposal

In this paragraph I will define some implementing policies that could better off the current procedure, relying on the basis of what learned from the analysis carried out thanks to the theoretical instruments.

These policies are not invasive since they will propose some optimal adjustments to the existing structure. However, it is not possible to avoid the distortion of the current procedure that will maintain its decentralized structure, but will assume a more centralized conformation.

2.6.1 What we Learned from the Theoretic Findings – Hints of Policy Advises

The theoretic findings showed that the decentralized matching mechanism at the core of the recruitment system is able to work efficiently. Most of all, it guarantees a stable result both if Candidates follow truncation strategies or truthfully reporting strategies, and for any information setting. The most important finding is the efficiency of the mechanism for the truthfully reporting strategies of the agents, since they are the more reasonable to be displayed into the real-world assignment. However, these specific stability properties rely on the respect on three conditions, that in the theoretic models have been presented as assumptions: 1. no costs on applications; 2. no restrictions on applications; 3. the chance of comparing different job offers and hold the most preferred (allowed by the one-sided commitment and the timing structure). All of them fail into the real market, and this statement is sustained by the market analysis presented at the beginning of this section at paragraph 2.3. The inefficiency that affects the real procedure is not caused by the decentralized structure or the lack of information on the job offers, neither by the matching mechanism – that then does not need to be changed. The failure of the assignment steams from a wrong management of the entire procedure. While the core of the recruitment perfectly works (the mechanism), the system built around

it is completely inadequate (the management and the rules). The hiring mechanism has been never analyzed from a matching point of view. It is clear that the lack of knowledge about the conditions that allow the matching mechanism to work properly, caused a failing system. Since the core of the procedure has the potential to be the gear of an efficient procedure, it is not necessary to replace it – unless other reasons are called¹³⁷. This means that the policy suggested to the MIUR would be just improving policies of the current system, and it would just necessary to adapt the managerial rules of the hiring process to the matching mechanism implemented. The tests carried out on the theoretic model showed that it is worthless to respect just one of the assumptions, since they are necessary but not sufficient conditions for the stability of the market for truthful reporting strategies. Even though the Ministry can decide to proceed one condition at a time, for reducing the degree of change, it is fundamental to act on all of them before claiming the efficiency of the market. However, the adjustments required will inevitably distort the current system, most of all its decentralized nature, since the central body would have more control over the procedure. In the next sub-paragraph, I illustrate the proposal of a design that solves the three points of failures simultaneously.

2.6.2 Re-Organization Proposal for an Efficient Decentralized Recruitment Procedure

The following proposal is not the only possible solution. It is based on the study of the International Market for Economists that involves the most important Colleges/Universities of the world each year. It is a decentralized market, since each Institution carries on its own hiring process. The procedure is not controlled by a central body, even though the Institutions found a way to organize the hiring process in a functional – and efficient – way. Since some of the solutions they adopted are interesting and relevant for our case study, I used it as a guideline model.

¹³⁷ As the beginning of Section III will show.

The following figure represents how the recruitment process should work after the implementation of this proposal.

Opening Vacancies	Academic Titles evaluation	INTERVIEWS and Final Evaluations	Closing Vacancies
Each University independently announces a job position	First screening of Candidates	Oral examinations of short-listed Candidates	Each University sends a job offer

Figure 5. Decentralized Matching Mechanism Proposal for the Italian Recruitment of Assistant Professors

Opening Vacancies: a determined period for all Institutions to publish a job announcement respecting all the mandatory requirements imposed by the Law in force. The duration of the call can vary by Institution, but its terms should be into the period designated for that operation. It would be not possible for any public universities to offer a position out of this timing period. During this phase, Institutions announce the vacancies and receive the applications. It reduces the timing costs experienced by candidates over the year to follow the different calls and make different applications at various timings.

Academic Titles Evaluation: by Law, the first screening of candidates is obtained by evaluating their academic curriculum. The ones who do not meet the mandatory requirements (e.g. holding a Ph.D.) are removed from the list, while the others are ordered and a short-list is achieved¹³⁸. The evaluation cannot last forever, but it should be ended before the deadline expiration. The deadline will be equal for all the Institutions into the process and decided by the Ministry.

¹³⁸ The total number of candidates onto the short-list must respect the norms included into the Art.24 of Law 240/2010 about the applicants admission to the oral examination.

Interviews and Final Evaluation: the Law in force requires Institutions to evaluate candidates during an oral examination. The new system would allow Skype Interviews, and/or the possibility of organizing an event as a big conference on Economics, where Institutions will have the chance to meet and interview the applicants. This is a crucial point of the proposal since it reduces the expenses of the travels necessary for sustaining the oral test at the University's site; it also avoids the restrictions on applications, since each candidate can be examined by multiple Institutions during the same day.

Closing vacancies: a determined period for all Institutions to send a formal job offer to the candidate they like the most. Candidates will have a determined amount of time for accepting or rejecting the offer. The aim is to give applicants the chance of comparing all the jobs offer and to choose the one they prefer. Also in this case, Institutions have to respect the timing indications of the Ministry, and cannot anticipate or delay the operation. As the winner of the competition accepts the composition, the hiring procedure is declared ended for that Institutions. Otherwise, the University goes on by offering the job to the second classified.

The suggestion is to implement a computerized program for the procedure in order to have a tight control over the respect of the timings. Moreover, all the operations would be electronically registered, and it would be easier and faster to have access to documents and files, and to check the accuracy of the process.

2.6.3 Other Solutions beside Re-Design: Incentives

The proposal above-presented is not the only possible redesign, as redesign is not the only possible solution to the inefficiency problem. An acclaimed theory – highly debated – considers incentives the best tool for achieving the efficiency. The incentives should be linked to the research indexes, quality, international recognition, etc. based on individual scores as number of citations, H-index, etc. The level of research produced into a department should drive the amount of financial resources it receives from the Ministry. This proposal goes toward an even more decentralized procedure than the current one, and aims to the complete

autonomy of Institutions in hiring procedures. The amount of money they will receive, will be the evaluation of their choices about the new entrants. It recalls the systems of the private institutions in countries as UK and USA – even if they also can rely on the expensive fees asked to students. However, the Italian universities are part of the public administration, and this is the reason why the access to a job position should be granted only by public competition. Moreover, the incentives' plans usually requires to have large financial funds available, and this is not really the case of the Italian Ministry of Education, University and Research. It also hard to exactly define how Institutions should be evaluated, since there is not a common idea about the factors that should be taken into account. Another aspect to take into consideration when designing the incentives' plan, is that by Law each University is economically sustained by the MIUR for a fixed amount of money each year. It would not unreasonable to believe that this fixed financial resource, can have the same effect of a minimum wage granted by the State to people without a job into the labor market. Some universities could find themselves accomplished by that amount, and not interested in raising more money by hiring promising researchers. Instead they could prefer to hire people they already know – even if they deserve the position or not – and to keep running the Institution with that fixed financial resource. The incentives' plan is just an *a posteriori* control over the performance of the Institution. Moreover, it does not solve any of the key conditions above-mentioned, necessary for obtaining stable results. Then, it is clear that this theory ignores the real key point of efficiency of the market, by focusing on the autonomy degree of Institutions and on financial resources. But are they or could they be the real factors of success and efficiency of this recruitment system? From a matching point of view, the answer is no.

SECTION III

PROPOSALS OF REDESIGN

The previous section ended with the presentation of a proposal for improving the decentralized recruitment system for Assistant Professors in Italy – the case study of this Market Design project. Even if the proposed re-organization of the procedure represents a valuable answer to the current inefficiencies, it is not the only possible solution. Instead of maintaining the decentralized structure, it could be formulated a different proposal based on a centralized procedure. The centralization process has been highly testified by the literature, and the success of the real redesign projects makes it a considerable alternative. Bettering of the recruitment carried out through a decentralized structure is a less invasive restyling of the assignment procedure, and still maintains that certain degree of autonomy of each University, considered particularly relevant. However, the decentralized matching could be difficult to manage – especially if the MIUR resulted quite mediocre in managing the system so far – and could be challenging to have the control over all the single hiring processes. The market could fail at any time, and it could be hard to keep the efficiency level established. On the other hand, a centralized structure guarantees the control over the entire procedure, and the simplicity of managing a clearly defined and transparent recruitment competition. Actually, the advantages brought by centralizing the system are numerous and will be listed in the next paragraph. The aim of this section is to illustrate a second potential solution for the Italian system, based on the total redesign of the hiring procedure. At first, it will be presented the traditional centralized model, such that the assignment process will be reported in the formal terms on a centralized matching market. However, a peculiar problem of this market will arise even after the centralization: the so-called Meritocracy Problem. Deeply explained in paragraph 3.3, it needs a tailored solution. The last paper contained into this

dissertation focuses then on the formalization of that problem – treated as an Agency Problem – into a matching mechanism. The paper shows how to recognize and include a meritocracy problem into the formalization of a recruitment procedure, and most of all, the danger of its potential consequences on the assignment. A suggestion of how to face this inefficiency in the case study is provided, and this last section ends with the proposal of a centralized recruitment procedure adapted for the meritocracy problem.

3.1 The Advantages of a Centralized Structure

By the word “centralizing”, we refer to a re-structural operation that re-organize a decentralized market – with a localized hiring procedure – to a system managed and control by a central body. A centralized structure is quite usual in School Matching markets, but also in some entry-level job markets (e.g. the Physicians’ assignment to Hospitals). The advantages brought by this structure are linked both to the managerial sphere and to the matching one. It allows a stronger control and a better monitor activity over inputs, outputs and procedures; it also make possible the implementation of some efficient matching mechanism as the Deferred-Acceptance algorithm (Gale and Shapley, 1962). So, just by centralizing – i.e. just by changing the structure of the system, the market will be better off under many points of view:

- A tight managerial control: a centralized structure allows a central body as the Ministry, to have a stronger control over the procedure. It is the opposite of being asked to monitor a multitude of single hiring procedures carried on by the various Institutions. In this case the MIUR is also the responsible of the design of the process, and to implement it – such that, it has not only the control but also the direction.
- A lower number of claims to Regional Courts: a better management and a tighter control should be sufficient ingredients for making the number of claims go down – together with an equal composition of national and local

committees. The reduction of complaints would avoid competitions to get stuck or to last more than expected.

- Reduction of waiting times: the centralized procedure entails determined period of time for any operation, and deadline must be respected for ensuring the functioning of the system. The first clear consequence would be the disappearance of waiting times between 1 and 2 years for knowing the response of a competition.
- Reduction of Universities' autonomy: at this point is already quite obvious that a centralized structure for Universities means to loose part of their autonomy into the hiring process. They will hold an important role in the selection of the candidate for the job position, but they will not be more free to decide when opening the vacancy, and all the timings about the hiring procedure.
- A unique contest in a sole location: the candidates will spend their money and their time for reaching only one location instead of multiple sites.
- A unique application for multiple vacancies: candidates will be asked to submit a list of preferences of the universities with an open position, and these preferences will be elaborated by the matching algorithm. The system asks them to complete just one application form, as to upload their documents one time, and then they will be sent to the institutions she reported into the preferences' list.
- A clear time-line, clear yearly academic job offer, clear composition of the committee: this makes the system transparent and equal as requested by the European Chart of the Researchers, and a market that could conquer gain the confidence of the players, a safe market.
- Safe market: candidates will feel free to express their true preferences; to reinforce this important feature the Candidates-proposing Deferred-Acceptance Algorithm will be used as matching algorithm¹³⁹ (Abdulkadiroğlu et

¹³⁹ According to the Theorem postulated by Roth (Roth 1985), it makes a dominant strategy, for all the candidates, to reveal their true preferences.

- al. 2005; Gale and Shapley 1962; Roth 2008).
- A thick market: the assumption is that once the system will be considered as reliable, more candidates will apply; the aim is also to avoid situations with a sole application for a job position in order to give at the university the possibility of a comparison. A recruitment procedure that is easy to understand could be also a potential attraction for foreign candidates.
 - Internationalization: through the survey, someone claimed the lack of an international large-scale effort of the current system. This perception could be confirmed by the very low level of hired foreign researchers. Even though the reasons could be different, the difficult of the application procedure could represent an entry barrier. The simplification of the entire system, thanks to the centralization, could represent also a first step of the Italian recruitment toward a European Researchers' market (that is the final aim of the European Chart).

3.2 The Formal Model of the Centralized Recruitment Procedure

As above-mentioned, part of the advantages of a centralized structure are linked to the matching mechanism that is possible to implement. The Deferred Acceptance or D-A algorithm, thanks to its excellent properties (Roth 2008), resulted to be the more suitable for this kind of market. Unfortunately, even for this algorithm, it is not possible to ensure, simultaneously, the strategy-proof also for both agents' sides¹⁴⁰. This paragraph presents in formal terms the Italian recruitment procedure for Assistant professors with a centralized structure. This formalization is necessary for describing the functioning of the algorithm illustrated by the following sub-paragraph. It is also necessary for comparing this structure to the decentralized one – at the center of the previous section – and to understand the differences between the two systems.

¹⁴⁰ “Impossibility Theorem: *No stable matching mechanism exists for which stating the true preferences is a dominant strategy for every agent.*” (Roth 1982)

Once centralized, the entry-level recruitment appears as a two-sided matching markets. It is constituted by public universities on one side and candidates for a job position on the other. As come out from the description of the market and of the current system, since institutions are not simply applying a regulation, they could be actually considered strategic players. The salary is the same for every academic institution, so it can be not taken into account. If we assume that each university has only one job role, it results as a one-to-one matching market.

Let be U the non-empty and finite set of Universities, $U = \{u_1, u_2, \dots, u_n\}$, and C the non-empty and finite set of Candidates, $C = \{c_1, c_2, \dots, c_n\}$. By assumption, the quantity of job positions of each university, denoted by q_i , is equal to 1, so that $q_i = 1$ ¹⁴¹. Each agent has complete, transitive and strict preferences over the individuals on the other side that will be reported in their preferences' orders: let be P_u the preferences of Universities over candidates, $P_u = \{c_1, c_2, \dots, c_n\}$, and P_c the preferences of candidates over universities, $P_c = \{u_1, u_2, \dots, u_n\}$ where the order represents a strong preference relation, e.g. $c_1 \succ c_2$. Let P denote the preference profile. The outcome of the game is a matching μ : a one-to-one correspondence from the set $U \cup C$ onto itself of order two¹⁴²; $\mu^2(x) = x$ means that if $\mu(u) = c$ then $\mu(c) = u$. Agent's preferences over outcomes are determined by their preference for their own mates at those outcomes. A matching is stable if it is not blocked by a pair or by an individual (i.e. individual rationality). On the contrary of a decentralized market, this is a revelation game, i.e. agents reveal their preferences by reporting them into a list that have to submit to a central clearinghouse. The clearinghouse will elaborate the preferences, usually by implementing a matching algorithm. Another difference with respect to a decentralized mechanism, in this case agents do not interact between each other, but just with the clearinghouse. Moreover, this is not a sequential game and so it is not dynamic, such that during

¹⁴¹ Even though a University would announce more than one job position, it can be treated as a one-to-one matching anyway by following the example of Gale and Sotomayor (Gale and Sotomayor 1985) who treated the College Admissions problem as an extension of the Marriage Stability (Gale and Shapley 1962). By maintaining the quota always equal to 1, the problems caused by false declaration about the universities' capacity, the ones demonstrated by (Sönmez 1997), are eliminated. Moreover, to maintain the quota equal to 1 is very realistic, there are just few exceptions.

¹⁴² (Roth and Sotomayor 1992)

the analysis of the mechanism functioning we look for pure strategies Nash Equilibrium. The mechanisms used into a centralized procedure can be various and also very simple, as the scrolling down of a rank, where the first choose her assignment and so on till the last of the list. The rival of D-A is usually the so-called Boston Mechanism¹⁴³. Each matching mechanism has its own properties about stability, efficiency and strategy-proofness on which the choice of a market designer relies.

3.2.1 The Candidates-proposing deferred-acceptance algorithm

After having deeply analyzed the Gale-Shapley theory and the practical works of Roth et al., the deferred-acceptance algorithm has been considered the right algorithm at the basis of a centralized recruitment procedure. Specifically, the deferred-acceptance algorithm takes into consideration the one formulated by Gale and Shapley in 1962, as well as the algorithm that Roth et al. used in 2003 for the New York City High Schools case (Abdulkadiroğlu, Pathak, and Roth 2005) and for the American Medical Labor Market (Roth 1984a, 2008; Roth and Peranson 2002).

- Step 0: Candidates and universities submit their preferences to the central clearinghouse and the lists are immediately updated by removing unacceptable matchings (i.e. from the researchers' lists are removed the universities that didn't rank them and from the universities' lists are removed the researchers that didn't rank them).

$$\begin{aligned}
 C &= \{c_1, c_2, c_3\} & U &= \{u_1, u_2, u_3\} \\
 P(c_1) &= \{u_2, u_1, u_3\} & P(u_1) &= \{c_3, c_1\} \\
 P(c_2) &= \{u_1, u_3, u_2\} & P(u_2) &= \{c_1, c_2, c_3\} \\
 P(c_3) &= \{u_3, u_1, u_2\} & P(u_3) &= \{c_2, c_3, c_1\}
 \end{aligned}$$

- Step 1: each candidate applies to its first-choice and universities hold the offers of their top-ranked candidate

¹⁴³ (Abdulkadiroğlu, Pathak, Roth, et al. 2005)

$$c_1 \rightarrow u_2 ; c_2 \rightarrow u_3 ; c_3 \rightarrow u_3$$

since $c_2 \succ_{u_3} c_3$, c_3 is rejected

- At each step the universities preferences' lists are updated: all the researchers that are less preferred than the one to whom they are tentatively assigned are removed. It is not possible to updated at the same way the candidates' lists because it could happen that after a tentative of assignment his/her proposal is rejected in favor of a more preferred one. In this case, at the next step the applicant who has been rejected goes on applying to her next preference, so their lists are updated only by removing universities from which they were rejected.

$$P(c_1) = \{u_2, u_1, u_3\} \quad P(u_1) = \{c_3, c_1\}$$

$$P(c_2) = \{\cancel{u_1}, u_3, u_2\} \quad P(u_2) = \{c_1, \cancel{c_2}, \cancel{c_3}\}$$

$$P(c_3) = \{\cancel{u_3}, u_1, u_2\} \quad P(u_3) = \{c_2, \cancel{c_3}, \cancel{c_1}\}$$

- Step 2: each rejected candidate applies to its next preference and universities hold the most preferred candidate and reject the others (i.e. if during this phase a university receives the application of an applicant that it prefers more than the one who applied in the first phase, this application is now rejected and the new is hold)

$$c_3 \rightarrow u_1$$

- The algorithm stops when all candidates are assigned, and/or there is no rejected candidate that can still apply to at least to one university, i.e. after that candidates' lists were updated, all the lists of the rejected ones (all the applicants who result to be not tentatively assigned) do report any university but only the candidate herself (since she has not been assigned to any institution, she is assigned to herself).

$$\mu = \{(c_1, u_2), (c_2, u_3), (c_3, u_1)\}$$

The deferred-acceptance algorithm demonstrated to produce stable matchings in cases quite similar to this kind of two-sided matching market. Also in this case it has revealed to be the best suitable algorithm that ends with a stable allocation (no agents who are not paired up together would both prefer a different matching) of candidates to institutions. Gale and Shapley (1962), demonstrated that the core of a two-sided market is never empty, such as there always exists a stable allocation – at the D-A will achieve it. One of the properties of the D-A is that the proposing side achieves its optimal allocation – that corresponds to the worse one for the other side, except when the game has a singleton core. Deciding the proposing side then means to give an advantage to one part of the market. Due to the guarantee of achieving the best outcome possible, the proposing side has the dominant strategy of truthfully reporting the preferences into the submitted list (Dubins and Freedman 1981; Roth 1982b). The same does not hold for the opposite side of the market, that can have the temptation to strategize the game. Then, when deciding what side will start to propose is not only about the final outcome, but also about which side of the market have under control from a strategy point of view. This is the main reason why Candidates have been chosen to start proposing in this version of the D-A. Traditionally, employers propose first in labor markets, but in my opinion it is more important to safeguard the safety of candidates in this peculiar case. More about this decision is illustrated in the final paragraph of this section.

3.3 The Meritocracy Problem

By “Meritocracy Problem”, I refer to any situation when an undeserving candidate has been declared as winner of a public competition. Real cases showed that this could happen due to family relations, or even simpler due to previous work relations or network of relations. Assuming that the nepotism due to family relations has been abolished thanks to the new rule on the family degrees between member

of the committee and candidate – even if Perotti showed that this is not enough (Durante et al. 2011; Perotti 2008; Perotti et al. 2009) – I will be more preoccupied on the case of so-called “internal candidates”. As internal candidates, commonly people refer to the ones who have already worked for the university that is opening the positions, and most of the times, they are still working there at the time of the job announcements. The internal candidates are usually preferred to external ones – because they are already well known and they are probably still working on ongoing projects – and sometimes, committee members could feel more comfortable in hiring one of them instead of the most deserving candidate. The perceptions about internal candidates – and what the unsafety they cause to the market – is well reported by the comments of some respondents of the survey launched in 2014 for measuring the satisfaction level of current researcher – who have already passed through this procedure. Since this is not a mechanism feature, it cannot be solved just by implementing the Deferred Acceptance algorithm, such that the centralization and the selected matching mechanism are not enough for solving the inefficiencies of this market. The next formal example shows the meritocracy problem in the context of a centralized structure that implements a researcher-proposing D-A algorithm.

- The market is a two-sided one-to-one asymmetric matching market and we assume for the example that it has already a centralized structure
- The regulations' guidelines of every university: choose the deserving candidate, based on their research productivity and academic curriculum, who is the more appropriate for the research program of the university. While the firsts are quite objectively estimable, the last needs a subjective judgment.
- There are five candidates, $C = \{c_1, c_2, c_3, c_4, c_5\}$ and two universities, $U = \{u_1, u_2\}$, each universities has one vacancy $q_{u_1, u_2} = 1$
- On the basis of the evaluations given by a central committee, candidates are ranked as c_3, c_1, c_4, c_2, c_5 ; c_4 has the mean score, c_3 and c_1 have high scores and c_2 and c_5 have low scores.

- The agents c_3, c_4, c_5 express the same preferences $P_{c_3, c_4, c_5} = \{u_2, u_1\}$; agents c_1, c_2 express the same preferences $P_{c_1, c_2} = \{u_1, u_2\}$
- Universities have to submit their preferences: $P_{u_1} = \{c_1, c_3, c_4, c_2, c_5\}$
 $P_{u_2} = \{c_2, c_4, c_1, c_3, c_5\}$
- The Candidates-Proposing D-A algorithm¹⁴⁴ runs:
 - Each candidate (c) proposes to his/her first preference (u)

U1 receives the proposals by c_1 and c_2 , since $c_1 \succ c_2$, u_1 keeps the proposal of c_1 and rejects the other ; U2 receives the proposals by c_3, c_4, c_5 and it holds the proposal by c_4 .

- All the rejected candidates propose to their second preference

U1 rejects all the proposes and continue to keep the one by c_1 that is its first choice; U2 receives the proposal by c_2 , holds it and rejects c_4

- The final matching¹⁴⁵ is: $\mu = \{(u_1, c_1), (u_2, c_2)\}$

The results are stable because, according to the lists presented, no one subjects in the matching would prefer another agent to be coupled with; but, actually, the preferences of u_2 do not correspond to what reported in its regulation. The implication is: if we simply look at the preferences' orders inserted into the mechanism the results can seem to be stable and efficient, only after an in-depth analysis (studying the regulation of the universities and looking at the choices operated by the committees) it is possible to understand that the market is suffering of misrepresentation of preferences.

- The hiring of one of the two candidate, c_2 , is not aligned to the regulative guidelines so there is a misrepresentation of its preferences that causes the market failure.

¹⁴⁴ I choose to use the candidates-proposing D-A algorithm in order to exploit the property of strategy-proofness for the side that starts proposing; in this way we are sure that for candidates it is a dominant strategy to reveal their true preferences (Roth and Sotomayor 1992).

¹⁴⁵ «A matching is a subset of $U \times C$, i.e. a set of matched pairs, such that any candidate appears in no more than one pair, and any university appears in no more than q_u pairs.» (Roth 2002)

3.3.1 The Meritocracy problem as a Failing Agency Relation

It was indispensable to translate the meritocracy problem into formal terms and to integrate into the matching model in order to analyze its effects on the functioning of the assignment mechanism. The next paper focuses on this research challenge: it formalizes the meritocracy problem, but it also shows how and where recognize it, and its potential (dangerous) consequences, together with a list of possible solutions.

The Agency Problem into Two-Sided Matching Markets

E. Quintilii

Abstract

The aim of the paper is to formalize the agency problem (Ross 1973b) into hiring procedures modeled as two-sided – one-to-one – matching markets, and to show what are the possible consequences in a matching procedure. Any Recruiter is intended as made up by two players – the formal and the representative – who act as being a sole agent into the game. The misalignments of interests between the two originates a failing agency relation that corresponds to a misrepresentation of preferences – defined as an unfair action committed by the representative – into the mechanism. I demonstrate that a market that suffers of unfair actions produces unstable results up to the real preferences – i.e. the hiring market is said to be unfair. The procedure of recruitment of Assistant Professors in the Italian Academic Job Market is used as a case-study to make the above statements clearer. The possible solutions are multiple and they vary according to the characteristics of the markets.

Keywords: *matching market, agency problem, unfair action, two-sided matching, misrepresentation, preferences, decision rules, recruitment process, Academic market*

(*) This paper has been developed during the visiting period at the Behavioral and Experimental Department of the University of Zurich in 2015. Special thanks to Prof. Roberto A. Weber for helping me focusing on the most important features that make this project interesting for the “matching community”, and for his “Behavioral Economics influence”, that enriches this paper.

Introduction

During the last thirty years, the practice of market design has been gaining a certain relevance in the field of labor markets. In 1984 A. Roth (Roth 1984) for the first time applied the matching procedures theorized by L. Shapley and D. Gale (Gale and Shapley 1962) to the American market for medical interns and residents. Their success paved the way to further studies both theoretical and practical. A matchmaker analyzes a problem of resources' allocation in markets where payments and prices are not possible, in order to find the matching mechanisms that will guarantee the best solution: an allocation that is at the same time stable and efficient. The stability condition portrays a final outcome where all agents are matched to someone they like more than being alone, and there are no two agents who would like to be paired with each other instead of their assigned mates. Despite all the research works carried out so far (Kojima 2015), there are still open questions and unexplored features that can characterize a matching mechanism. The aim of this paper is to bring to light one of them in the framework of the two-sided matching markets in the setting Recruiter-Candidate, by starting to focus on the decision rules implemented by the Recruiters. I detail three kinds of procedure that differ for the degree of freedom of choice exerted: 1) regulated by rules – as it happens for Boston schools (Abdulkadiroglu et al. 2006; Abdulkadiroğlu, Pathak, and Roth 2005); 2) personal choice (Marriage Problem Gale and Shapley 1962); 3) semi-regulated, i.e. relies on a set of general rules and on personal judgments. I define Semi-Regulated the two-sided matching markets where the Recruiters' side implement the latter decision process and I formalize its model. The difference with a traditional model stays in how the Recruiters create the preferences' ranks. The semi-regulated decision rule implies a two-step process carried on by two players who act as being one (the Recruiter). Specifically, in the first step Recruiter's rules on recruitment are applied to identify the acceptable Candidates and to rank them in classes of indifference on the basis of how well they satisfy the requirements contained in the regulation. In the second step an active player – who acts in behalf of the Recruiter – has to intervene to break the ties and to rank the applicants in an order of strict preference. The similarity with the agency

theory (Eisenhardt 1989; Ross 1973) drove me to implement it for a better understanding of the nature of agents' relation and actions. I will refer to the set of hiring rules as the formal (principal) since they embody the expression of the Recruiter will, and to the one who actively order the Candidates as representative (agent). As stated by the agency studies (Donaldson and Davis 1991), it could happen that the interests of a representative are misaligned to the formal's ones. In this case the former could choose to ignore the mandatory rules expressed by the latter, and to formulate a list of preferences based only on private judgments. This would be a violation of the agreement between the two parts but most of all a misrepresentation of Recruiter's preferences that I define as an "unfair action". As recognized by the main authors in the field of market design (Abdulkadiroğlu et al. 2005; Roth 2007) , the truthfulness of agents' declaration is a key element for the well-functioning of the matching mechanism, so either the presence of an unfair action could represent a dangerous threat of failure for the entire market. From the literature we already know that strategic behaviors of Recruiters can be the causes of manipulations (Abdulkadiroglu and Sonmez 2010; Roth and Sotomayor 1992; Roth 1982; Sonmez 1997) besides strategic actions as prearranged matches (Kojima and Pathak 2009), but in this case the preferences' misstatements of the Recruiter is totally involuntary. This is why it is so important to identify the structure of an agency relation: treating the Recruiter as a sole player hides the true responsible of the misrepresentation. Moreover, the reason at the basis of the misrepresenting behavior is that representatives are manipulating the lists for obtaining the best for themselves. I briefly describe the Italian recruitment procedure for Assistant Professors as a case-study, in order to present a real-world market to which the theory of semi-regulated market applies. Furthermore, the case study gives me the possibility of highlighting the implications of a failing agency relation, especially when the Recruiter is a public Institution. In this case the rules of recruitment are known by anyone and the unfair actions committed by the representatives are easily detectable, making the applicants aware of the vitiated mechanism. So, even though a Candidate-Proposing Deferred-Algorithm algorithm allows to obtain stable outcomes up to the revealed preferences, the

instability up to the real preferences could emerge. I show that the agency failure has consequences not only directly on the mechanism – whose outcome is no longer stable up to the real preferences – but on the market as whole, either on individuals' behaviors, together with the risk of compromising the entire system due to distortions it created. It is not possible to propose a single solution since the solution will vary by market but a list of possible actions is provided before conclusions.

In the first section I briefly go through the theoretical background of the matching mechanisms and of the market design. I introduce the different kinds of decision rules in the second section, with a special focus on the principal-agent decisional process and a brief characterization of two-sided matching market based on them. I formalize the agency problem and unfair actions in the third section and I illustrate the case-study in the fourth as application of the theory. I show the possible consequences in the fifth section with a special attention for the case of public Institutions on the Recruiters' side. I detail the multiple solutions that can be employed in the last section and further developments of the work are briefly presented in the conclusions.

I. Theoretical Framework

In the so-called matching markets, since payments and prices are not involved, the process of allocation relies on the reciprocal choice of the agents: they can be coupled if and only if both have signaled the other in his/her preferences. During the 1950's L. Shapley started to analyze this kind of markets by focusing on the outcomes' characteristics, such as stability and efficiency, obtained by the process of allocation. In the paper "College Admissions and the Stability of Marriage" (Gale and Shapley 1962) for the first time, concepts stemmed from mathematical matching problems and from game theory, were theoretically applied to markets. The authors demonstrated that it was possible to build up processes of allocation (matching mechanisms) that would guarantee stable outcomes and efficient results in terms of allocation. The work started by Gale and Shapley has been

carried on by different authors, but the one who mostly contributed to the definition of the theoretical basis on this new field of studies was A. E. Roth (Roth 1985). The theorems postulated by these three main authors established the robust basis not only for the matching theory but also for the new field of *economic engineering*. The practice of *market design* (Roth 2010, 2007) is one of these main activities: economists are called not only to analyze matching markets, but also to re-design the structure of the inefficient ones (Roth 2002). This area of study represents the practical realization of what Gale and Shapley proposed only from a theoretical point of view, and Roth has been recognized as the pioneer of this new branch. Themes from microeconomics and game theory are used as tools together with the theories of matching (Roth 2000). Particularly, from game theory stemmed the idea of building up efficient markets that rely on a set of detailed rules. The successful applications on real-world matching markets (Jackson 2013), designed by Roth in collaboration with other authors (Abdulkadiroğlu, Pathak, and Roth 2005; Abdulkadiroğlu, Pathak, Roth, et al. 2005; Roth 1984; Roth et al. 2005), revealed the practical value of the field and the great impact on the human welfare that could be obtained by a well-functioning re-structure of inefficient markets. These real cases are integral part of the literature and, mostly, they were able to foster further studies. The markets at the center of the research are usually the college/school admissions and the entry-level labor markets (Roth 1991a), the branch to which also this paper belongs. Nevertheless, each example, even theoretic or practical, brings out something new and points the attention on characteristics that could be then generalized.

How players building up their preferences' lists is the starting point of this research study and the concepts of affirmative actions (Abdulkadiroğlu 2005) and representativeness (Abdulkadiroğlu 2005; Martinez Ruth et al. 2000) properties have been fundamental for the development of the formalization of the semi-regulated decision process and of the unfair actions. The probability of manipulation of a matching by the misrepresentation of preferences determines the well-functioning of the market: if the outcomes are based on false declarations,

they will not be truly stable and so the market will fail in its optimal allocation mission (Roth 1982; Roth 1984b; Roth 2007; Roth 2008). The paper shows that other kinds of manipulation are possible in addition to the ones already known (Roth and Rothblum 1999; Kojima 2015; Roth 1984b), and that the sole implementation of the traditional tools, sometimes, could be not sufficient. When the matching mechanism is not able to stop manipulations, it is necessary to build up the suitable market structure to impede them. In these cases is also useful the implementation of behavioral theories for a better understanding of the agents, and of experimental economics techniques (Kagel and Roth 2000), mostly to confirm the conjectures when a real application is not achievable and/or to test the design before practically realize it.

The value of the paper stems from adding something more at the literature debate from a theoretic and a practical point of view about two-sided matching markets, involving also elements from other economic branches as corporate theories, behavioral theories and laboratory experiments.

II. Identifying the origin of the Agency Problem

2.1 The Different Kinds of Decision Rules

The expression *decision rules* refers to how agents build up their preferences' orders – also called *decision process*. It is interesting to focus on the degree of freedom (δ) that the Recruiters exercise during this process. Some players are completely free and their choices are totally based on personal interests, others suffer from the constraints of a regulation that guides them or dominates the process. It is possible to distinguish between three kinds of decision-rules:

Definition 1. "Private decision-rules": the rules followed by the agents during the choice are related to the personal sphere and the agents respond only to personal tastes [maximum degree of freedom, $\delta = 1$].

The criteria are completely subjective, they are totally unknown by others.

Definition 2. “Regulated decision-rules”: the rules followed by the agents during the choice are detailed by their own regulations (considered as a complete set of commands to be aligned with) and the agents have to strictly apply their requirements [minimum degree of freedom, $\delta = 0$].

Under these conditions, the act of listing preferences could be carried out by building up a suitable algorithm, i.e. individuals are not necessary. This is one of the forms usually implemented by public Institutions as schools. Due to the impersonality of the preferences' ordering process, doubts about the role of the players who follow this decision rule could arise. This already happened during the evaluation of the Boston school mechanisms, and the authors of its redesign concluded by retaining schools as active agents that did not strategically play (Abdulkadiroğlu et al. 2005). Starting from this statement, we can describe the subjects who rely on regulated decision rules as non-strategic players. This means that they will always express their true preferences and will never try to manipulate the mechanism in anyway. Moreover, their preference profile is common knowledge.

Definition 3. “Semi-regulated decision-rules”: the rules followed by the agents during the choice are related both to a less-restrictive regulation and to personal judgments. They have to follow the guidelines contained in the regulation but they are also required to implement them with their own opinions [medium degree of freedom, $0 < \delta < 1$]. This process is deeply explored in the next section.

2.2 The Semi-Regulated Decision Process

The Semi-Regulated Decision Rule, entails a two-steps decision process:

I step. Regulated decision-rules represented by a set of mandatory guidelines defined by the Recruiter¹⁴⁶: they allow to restrict the number of agents who can be chosen by defining characteristics they must possess. The regulation identifies the

¹⁴⁶ It can be both a private or a public Institution.

kind of agents on the other side of the market that they prefer but they are not able to individually rank them but only to create classes of indifference.

II step. Private decision-rules represented by personal opinions of the agents involved: they have to judge the subjects who satisfy the regulation constraints in order to strictly rank them in a list of preference. Choices have to be aligned with the guidelines given by the regulation (i.e. they have to break the ties into the classes of indifference).

The entity “Recruiter” or Institution is formally the player involved in the game but it is not able to complete all the tasks required by the matching mechanism when its regulation is made up by only guidelines and not detailed rules. For this reason, the two-steps process is formally realized by the entity Recruiter but practically it completes just the first step by providing the regulation. The second step requires the help of its members who will act in the name of it, following its instructions. Then it is possible to identify two different players inside the same agent for carrying on this decision-process:

1. The first step is dominated by the set of mandatory guidelines. They embody the spirit and the general preferences of the agent and represent its direct action. The player is the Institution and I will refer to it with the term “*formal agent*”.

Definition 4. We define the **formal agent** (or simply *the formal*) as an active but not strategic player who is directly involved into the game but that cannot fully play the role by herself.

2. The second step is dominated by the individuals who have the task of implementing the regulation, they are “*representatives*” of the formal agent.

Definition 5. We define the **representative agent** (or simply *the representative*) as a strategic player who is indirectly involved into the game and who is active only in a dependent way.

With the expression “*active in a dependent way*” I want to underline that their decisions depend on formals' rules besides the extent of their actions (i.e. the

degree of freedom that they exercise) and their interests have to be aligned with those of the subjects they represent.

It is important to highlight that each regulation is different, so the formals' rules could be more or less specific and, as consequence, the degree of freedom of the representatives could get any value between 1 and 0¹⁴⁷, and it has to be evaluated by each situation.

It seems obvious to make a recall of the agency theory (Donaldson and Davis 1991; Eisenhardt 1989; Ross 1973), one of the main theory in the field of corporate governance. As reported in (Ross 1973) an agency relationship arises *“between two parties when one, designated as the agent, acts for, in behalf of, or as representative for the other, designated the principal, in a particular domain of decisions problems”*. It is quite simple to recover the same scheme in this model. Given the similarity, the agency theory is taken into account for a better understanding of the relation between the Institution (or formal) and the committee (or representative).

The core question is: what are the incentives that bring the representatives to act or not in accordance to the formals' interests? The answer could be fundamental in order to understand if a blocking-action is necessary – i.e. the incentives are not strong enough for representative to act fairly. In the corporate governance field, the agency theory is generated when ownership and control are separated: referring to the case of matching market we could say that the “ownership” is exercised by the formals that establish requirements for the hiring procedures and the “control” by the representatives who practically select the subjects and rank them in a preference list. According to the Agency theory *“if both parties to the relationship are utility maximizers, there is good reason to believe the agent will not always act in the best interests of the principal”* (Donaldson and Davis 1991).

Since being rational and utility maximizers are the basic assumptions of any strategic game, the theory tells us that sometimes (not always) the representatives

will act following their own interests instead of the formals' ones. In a matching procedure, the utility value depends on with whom you have been paired: the higher her position in the preference list, the higher the value of utility obtained (i.e. when agents are linked with their first preferences their utilities have their maximum value). If the representatives want to maximize their personal utilities (and not the formals' utilities) they will misrepresent the agent's personal preferences by going over the boundaries marked by the regulation, and submitting their preferences. The outcomes will be stable up to the preferences presented, but actually unstable because based on false declaration. In the next section these statements are formalized in the model of a Semi-Regulated two-sided matching market.

2.3 Characterizing Two-Sided Matching Markets by Decision Rules

According to the different kinds of decision rules that the Recruiters could implement to create their preferences' lists, it is possible to characterize the two-sided matching markets as:

- **Private:** the Recruiter relies upon personal decision rules – (e.g. Marriage problem¹⁴⁸).
- 2. **Regulated:** the Recruiter relies upon regulated decision rules, the criteria are completely objective and pre-determined.
- 3. **Semi-Regulated:** the Recruiter relies upon semi-regulated decision rules, the criteria are both subjective and objective.

Furthermore, in the case of the treatment of the agency problem, it could be useful to specify when the Recruiters' side is made up by Public Institutions, because more information on the market could be provided:

2* Public Regulated: the rules are totally known or knowledgeable by the agents on the other side – (e.g. Boston School Mechanism¹⁴⁹).

¹⁴⁸ (Gale and Shapley 1962).

¹⁴⁹ (Abdulkadiroglu et al. 2006).

3* Public Semi-Regulated: the rules are partly known or knowledgeable by the agents on the other side– (e.g. Italian recruitment procedure for Assistant Professors¹⁵⁰)

While the firsts – private and regulated – have been deeply studied along the years, the model of the Semi-Regulated or SR matching markets is presented in this paper for the first time in the next section. Recognizing its structure in a real market allows the authors of the new design to ask themselves about the agency relation and about its possible effects on the matching procedure.

III. The Formalization of the Agency Problem

3.1 The model of the Marriage Problem

Since the matching process of the case-study involves Universities and Candidates for an assistant professor position, this framework will be used for building up the model. Traditionally, one-to-one two-sided matching market is formalized as follows.

Let be U the non-empty and finite set of Universities, $U=\{u_1, u_2, \dots, u_n\}$, and C the non-empty and finite set of Candidates, $C=\{c_1, c_2, \dots, c_n\}$. By assumption, the quantity of job positions of each University, denoted by q_i , is equal to 1, so that $q_i = 1$ ¹⁵¹. Each agent has strict preferences over the individuals on the other side that will be reported in their preferences' orders: let be P_u the preferences of Universities over Candidates, $P_u = \{c_1, c_2, \dots, c_n\}$, and P_c the preferences of Candidates over Universities, $P_c = \{u_1, u_2, \dots, u_n\}$ where the order represents a strong preference relation, e.g. $c_1 \succ c_2$. A Candidate $c \in C$ is acceptable for $u \in U$

¹⁵⁰ Section V of this paper.

¹⁵¹ Even though a University would announce more than one job position, it can be treated as a one-to-one matching anyway by following the example of Gale and Sotomayor (Gale and Sotomayor 1985) who treated the College Admissions problem as an extension of the Marriage Stability (Gale and Shapley 1962). By maintaining the quota always equal to 1, the problems caused by false declaration about the Universities' capacity, the ones demonstrated by (Sönmez 1997), are eliminated. Moreover, to maintain the quota equal to 1 is very realistic, there are just few exceptions.

if $c \succ_u \emptyset$; a University $u \in U$ is acceptable for $c \in C$ if $u \succ_c c$, i.e. u is preferred to stay unmatched.

The outcome of the game is a matching μ : a one-to-one correspondence from the set $U \cup C$ onto itself of order two¹⁵²; $\mu^2(x) = x$ means that if $\mu(u) = c$ then $\mu(c) = u$. Agent's preferences over outcomes are determined by their preference for their own mates at those outcomes. A matching is stable if no agents involved prefer a different mate respect to the one they ended with, i.e. a matching is stable if it is not blocked by any pair of agents.

3.2 The model of the Semi-Regulated Matching Market

Let consider each u as a combination of two inside players, the formal $f \in F$, and the representative $r \in R$, such that $u = (f, r)$. Let define as F the non-empty and finite set of formals, $F = \{f_1, \dots, f_n\}$, where f_i is the one related to u_i , and P_f is the set of preferences expressed by each formal. Due to the general preferences they express on the Candidates, in their lists they order classes of Candidates, $P_f = \{[c_1, \dots, c_n], \dots, [\dots, c_{n+n}]\}$. Each class is characterized by the relations of indifference between all the subjects in the class represented by the square brackets, such that $c_1 \sim c_n$. Let be $C' \subset C$ the finite set of most preferred Candidates, $C' = \{c_1, \dots, c_n\}$ and let identify the following less preferred class of Candidates as C'' , C''' and so on. Then the rank of Candidates realized by the formal could be written as $P_f = \{C', C'', \dots, C^n\}$ for $C', C'', \dots, C^n \subset C$, $c_1, \dots, c_n \in C'$ and $C' = (c_1 \sim \dots \sim c_n)$, such that for every $c' \in C'$ and $c'' \in C''$, $c' \succ_f c''$ and $C' \succ_f C''$. I impose to the formals' preferences the following property:

Individual Affirmative Action (IAA) Property¹⁵³: for $f \in F$, P_f satisfies IAA if for every $c', c'' \in C$ such that c' respects (all) the requirements at f and c'' does not respect (all) the requirements at f , $c' \succ_f c''$. If c' respects (all) the requirements as well as c^i , then $c' \sim_f c^i$ and $c', c^i \succ_f c''$.

¹⁵² (Roth and Sotomayor 1992).

¹⁵³ This is a restriction of the one Affirmative Action property introduced by A. Abdulkadiroglu (2005); the class does not need to respect type-specific quotas but just type-specific requirements required by the regulation represented by f .

The capacity constraints, considered in the original paper by Abdulkadiroglu (2005), is excluded since $q = 1$ is imposed and the affirmative action constraint refer to the mandatory/non-mandatory requirements imposed by f only. Furthermore, the original concept refers to affirmative action over set of students, where a set is a combination of students which respects type-specific quotas constraints. In this case each Candidate has to accomplish to the requirements of f , so the affirmative action is respected directly on individuals.

Once the relations between all the Candidates are established, the classes are created by a

Class-Maker Action (CMA): for $f \in F$, P_f is build up by a class-maker action if for every $c', c^i, c'' \in C$, such that $c' \sim_f c^i$ and $c', c^i \succ_f c''$, c' and c^i are assigned to the same most-preferred class C' and c'' to a less-preferred class C'' , for every $C', C'' \subset C$ ¹⁵⁴.

For f , any $c' \in C'$ is perfectly substitutable to any other $c^i \in C'$ and always preferred to any $c'' \in C''$. The Candidates who do not accomplish at least one of the mandatory requirements are considered unacceptable and removed from P_f , such that P_f ranks in classes only acceptable Candidates, for any c' ranked in P_f $c' \succ_f \emptyset$. The preferences of the formals have to respect the property of restricted responsiveness (Abdulkadiroglu 2005), i.e. for $c' \in C'$ and $c'' \in C''$, $C' \succ_f C''$ if and only if $c' \succ_f c''$.

Let define R the non-empty and finite set of representatives, $R = \{r_1, \dots, r_n\}$, where r_i is the one related to u_i , and P_r is the set of preferences expressed by each representative. The representatives formally have the role of breaking the ties within the classes $C', C'', \dots, C^n \subset C$ identified by P_f . They substitute the relations of indifference generated by the incompleteness of the regulation with a strict preference order such that $C' = (c_2 \succ_r c_1 \succ_r \dots \succ_r c_n)$ instead of $C' = (c_1 \sim \dots \sim c_n)$, but

¹⁵⁴ Special cases as one Candidate per class or all Candidates in a sole class are admitted while empty classes are excluded.

the classes' orders have to be respected, $P_f = \{C', C'', \dots, C^n\}$. I impose that P_r have to show the

Prior Responsiveness (PR) Property¹⁵⁵: for $r \in R$ and for $f \in F$, P_r satisfies PR if for every $C', C'' \subset C$, for some $c' \in C'$ and $c'' \in C''$, we have $c' \succ_r c''$ if and only if $C' \succ_f C''$.

Definition 6. A representative $r \in R$ is prior-responsive if for any two Candidates $c', c'' \in C$, the Candidate who belongs to the most-preferred class identified in P_f where $C' \succ_f C''$, for $f \in F$, is ranked before the Candidate who belongs to a less-preferred class into P_r , such that for $c' \in C'$ and $c'' \in C''$ $c' \succ_r c''$.

PR imposes responsiveness of P_r to the previous rank contained in P_f . This definition reverses the concept of responsiveness (Martinez Ruth et al. 2000) since a Candidate is preferred to another for the classes she belong to – while in the above mentioned paper it expresses the opposite – and adapts it to a framework with an agency relation. The agent is no longer responsive to her personal preferences over individuals/subsets of them but to the rank of the previous player (“prior”) with whom she plays as a sole entity (the University).

Proposition 1. When the Universities' preferences satisfy the property of IAA, RR (the formals) and PR (the representatives) they are truly reported and the matchings are truly stable.

3.3 Unfair Actions as a new kind of Preferences' Misrepresentation

As seen in the previous section, according to the agency theory, there is the risk that the agents will act for maximizing their utilities instead of the principals' ones. Specifically, there is the probability of a misalignment of interests between the principals and the agents that drives the latter to follow private profits. The same could happen in the matching framework: the judgements of the representatives could be affected by personal interests or relationships and they could be brought

¹⁵⁵ Recall the Restricted Responsiveness by A. Abdulkadiroglu (2005) and the q_F -responsiveness by Martinez et al. (2000)

to modify the preferences' insertion. The first main consequence of the failure of an agency relation into a matching framework is then the misrepresentation of the Recruiters' preferences by the representatives. Specifically, the ranks are misstated when at least one of the properties above mentioned – IAA, RR, PR – are not respected.

Proposition 2. When the Recruiters' preferences do not satisfy at least on the properties IAA, RR, PR they are not truly reported and the final matchings are unstable up to the true preferences.

If IAA is not satisfied: c'' is preferred to c' even though she does not accomplish the requirements as well as c' , then the CMA assigns c'' to a most-preferred class and c' to a less-preferred one such that at the end of the decision process c'' will be ranked higher than c' . If RR is not satisfied: C'' is preferred to C' even though for some $c' \in C'$ and $c'' \in C''$, $c' \succ_f c''$; being P_r prior-responsive, at the end c'' will be ranked higher than c' . If PR is not satisfied: for every $C', C'' \subset C$, $C' \succ_f C''$ such that for some $c' \in C'$ and $c'' \in C''$, $c' \succ_f c''$, $P_r = (c'', c')$. The outcomes of the matching mechanisms will be stable up to the stated preferences, but unstable up to the real ones. As highlighted by Roth and Rothblum (1991), the misrepresentation causes a chain of reactions of rejections, and lead the results to be unstable because under the true preferences, any agent could have been coupled with an higher ranked player of the other side.

Since I have defined the formals as agents who do not play strategically¹⁵⁶ – with the term formal I refer to a set of rules that could be systematically applied – the properties of IAA and RR are theoretically always satisfied. Actually, it depends on how the regulations are applied: the extreme cases are when a) there is a computerized procedure which ends with the creation of the classes; b) it is a task of the representatives to apply the rules and to carry on the all process¹⁵⁷. Unfortunately, even in the better case the representatives could have opportunities

¹⁵⁶ Paragraph 2.1

¹⁵⁷ In this case if IAA and RR are not respected depends on a mistake of the representatives that will be reflected in the final ranks and so this situation could be easily assimilated to the case of unfair action.

of reversing the formals' preferences and let their preferences to be non-prior responsive¹⁵⁸. I identify this situation as an unfair action or behavior by the representatives:

Definition 7. It is possible to identify the presence of an **unfair action or behavior** every time a representative fails to act in behalf of the formal during the decision process, and expresses preferences based on personal interests. For $r \in R$, P_r does not respect the constrained imposed by P_f , for $f \in F$, if for every $C', C'' \subset C$, for some $c' \in C'$ and $c'' \in C''$, $C' \succ_f C''$ but $P_r = \{c'', c'\}$, i.e. the Prior-Responsiveness property is not satisfied.

An unfair action is registered when the representatives do not simply break the indifference ties contained by the subgroups C' and C'' , but invert the positions of Candidates within the list of preferences. Note that the formal will not know if unfair behaviors have modified its dispositions. The representatives, who have to act in behalf of it, are not following the rules of the game imposed by the regulation. They are playing as they were the real agents in the market by implementing their strategy to reach the results according to their personal interests. The misrepresentation of preferences is a consequence of the divergence of their aims. The consequences due to an unfair action, could be minimal when the choices based on personal interest quasi-overlap the decisions based on impartial judgments or when the differences between the Candidates are almost unimportant. Problems could arise when the decision taken from a personal point of view are very far from the requirements of the regulation. Beside the instability of the matchings there could be various potential consequences that could affect the market under different point of view. Based on the previous statements I define:

Definition 8. A semi-regulated matching market is defined **unfair** if the Recruiters' side suffers of unfair actions or behaviors.

¹⁵⁸ Unless they are technically allowed to only operate on one class per time or they are technically impeded to be not prior-responsive.

Proposition 3. A matching in a semi-regulated matching market is unfair if and only if the Recruiters' preferences do not satisfy the prior-responsiveness property, i.e. the Recruiters' preferences have been misrepresented unbeknown to them.

It is important to notice that an unfair matching is still stable up to preferences inserted in the mechanism, and the definition of unfair implies the instability of the matchings up to the real preferences.

Moreover, the proposition wants to outline a peculiarity of this way of misrepresenting preferences. What differentiates this manipulation from the others already treated in the literature (Roth 1982; Roth 1984; Roth and Rothblum 1999) is that the Recruiter – interpreted as the entity/Institution – is not aware of it. Due to features well known as delegation (Alonso and Matouschek 2008) and asymmetry of information (Harris and Townsend 1981), the representative has the possibility of hiding the unfair activities to the Recruiter. Then, the unfair action could be seen as an involuntary misstatement of preferences by the Recruiters who are unaware of their representatives' behaviors. This characteristic could make the unfair action more dangerous than the other kinds of misrepresentation with direct and indirect potential consequences on the market in addition to the traditional instability. They will be shown in the fifth section.

The next section presents the Italian recruitment procedure of assistant professors as a case-study where it is easy to detect the model of a semi-regulated matching market affected by unfair actions. Furthermore, the real case will make clearer the following discussion on the potential consequences and possible solutions.

IV. Potential Consequences

4.1 Not Only Instability

Despite the general consequence of instability up to the real preferences that will affect any SR matching mechanism with a failing agency relation, there could be a multitude of potential effects that differ by market.

1. Hiring of less-deserving Candidates. Due to the involvement of the representatives' personal interest in the selection process, this could be a quite usual consequence. Pre-existing relationships could distort the evaluation of the applicants by overestimating the value of their research and their work respect to the others or by weighting more the value of knowing them and having already worked together. The psychological processes that could be in force during the selection are various but are not the matter of this paper. The hiring of less-deserving applicants has a direct impact on the productivity of the Recruiter and an indirect impact on the Candidates' behavior and on how the Recruiter and the market are perceived. As the number of undeserving Candidates hired in the market increases, the reputational level of the mechanism and of the Recruiter decreases.
2. Loss of confidence. A market perceived as unfair will lose credibility and reliability. This could lead the potential applicants to renounce and to abandon the recruitment process. Moreover, the mechanism is judged either by the "outsiders", the community of agents who do not participate into the market. The bad reputation based even only on few events, could modify the behaviors of future potential Candidates making them leave the market – and generating a consequent thickness problem (Roth 2007)¹⁵⁹ – or misrepresent their preferences by avoiding to apply for most-preferred but unreliable Recruiters – safety problem (Roth 2007) – or driving them to adapt to the unfairness of the market.
3. Exploiting unfair actions. If the unfair behavior of the representatives become known and it can effectively ensure good results, more can try to exploit this wrong conduct. On one side, the applicants could put more efforts on building up the right relationships' networks instead of focusing

¹⁵⁹ E.g. Italian Academic Job Market – When the contests are perceived as "closed" the number of applicants is very low, there are either cases with a sole Candidate. (data from MIUR website – E. Quintilii 2017 "The Role of Information into Matching Markets", Ph.D. Thesis).

on preparation¹⁶⁰. On the other side, Recruiters' agents could feel incentivized by the actions' of the others to act in the same way in a sort of herding behavior (Raafat et al. 2009). Then, Candidates and representative could stipulate informal agreement before the recruitment process starts (Kojima and Pathak 2009) that in worst conditions could cause also a congestion problem (Roth 2007).

The potential consequences connected to the agency relation failure are, at the same time, even potential causes of dangerous problems as thickness, safety, congestion that deeply affect the efficiency of a matching mechanism. While great examples (Abdulkadiroglu et al. 2006; Roth 1984a, 2007, 2010) show how the centralized structure and the implementation of the Deferred-Acceptance Algorithm are excellent solutions, they cannot solve unfair actions. None of them can block or limit the power of the representatives and there is no possibility of carrying the Recruiters reveal their true preferences, most of all if they charge agents to do it. This is one of the key point of the SR matching markets affected by an agency problem: if market designers do not take into account the agency relation on the Recruiters' side, they will treat the market as usual by implementing traditional tools that are able to fix only the superficial inefficiencies without solving the relevant problem on the basis. Even if the centralization and the DA are fundamental pieces of the restricting, in this case they are not enough.

4.2 Public Institutions on the Recruiters' Side

In the case of a Public Institution as Recruiter in the matching process the potential consequences above mentioned could be more intensive due to some features linked to the "public" nature of the entity:

1. *Partial information over the Recruiter's preferences.* The regulation is known (or at least knowledgeable) by anyone because the documents

¹⁶⁰ E.g. Italian Academic Job Market – The Candidates of contests for Associate and Full professors positions, exerted less effort in their research works after the switch from a national contest to the local ones because of relying on the "misconception" of more favorable committees and more personal evaluations. (Checchi et al. 2014).

on the hiring procedure have to be available and freely consultable. This means that the mandatory and the preferential titles and skills required to the Candidates are known by everyone, and so the applicants have a partial information on the preferences of the Recruiter.

2. *Partial information over the other Candidates.* The name of the participants are public, as the evaluation of any kind on them. In the Italian case showed in section 4, for example, any Candidate has the right of asking to review all the title/papers/documents presented by the others for applying the job position.

The opportunity of having these partial information makes the applicant able to identify unfair actions, at least the most notable ones – e.g. the winner misses one the mandatory requirements or does not satisfy any preferential requirement. The Italian case is full of good examples¹⁶¹ on this situation, and most of all, full of complaints by Candidates to the Administrative Regional Court¹⁶² and of contests nullified because affected by unfairness. It is clear that the private market cannot be theatre of claims, first because there are not sufficient information available, second, because a firm could have particular systems of evaluation linked to the job position – not entirely based on titles and merit. Furthermore, even if a perceived unfair hiring in a private Institution could generate disdain, it is not comparable to the indignation raised up by the same event in a public Institution. Looking at the Italian market, the resentment is based on the idea that the public entity is sustained by the taxes so it is property of the people, and its task is to pursue to well-being of the society and to try to increase the welfare through its work and activities. The hiring of a Candidate who is not deserving or not suitable

¹⁶¹ The most famous over the last years concern the University of Insubria and the University of Milan (2011-2012) (“Concorsi pilotati, Secs Team lancia una nuova raccolta fondi” n.d., “FABIO SABATINI – La de-feudalizzazione dell’Università» LA PAGINA DEI BLOG - MicroMega” n.d., “FABIO SABATINI – L’incredibile odissea di una ricercatrice contro i muri di gomma universitari» LA PAGINA DEI BLOG - MicroMega” n.d., “FABIO SABATINI – Università” n.d.).

¹⁶² TAR – Tribunale Amministrativo Regionale.

enough for the job positions is perceived as threatening the whole system and reveal the selfish interest of someone inside who cheats the entire community.

At the light of these statements, it is easy to understand why the three potential consequences illustrated in the previous paragraph are most dangerous: the partial information owned by anyone a) generates awareness of the level of the hired applicant and comparability to the level of the others and to the legal requirements; b) the loss of confidence and reliability is stronger and linked to the feeling of indignation; c) if widely diffuse, the exploitation of unfair actions could become the typical way of obtaining a public job position, instead of the exception. It is important to know that if the agents in the market are able to identify the unfair actions, they are practically discovering the misrepresentation of the Recruiter's preferences and the instability of the market up to the real preferences.

V. Application - When Public Institutions are the Recruiters' Side

5.1 The Recruitment Process of Assistant Professors in Italy¹⁶³

In the following study, with the generic term "University", I will refer exclusively to Public Institutions. The assistant professor (or researcher) represents the position of entrance in the Academic job market and it is the starting point of the academic career path. The process of recruitment is defined by the Law n. 240/2010 and the mandatory requirements imposed by Law are contained in the regulation of any University. To obtain the role, Candidates have to participate to a public contest that is publicly announced. The current academic job market in Italy has a decentralized structure, i.e. each University makes an announcement for vacancies and elects among the Candidates following their own regulation. The proposal of call must be approved and the number of calls are constrained to the amount of financial resources available for the University – usually Universities do

¹⁶³ The market presents different inefficiencies and its redesign it will be deeply treated inside the PhD Thesis. In this paper the case-study is generally illustrated as long as it proves the real application of the model.

not offer more than one job position per SSD – Disciplinary Scientific Sector. A local committee will judge the applicants on the basis of the value of their researches' works and other skills specifically indicated in the announcement – they vary by University and by job position –, and will declare the winner of the competition. By Law, the recruitment process is based on the evaluation of scientific publications, personal curricula, teaching activity, linguistic and other skills that could be requested.

It is important to underline that the call is made by the Department where the vacation is offered, and the Department will carry on the procedure¹⁶⁴. The announcement specifies the SSD of the position to which Candidates will have to demonstrate to belong to, through their scientific production. When the deadline for sending applications has passed, the committee is selected. The composition of the committee has to be aligned to the guidelines of the European Charter¹⁶⁵ integrated into the Law 240/2010: three members, one of the Department that offers the job position, and two randomly selected out of the University (better if one of them is from another country). The preliminary evaluation of the Candidates involves the academic titles – PhD degree or an equivalent academic title is a mandatory requisite of access from 2015– curriculum vitae and scientific production (each University will establish a maximum number of papers that can be presented), including the PhD's thesis. Internationally recognized criteria and parameters are implemented, even though the bibliometrics references are actually applied mainly by Scientific Departments and less considered in the Liberal Arts' sectors. The best Candidates¹⁶⁶ are convened to a public dissertation of their scientific production and the knowledge of a foreign language is tested – if required in the announcement of the University. Then, the committee writes a brief judgment of all the attendee and select the winner who will be proclaimed with an official document. All the documents related to the procedures are public and they

¹⁶⁴ The term “University” is used in a general way to indicate the Institution, so sometimes it will be used also in substitution of the term Department.

¹⁶⁵ European Charter for Researchers – The Code of Conduct for the Recruitment for Researchers 2005

¹⁶⁶ Their number have to be between 10-20% of the total Candidates and in any case not less than six, they are all admitted when the total number is less than seven.

have to be published on the website of the University so that anyone have the possibility of consulting them.

The outstanding problem is related to the sort of “advantage” someone profited by. Cases of so called “influence peddling” have been registered (Perotti 2008) during the years, highlighting a meritocratic problem. The Law establishes a relation over the fourth grade of family affiliation between a Candidate and a commissioner, but Perotti illustrates as a network of friends can easily overcome it, by simply recurring to mutual favors. The advantage given by the family relationship is not the only one that could be find in this market. The so-called “internal Candidates”, the ones who are working at the Department when the job position is offered¹⁶⁷, are the main characters of this “scandal”. The consolidated relation with the members of the academic staff seems to give them a preferential treatment¹⁶⁸. Complaints to the TAR¹⁶⁹ are often executed by external Candidates and the nullification of contests due to these reasons are always under the attention of the media and of all the national and international academic community¹⁷⁰.

5.2 The Model in the Real Case

Once centralized, the Academic Italian entry-level recruitment appears as a two-sided matching markets. It is constituted by public Universities on one side and Candidates for a job position on the other. As come out from the description of the market and of the current system, since Institutions are not simply applying a regulation, they could be actually considered strategic players. The salary is the same for every academic Institution, so it can be not taken into account. I assume that each University has only one job role – close to reality, it results as a one-to-one matching market. The preferences of the Universities are build up by a

¹⁶⁷ Usually these Candidates spend most of their academic path in that Department (PhD degree, research fellowship, post-Doc programs)

¹⁶⁸ <http://blog-micromega.blogautore.espresso.repubblica.it/2012/05/23/fabio-sabatini-la-de-feudalizzazione-delluniversita/>

¹⁶⁹ TAR is the acronym for Tribunale Amministrativo Regionale – Regional Administrative Court

¹⁷⁰ Following an episode of “corruption” in 2011, it was created an online page to collect money for sustaining the claim to TAR of the Candidate who was affected by the unfairness of the procedure. Both national and international academic professors joined the cause by sending money through this page.

committee who has the task of following the mandatory and non-mandatory requirements published in the announcement of the job position for making their choices. Since there is an agency relation – Universities’ regulations as formals and the committees as representatives – it can be defined as a semi-regulated matching market. The cases of influence peddling are the unfair actions operated by the committees. They result from a misalignments of interests between the Institution – which desires the more deserving and suitable researcher – and the Committee (or someone into the committee) – who prefers to select someone they already know even though she is not the best Candidate¹⁷¹. At the same time, deserving Candidates are forced to stay unemployed, to accept a position if a less preferred University or to go abroad. Sometimes, Candidates decide to not apply for job positions that are perceived as “closed”¹⁷². It is quite clear how unfair actions can create distortions in a market. The Department and the University – as impersonal Institution – do not have the information to evaluate the selections of the committees neither they follow the procedure in some way. It would be a mistake to consider the University as the strategic player on the Recruiter side: it is not voluntarily misrepresenting preferences as the case of colleges which manipulate quotas, for example, for trying to obtain/avoid who they want/dislike. The model of the semi-regulated market allows to identify the real responsible of the misstatement of preferences and this is a priority in order to solve the problem while restructuring the matching mechanism. In fact, even if the market would have been centralized and managed by a central body that implements the D-A algorithm for obtaining the matchings, still the Universities’ preferences will be misreported by the committees.

¹⁷¹ Even without going deeply in analyzing the number of publication, citation and h-index of Candidates – actions that is actually not possible for all the contests since a list of all Candidates is not always provided on the MIUR websites – it is sufficient to refer to all the claims addressed to TAR for winners who were the less academic titled of all the others.

¹⁷² Reserved to internal Candidates. This “perception” – and more than this – has been captured by a survey conducted in 2014 on the current Assistant Professors who went through these hiring procedure. Data of the surveys are partially available in the Appendix and fully available in the Ph.D. Thesis “The Role of Information into Matching Markets” by E. Quintilii, 2017.

VI. Multiple Solutions for the Agency Problem

6.1 A Set of Possible Solutions

As anticipated in the previous section, even for the redesigning of a SR mechanism, the centralization of the market and the DA algorithm are the fundamental pillars. Since they have been deeply analyzed and explicated in the literature, they will be not illustrated in this paper. I will only to specify that the Candidate-Proposing DA algorithm is the best choice thanks to its properties – since it is strategy-proof and formulate the best outcomes for the proposing side of the market, it is quite robust to manipulations¹⁷³. Note that a Firm-Proposing DA (Roth and Rothblum 1999) will incentivize representative to reveal the preferences they like most but there are no incentives for them to reveal the true Institutional preferences. These tools together allow to solve traditional matching inefficiencies, but, as already outlined, they are not able to block or limit unfair actions.

Actually there is not a unique solution, but multiple possible solutions. The choice of what kind of solution implement or what kind of tool add to the new design depends upon the level of inefficiency and reputation of the mechanism. If it is strongly perceived as unfair and this perception created a distort view of anything related to the market, it is necessary to act directly on the source of the problem as heavy as possible. The basic idea is that an invasive intervention has the ability to affect the opinion and to rescue the confidence in the system¹⁷⁴: more is the level of distrust, more rigid have to be the measures. Shown below a list of possible solutions identified until now:

- Reducing or controlling the representative tasks (Alonso and Matouschek 2008; Holmstrom et al. 1982; Holmström 1977). In the so-called delegation problem the principal establishes the set of decisions the agent is allowed to make and there are no ways for him to make any decision out of this set

¹⁷³ Gale and Shapley, 1962; Pathak and Sönmez, 2011; Roth and Sotomayor, 1992; Roth, 2008

¹⁷⁴ E.g. Italian case – A survey revealed that there are doubts about the well-working of a central structure because the evaluation system (i.e. the preferences expresses by the Institutions) will continue to be affected by wrong behaviors (subjects: 150 assistant professors over 4 different Universities; year: 2014; data unpublished)

- e.g. using technological constraints. One of the proposal of this theory is then to reduce the dimension of the set until the size that allow the principal to benefit from the delegation. This is a quite invasive solution since it shrinks the autonomy space of the agent. An example of how could be applied to the SR mechanism is illustrated in the next paragraph.
- System of incentives (Ballwieser et al. 2012; Laffont and Martimort 2009; Maskin et al. 1982). The well-established theory of incentives could be applied to the agency scheme but the possibility of implement a good system of incentives depends first on the financial availability of the market. While the incentives have been applauded as a good resource of the corporate governance policies, research studies in behavioral economics show us that they do not always work as we want and that their power depends on how well they have been designed (Charness and Gneezy 2009; Gneezy et al. 2011; Gneezy and Rey-Biel 2014). Note that “incentives” is not synonymous of “monetary incentives” and that particular situations could require particular incentives’ design.
 - Eliminate the representatives (Alonso and Matouschek 2008). These proposal stems from the theory of optimal delegation. If the misalignment of interest is relevant, the principal could benefit more by eliminating the agents. Then, it highly depends on the level of inefficiency of the agency relation and on the degree of the consequences it is causing to the mechanism. There are some situations where removing the representatives is not allowed. The Italian case is an example: the recruitment process is regulated by the Law 240/2010 which establishes the role of the committees and removing them will eliminate completely the autonomy of Universities to choose their preferred Candidates.

6.2 Application - A Proposal for the Italian Case-Study

The proposal for avoiding unfair actions in the Italian case, is to control the submission of the preferences' list by Universities: they can express a choice among the Candidates in the same class of indifference identified on the basis of

the characteristics requested by the regulations. A central committee will evaluate the Candidates on the mandatory requirements and will remove the ones who do not satisfy them. Then, it will assign scores to applicants for each main category of evaluation – academic title, research activities, teaching activities and so on – and subcategories – e.g. innovation, originality, relevance for the research works. At the same time the local committees will judge the research curriculum of any Candidate for whom they are an acceptable mate¹⁷⁵. The task is to evaluate the academic path of the Candidate at the light of the research project linked to the job position¹⁷⁶, in order to identify the most suitable applicants. Moreover, usually Departments add some preferential requirements into the announcements, they will be treated as adding-points requirements. Applicants will be asked about them during the application procedure so that the “extra-points” for the specific Universities could be added directly via technical tools. Once all scores are summed¹⁷⁷, the classes of indifference for each Universities can be created automatically by computers: using scores allow to divide the Candidates using a percentile rank method, such that the ones with highest scores are the first class and so on. The commissioners will act only within these identified classes, through a computerized format which will allow them to visualize a class per time. In this way the mechanism will not allow them to list a low score Candidate before a high score Candidate. This measure ensures the fairness of the procedure consistent with the requirements of a typical regulation that involves public Institution, but, at the same time, it allows the committees to participate at the evaluation and to select the one who is considered more appropriate for the research project linked to the job position announced. The tasks of the principal/formal and of the agent/representative are respected through the practice of directly reducing and indirectly controlling the tasks of the latter.

¹⁷⁵ Any Candidate who satisfy the mandatory requirements is ranked in the preference list of any University.

¹⁷⁶ The job positions offered are titled with the name of the project and are addressed to researchers of specific scientific sectors.

¹⁷⁷ Note that the score of the central committee will have a higher value respect to the one of the local committees; the preferential requirements will assign a restricted level of points to impede the so-called “tailored announcement” for specific Candidates.

6.3 Further studies

Further studies are focused on the possibility of solve the problem of the agency relation in matching market by trying to manipulate the wrong behaviors (i.e. indirect control over the representatives):

1. Exploit the centralized structure. Studies in behavioral economics as social preferences, specifically in prosocial behaviors (Bar-Tal 1976), have identified a change in choices by the two treatments where the controlled variable is if the choices are visible to anyone or not, i.e. manipulating reputational opportunities affects prosocial behavior (Haley and Fessler 2005). When they are observable and comparable, people want to signal to others they are fair, generous and so on and to maintain a reputation on who they are (Bénabou and Tirole 2005). Starting from these theories, I hypothesize a specific solution for agency problem in matching markets¹⁷⁸. Instead of having a clearinghouse which simply coordinates and manages the matching mechanism, the idea is to create a central body who shares some of the tasks or all the tasks of the representative. The former will carry on the activities in an objective and general manner while the latter will be more subjective and specific. The evaluations of both will be knowledgeable to the Recruiter – in a private market – or to anyone – in a public market. The hypothesis is that the comparison puts pressure on the representative driving them to be sincere; moreover, the system could be thought in a way that takes into account the judgments of both such that excessive scores by agents could be mitigate by the ones of the central body. In this case the solution would be less invasive. On this hypothesis a 2x2 matrix of treatments has been building up and the test will be conducted in a laboratory. If the results will be positive, in the Italian case, for example, it would not be necessary to reduce the committees' autonomy – avoiding the dissatisfaction of the Departments.

¹⁷⁸ Current research.

Current studies are focused on the implementation of behavioral and experimental theories for finding suitable solutions that allow to obtain the best outcomes through the less-invasive controlling actions on the agency relation into a matching market.

VII. Conclusions

The entry-level job markets, besides the great developed problem of the school/college admissions, have been capturing attention of market designers since the arising of this new branch of study. The real applications and the theoretical examples offered by the literature, showed the distinctiveness of each case. The analysis of different markets effectuated during these years, brought to light characteristics of the matching mechanism that left a mark on the path of the market design practice/theory. In the first part of the paper I listed the different decision processes that could be used by Recruiters to rank their preferences and I highlighted in particular a semi-regulated process, i.e. when choices are made implementing rules and individuals' judgments. The main feature of this process is that it is a two-step procedure carried out by a principal or formal and an agent or representative who act as being a single player (the Recruiter). I called a matching market when the Recruiters' side relies on this decision method as Semi-Regulated matching market and I formalized its model. The aim of the paper, then, is to introduce for the first time the scheme of the agency problem into a matching framework and to highlight its characteristics and its potential power of generating inefficiencies that traditional tools cannot solve. I show that the failure of an agency relation on the Recruiters' side leads to an uncontrolled misstatement of preferences identified as unfair actions, since they are caused by the wrong behavior of the players who act in behalf of the Recruiter entity. Then the mistake of considering these Institutions as a unique body during the decision process while it is carried on by two different agents who act as being one, hides the real actor of the preferences' misrepresentation. As outlined by the principal theory (Eisenhardt 1989; Ross 1973b), the misalignment of interest drives the agents or

representatives to follow their own. In the matching mechanism, specifically, they submit personal preferences' lists that represent their personal choices instead of representing the interest of the Recruiter. This is the cause of the involuntary choices' misrepresentation of the Recruiters that ignore what happens at the representatives' level. The potential consequences could be various and could trigger a vicious-cycle of inefficiencies: the effects of unfair actions could distort the market till to become causes of thickness, safety and congestion problems. The Italian procedure of recruitment for Assistant Professors has been presented as a case-study in order to show how the theory applies to a real-world market and to make clearer the dangerous effects connected to a disastrous agency relation that could drive to ruin the entire mechanism. A set of possible solutions from the fields of delegation problem(Holmström 1977), optimal delegation(Alonso and Matouschek 2008) and theory of incentives(Maskin et al. 1982) is detailed. Unfortunately, there is not a way to solve the problem that is valid for all the matching mechanisms. The selection of the right solution will depend upon the current state of the market and its regulations, on the level of confidence and reliability that it still preserves and on the behaviors of all the agents involved. An application of a "controlling solution" for the above mentioned Italian market is showed together with an explanation of why it is retained the most suitable for this case. Current studies are focused on the possibility of exploiting the centralized structure for trying to "manipulate" the wrong behavior of representatives driving them to be fair. Further investigation will involve the public opinion into matching mechanisms that involves public Institution. It will be treated as an "external agent" with the capability of having an influence on the players, and so, on the game. Many psychological theories will be implemented. Moreover, if there will be results that confirm an effect of the public opinion on the agents, the next step will be focused on how the effects of the public opinion could change according to the different country cultures.

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Appendix

Scheme of the Semi-Regulated Decision Process

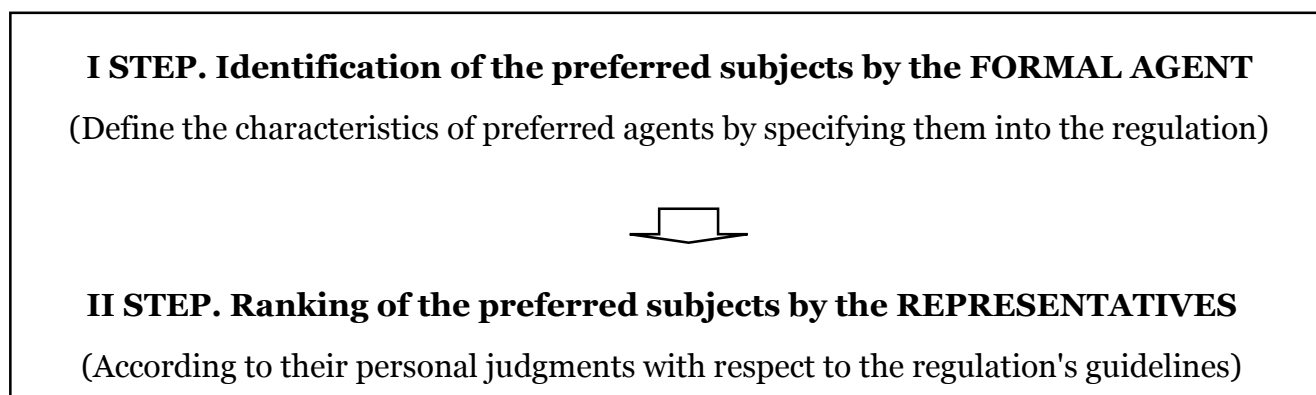


Figure 6. The Semi-Regulated Decision Process

3.4 Final Proposal of Re-Design: a Candidate-Proposing (Deferred) Algorithm adjusted for the Agency (Meritocracy) Problem

3.4.1 A Solution for the Agency Problem: Control over the Institutions' Preferences

To find a solution of unfair actions is the prerogative of this research work that will be carried on by using the process of assistant professors' recruitment in Italy as exploratory case. The Candidate-Proposing D-A algorithm is the best choice thanks to its properties – it ensures stable outcomes, is strategy-proof for at least one side of the market, is quite robust to manipulations¹⁷⁹ – with respect to other algorithms as the priority matching: a mechanism that starts by matching the agents that mutually choose them as first preference, will foster prearrangement between universities and candidates. Nonetheless, as demonstrated in the previous section, the unfair behavior of the committee could not be blocked by the algorithm in this form.

The first proposal for avoiding unfair actions is to control the submission of the preferences' list by universities: they can express a choice among the candidates in the same class of indifference identified on the basis of the characteristics requested by the regulations. The rank realized by the central committee will be essential since this first selection will involve only the value obtained during the national evaluation. The “class of indifference” highlights the grade of suitability of the candidates (e.g. C' contains the high score candidates, C'' medium-high score candidates, and so on) and it is not possible to forecast how many categories there will be, because it depends on the values obtained by the applicants and their responsiveness to regulation requirements. The two extreme cases are (a) all the candidates fall in the same class (b) each candidate constitutes a class. Neither of the two represents a problem for the mechanism. The first case could happen

¹⁷⁹ Gale and Shapley, 1962; Pathak and Sönmez, 2011; Roth and Sotomayor, 1992; Roth, 2008

when all the candidates obtained the same marks by the central committee and they all possess the same skills, so they are all suitable to be selected and never mind who, among them, will be hired. In this situation a control over universities' preferences would not be necessary but, since the results will be the same in both cases, the presence of the provision has no influence. The opposite is for the second case: the differences among marks and skills are indicate that only few applicants – the ones at the top positions – are able to be elected. If the measure on the universities' preferences would be absent, anyone could be hired by unfair committee. By let the commissioners acting only within these identified classes, the University rules are respected and there is not the risk of overturning its preferences. The mechanism will not allow them to list a low score candidate before a high score candidate. This measure ensures the fairness of the procedure consistent with the requirements of a typical regulation that involves public institution, but, at the same time, it allows the committees to select the one who is considered more appropriate for the research project linked to the job position announced. The main critique at this system is about the autonomy of universities that are forced to follow a path for submitting their preferences, but the procedure is just the practical application of the regulative guidelines. This makes more real the involvement of the institutions in the game and makes easier for the representatives to remain in the boundaries outlined by the regulation. Another problem could arise about the scores of the candidate, e.g. on how to obtain them, but this is not the matter of this paper.

Let illustrate the functioning of the control on preferences:

- There are five candidates, $C=\{c_1, c_2, c_3, c_4, c_5\}$ and two universities, $U=\{u_1, u_2\}$, each universities has one vacancy $q_{u_1, u_2} = 1$
- On the basis of the evaluations given by a central committee, candidates are ranked as c_3, c_1, c_4, c_2, c_5 ; c_4 has the mean score, c_3 and c_1 have high scores and c_2 and c_5 have low scores.
- The agents c_3, c_4, c_5 express the same preferences $P_{c_3, c_4, c_5}=\{u_2, u_1\}$; agents c_1, c_2 express the same preferences $P_{c_1, c_2}=\{u_1, u_2\}$

- Universities have to submit their preferences :
- formals (of both u): according to academic curricula and objective evaluation of scientific production, c_3 and c_1 result to be the most preferred, followed by c_4 , while c_2 and c_5 are the lasts. (By assumption, the two universities involved have the same guidelines)

$P_{F1, F2} = \{[c_3, c_1], c_4, [c_2, c_5]\}$, where the square brackets identify a relation of indifference between the subjects inside.

- representatives: break the ties according to personal judgments on the candidates – evaluate who is more suitable for the research project, etc.

For blocking unfair actions, the representative will express preferences for each class of candidates (high/medium/low score). In this way they cannot do more than breaking the ties. The power of the institutions' committees is reduced.

$$P_{R1} = \{c_1, c_3, c_4, c_5, c_2\} \quad P_{R2} = \{c_1, c_3, c_4, c_2, c_5\}$$

The representatives of u_2 now, to sustain the candidate for whom they have a personal interest, can only put him/her as first in his/her category.

- Candidates-proposing D-A algorithm:
 1. c_3, c_4, c_5 propose to u_2 that keeps the application of c_3 and refuses the others; c_1, c_2 propose to u_1 that chooses c_1 in accordance to its list and refuses the other.
 2. The non-matched propose to their second preference: c_4, c_5 propose to u_1 but, since it already obtained the first candidate in its list, refuses all; c_2 propose to u_2 but, since the offer that it is holding is better according to its preferences' order, the university refuses.

- The mechanism ends up with the following matching:

$$\mu = \{(u_1, c_1), (u_2, c_3)\}$$

It is quite evident that in this way the deserving candidates will have more possibilities to be hired respect to the ones who are at the bottom of the ranking.

The classes of candidates are not previously defined, but they will depend on the scores obtained by the subjects: starting from the highest score a percentile rank will be built up.

Essential elements of this system is the rank of candidates and the procedure to assign them objective scores that reflects their value that risk also to be the most discussed.

The proposed solution could appear quite invasive and restrictive with regard to the autonomy of the departments who currently carry on the recruitment process. I want to underline that this is not the sole solution, many other possibilities exist, as a robust system of incentives for example. One of the greatest problems, in this case, will be linked to the availability of financial resources – that at the moment are scarce – for building the system of incentives, so the proposed measure would be less expensive. Moreover, the choice of what kind of solution the market needs, depends upon its current state of inefficiency and, especially, on how it is perceived by the agent. If it is strongly considered unequal or non-meritocratic, for example, it could not benefit so much of an incentive system that could appear ineffective in the short term. The basic idea is that an invasive intervention has the ability to strongly affect the public opinion by acting directly on the problem of loss-confidence. The regulation expressed by the new design for the recruitment process could appear very strong but probably necessary. When a market suffers from a so great loss of confidence from the players, one of the most important things to do is to give it back a reliable reputation: more is the level of distrust, more rigid have to be the measures. One of the aims of the laboratory experiments will be to demonstrate that the simple implementation of the D-A is not sufficient to solve unfair actions, and so the meritocracy problem¹⁸⁰.

I want also to underline that the main objective of this work is to create a system that increase the well-being of the players: if even one subject is better off and nobody's condition gets worse, in the respect of the market rules, than the aim is

¹⁸⁰ Some experiments were already conducted, but they were based on low numbers and on paper calculations. Results reported in the fifth chapter in "How to design efficient..", Quintilii E., see references.

achieved. The mechanism is expected to improve the candidates' well-being and, at the same time, to regulate the actions of the universities in the respect of the law in force. If the system will achieve its goals of “meritocratic matchings” it could be registered also a decline of the complaints' number to the TAR, reducing the times of contests that often are stretched by the TAR actions of reviewing of all the documentation.

3.4.2 The Final Re-Design Proposal for the Italian Recruitment System of Assistant Professors

At this point, it is possible to present the complete final redesign proposal by putting together all the elements illustrated into this last section.

1. Centralized Structure

The recruitment system will be carried out through a unique process (instead of by a multitude of single Institution processes), under the control and supervision of the MIUR – or specific entities part of MIUR. The centralized structure allows an easier management of the entire procedure, mostly about the timing of the hiring process that is one of the most critical aspects of the current mechanism. Moreover, it is not necessary to check that each Institution respects the Law, or that each regulation is completely aligned to the Law. It also reduces the costs both for Candidates and Universities, and the waiting times that could not last more than a predetermined period. The costs reduction should foster the number of applications per Institutions, providing the right thickness level per single Institution. It is unequivocal the managerial advantages connected to this kind of structure, and the clearness and transparency that would be gained by the process. The uncertainty degree about the total job offer over the entire year is eliminated, together with the state of anxiety and concern.

In practical terms: it will be created a specific web-page for the Recruitment Process, to which access directly or through the MIUR website. On this page, each candidate will be asked to register herself, and to complete a format of her

Academic Titles, as to upload all the necessary documents (e.g. ID, papers, research statements, etc..). Applicants would have also the chance to report particular skills about languages, IT skills, teaching activities and so on. The first class of requirements will be denoted as “mandatory”, and it will not possible to skip them – since it is not possible to be hired as Assistant Professor in absence of that requirements¹⁸¹. The second class is formed by “additional” requirements that could help Universities to differentiate between candidates, or to identify the most suitable for the position they are opening. Then, they will choose the Institutions for which they apply among the ones that have a vacancy¹⁸², and will rank them from the most to the least preferred. Institution receive the applications on their personal page on the recruitment online-platform, such as they can have access to the candidate profile with a click. They will be asked to rank the candidates after the evaluation process.

2. Candidate-Proposing Deferred Acceptance Algorithm

The centralized structure allows to implement the Deferred Acceptance algorithm(Gale and Shapley 1962) as matching mechanism. Beside the guarantee of obtaining a stable result, it ensures that the proposing-side of the market has the dominant strategy of truthfully revealing the preferences- list (Dubins and Freedman 1981; Roth 1982b). This is the reason why I opted for a Candidate-Proposing D-A algorithm. Since they will achieve their best outcome (Gale and Shapley 1962), there is no reason to strategize or to play the game, it is sufficient to participate into the procedure and to truthfully report the preferences. Candidates will then experience a safe market. An objection to this choice could rely on the fact that the final outcome will be Candidate-Optimal, such as Institutions are penalized, while usually the literature gives the advantage to the employers’ side. The shift to an Institutions-Proposing D-A algorithm could drive candidates to try strategies for obtaining their most preferred result. However,

¹⁸¹ The ones who does not meet the mandatory requirements are automatically excluded by the procedure.

¹⁸² By one application, the candidate competes for different open positions. Notice that, the contests are differentiated by SSD, so also the application is specific per each SSD – since the candidate should demonstrate the belonging to that research category through her papers, then the submissions are expected to be different per SSD application.

recent laboratories experiments demonstrated that it is hard for agents to identify the right strategy that will make them achieve the best outcome (Featherstone and Niederle 2011). It is more reasonable to expect candidates to worse off their result instead of to obtain their desired assignment. The outcome of the game would be unstable, and it could appear also unfair, because due to a wrong strategy, candidates could also lose the chance of being assigned in favor of less-deserving candidates who truthfully revealed their preferences. Even if the sole authors of this result would been the wrong choices of the agents, it could be difficult to explain that the fault is in agents' actions and not in the system. It is particularly relevant to avoid this kind of situation in a system that is already at the center of an intense debate and mass-media attention. The new design should guarantee to the market a situation far away from prosecutions of any type, in order to restore its reliability into the Academic environment; but also into the Public Opinion.

In practical terms: the algorithm would be designed such that candidates send a proposal to institutions: the algorithm is performed by a computerized system.

3. National and Local Committees

The evaluation process is not really responsibility of this Market Design project, and this is why it has not been deeply analyzed in the dissertation. Actually, it is not a task of the market designer – whose work is more concentrated on the matching mechanism and on the rules of the game that allows it to work properly. However, it has to be incorporated into the mechanism, so the best solution would be a collaboration of the market designer together with the individuals who have the best knowledge about this part of the process¹⁸³. The aim of this brief introduction is to underline the complexity of determining how the evaluation should work, and that the next proposal is not definitive but functional to illustrate the recruitment system in its entirety.

¹⁸³ For example, the people who have been responsible of the recruitment design previously, someone who is part of the MIUR or of the ANVUR.

Given the centralized structure, and the problems related to the composition of committees and to meritocracy, it comes natural to think about a National Commission¹⁸⁴. The task of the national commission is to evaluate all the candidates from an academic point of view. The idea is that the national commissioners will assign a score¹⁸⁵ to each candidate, and they should represent a sort of “objective evaluation” of applicants¹⁸⁶. The inspiration is to build up a committee whose work would not be easily called into question by applicants, with the final goal of reducing the claims to the TAR. The Local Committees, on the contrary, will be asked to evaluate candidate under a more subjective point of view. Specifically, they will judge the suitability of candidates to the open position, their research work¹⁸⁷ and the teaching activity. At the end, they will assign a score too. The two final scores will be summed – however the score of the national and local committee will have different weights – and a rank will be realized for each Institution. Since the Law prescribes an oral examination for testing the foreign language skills (English), and also the knowledge of the research field, it should be included at this step. The proposal is that the National Committee will conduct the oral test and will assign the score, while the local committees will have the chance to participate and ask questions, but not to evaluate them for this test.¹⁸⁸ The rank is necessary for implementing the policy of control over the Universities’ preferences, as it will be clarified at the last point of this list. The double evaluation is necessary for giving to the procedure a more reliable aspect and for guaranteeing a meritocratic hiring process (national committee), and for letting Institution having the chance of knowing the candidates and express their preferences. The autonomy of Universities would be already highly compromised

¹⁸⁴ The National Commission is a unique centralized commission that will evaluate all the candidates of the related SSD, for this reason it is called “National”; about the commissioners, the idea is to choose external individuals from foreign universities.

¹⁸⁵ The composition and the scale of scores is a very sensible argument and it will be not discussed in this work, since, as outlines, the evaluation procedure is not the interest of this research work.

¹⁸⁶ Aligned to the principle of transparency, it is also possible to provide a single score for each class of evaluation and then, the total score obtained for the “academic evaluation”.

¹⁸⁷ Notice that the research work will be evaluated by both committees, but the national one will be asked to follow a bibliometric evaluation, while the local commission should evaluate parameters an innovation and originality.

¹⁸⁸ The score assigned to an oral examination is always a critical point of any evaluation process. In fact, it is not possible to check if it is appropriate, and it could be also determined by subjective judgments that are hard to justify. For these reasons, in my opinion it should be a task of the National Committee.

by a centralized structure, so it is mandatory to involve them into the evaluation process. Moreover, they are the future employers of that applicants, and they should have an active role in their evaluation.

4. Computerized procedure for score elaboration

Since part of the score can be assigned mechanically (e.g. points related to academic titles), it is possible to free the national committee of this task and to use a computerized procedure. This would imply to ratify a clear point system about the academic curriculum of candidates. Similarly, the same program would be implemented for elaborating the final score as a weighted sum of the national and local committee's score. All the scores will be then uploaded on the recruitment website – and on the personal spaces of candidates and institutions. Since the competition is public, all the documents about it must be publicly available, such as the score should be accessible by anyone.

5. Institutions' Ranks and Groups of Preference

This point is part of the policy on the control over Universities' preferences, although it is exactly what should be already happening in a recruitment procedure¹⁸⁹. As already extensively illustrated, the Institutions' regulation and job announcement are specific enough about the perfect candidate for the position to identify groups of applicants – since they report characteristics the winner should have. As above-mentioned, each University receives a different ranking – the difference is traced by the additional requirements included into the job announcement, by the applications received, and so on. This rank is then divided by a computerized procedure that follows a percentile ranking method. A reasonable question at this point could be: why is it necessary to build up groups when having already a ranking? The answer is based on two elements: first of all, part of the score is influenced by subjective judgments. When the score difference

¹⁸⁹ Instead of being externalized, this step should be followed by the committees by aligning their judgments to regulation and announcement guidelines.

is almost meaningless, and there are not only objective parameters in its construction, it is not possible to robustly sustain that one candidate is best than one another. Building up the groups of applicants with very similar scores means to leave Institutions the final judgment on them – since they are the employers, it seems reasonable that they have the last word on the hiring procedure. The only rule about the formation of the groups is the percentile ranking rule, such as there are no limits about the dimension of the group.

In practical terms: once a ranking of candidates is realized for each Institution, a computerized procedure based on the percentile ranking rule will divide the applicants into groups, on the basis on their final scores. The groups are ordered and identified by alphabetic letters A B C and so on till the last one, where A is the group of the most preferred candidates.

6. Local Committees realize the List of Preferences

Once the groups are ready, they are submitted to the local committees. Their task is to create a list of preference by ordering the candidates into the same group. They will transform an indifference relation into a strict preference relation into groups, and at the end they will obtain a list of preferences over candidates that respects the regulation and the job announcement, and whose process of realization respected the Law in force. As the lists are submitted into the assignment procedure, the algorithm can finally run. The (stable) outcome will be official and definitive – since it is not possible that any agent receives more than a mate, so there are no choices or ties to solve.

In practical terms: the local committee will receive the composition of any group one at a time, starting from group A. They will have to rank the candidates inside group A, once the operation is concluded, it will appear the screen with the names of applicants in group B. As before, the local commissioners list the components of group B and goes on till the last group. It is clear, that a candidate of group A cannot be listed after a candidate of group C, because this computerized procedure would be structured exactly for impeding this kind of actions. Even the

timing for each operation will be predetermined and will have to be respected – it is necessary that one Institution is late for having a delay in the entire procedure, so penalties should be designed for avoiding such situations. Notice that, during the evaluation process the National Committee score has a larger weight than the one assigned by the local committee, but the latter has a second chance of influencing the ranking, even if under a special kind of control.

As said at the starting of this paragraph, this is just a proposal of how a centralized recruitment process for Assistant professors could work. The solutions that could be implemented are various, and it would be interesting and helpful to discuss them with the experts of the educational field who have been in charge as responsible of this procedure so far. The centralized structure gives the chance to provide tailored systems, it is just necessary to implement the right matching mechanism at its core. The aim of this proposal is to show that a new and efficient recruitment procedure is possible, realistic and ready to be implemented.

Conclusions

Resume of the most important findings

This dissertation focused on the role of information into matching markets, both with a decentralized and centralized structure, while analyzing the case of the hiring procedure for Assistant Professors in the Italian Academic market. The investigation started from the hypothesis of a key role of information on the functioning of matching mechanism, with the possibilities of distortions of results specially in decentralized structures. However, the study divided into the three paper that build up this thesis, showed unexpected results.

The hiring procedure for Assistant Professors in Italy works by public competitions carried on independently by each University (decentralized market). Besides some managerial remarks, the system suffers of a great lack of information that affects both sides of the market in different ways. The starting hypothesis was that this information gap could be a cause of inefficiency, leading agents to display misrepresenting strategies and producing the instability of the final outcome. The dissertation presents the “market design” work that has been carried out on this real-world matching market – never explored before in the literature. The case study was a starting point – and for sure an inspiration – for the in-depth analysis on the many faces that information has into a matching market and mostly, on if they are able to sabotage a mechanism and how.

The first paper introduced the formalization of a decentralized matching mechanism that matches Institutions and Candidates and works “per rounds” – i.e. candidates cannot address all the institutions at the same time, but as soon as they open a position and the positions are announced at different timings. The theoretical model allowed to identify the strategy profile that identifies a Subgame Perfect Equilibrium of the game whose outcome is proved to always be stable. While Institutions have a dominant strategy in revealing their real preferences – thanks to the *institutionally acceptable* condition – candidates are beneficiated by

reporting a truncation of their lists, where the last element correspond to their best possible mate in a stable outcome – i.e. Candidate-Optimal mate. This kind of strategy requires a lot of information, as confirmed also by other authors (Ehlers 2008; Roth and Rothblum 1999; Roth and Vate 1991), such that it is perfectly implementable in a complete information setting. However, real markets do not offer conditions of this kind. It Due to the results of some laboratory experiments (Featherstone and Niederle 2011), I considered interesting testing the consequences of displaying non-optimal truncation strategy¹⁹⁰. In a contest of numerical symmetry there are no consequences but the slowing down of the mechanism functioning that need more rounds for achieving the final results. In a realistic context of numerical asymmetry – where the number of Candidates is above the number of open job positions – the truncation strategy become particularly risky and the matching result can be unstable to unassignments. Then, by assuming all candidates are risk-averse – and so truthfully revealing their preferences – the mechanism is still able to achieve stable results, that are Institutional-Optimal. In this case the absence of information could trigger the players to display a strategy that is different from the equilibrium strategy. However, the mechanism showed to be robust enough for ensuring stable results also under a strategy profile that is not an SPE. From this results, I concluded that the bad-functioning of this mechanism is mainly due to its own characteristics. Indeed, the results – for all agents truthfully revealing their preferences – heavily relies on three key assumptions: 1. One-sided commitment of Institutions to temporary assignment realized at the end of each round (while candidates do not commit); 2. Costless applications; 3. No restrictions on the number of applications per single round and over the entire game. It is sufficient that one of these assumptions is not respected for incurring in unstable outcomes. All of them are necessary but not singularly sufficient conditions for the results about stability to hold. In a system where these assumptions do not hold, a stable outcome it is only

¹⁹⁰ Any candidate is always better off by displaying a truncation strategy instead of revealing her real preferences.

possible if any candidate apply just to one of her stable mate (preferably to her C-Optimal mate).

As the theoretical model of the Decentralized Multiple Round Mechanism is applied to the real case study, it is easier to make some statements about its current functioning. None of the three assumptions has been found to work in the market: the application (or better participating in a contest) is costly; the overlapping of contests' dates force candidates to renounce to some of them; all contests end at different timings so candidate cannot compare all the job offers and cannot just leave a position she accepted. Since the amount of information for applying only to a stable mate – or to display an optimal truncation – is not available, it is possible to assert that the outcome of the current procedure is not stable. This is a starting point for demonstrating the inefficiency of the hiring procedure and for highlighting the causes at the basis of its failure. In fact, the huge lack of information that the market suffers have been not explored by the model. In the first paper I just made some considerations about the amount of information necessary for displaying a truncation strategy – as the other cited authors did before me – and that it could not be available. Though, the missing of few information – as for example the preference profiles of Institutions – could create a distortion in the choice of the candidate's strategy. This left space to some open questions: what would happen if – as in the real case – there is an almost complete lack of information? Would the truncation strategy be really the best strategy for any candidate in a setting with almost no information? And how much lack of information could this decentralized mechanism handle? The answers are offered by the research work the second paper focuses on, and they are some very interesting ones.

The second paper presented a “stress-test” of the Decentralized Multiple-Rounds mechanism from an information point of view. At first, it has been studied an incomplete information setting as the ones already treated in the literature – i.e. by removing the preference profiles of the other players from the common knowledge set. The assumptions outlined in the first paper are then confirmed as even in this

setting the truncation strategy is still the best for candidates, but its risk level incredibly increases. The conclusion, then, is the same that Roth and Rothblum elaborate for the Deferred-Acceptance algorithm (Roth and Rothblum 1999): not have knowledge about the preferences of the Institutions does not exclude the truncation strategy, just makes it riskier. It is even more clearer how much the adoption of this strategy depends on the risk-profile of any single candidate. This consideration is stronger if the assumption of numerical asymmetry is taken into account. Since the preferences profiles of Institutions are unknown, it is not possible for candidates to display the optimal truncation strategy presented in the first paper – where the model is studied in a complete information setting. For this reason, the results entail truncation strategies denoted as any strategy that does not reveal at least the last preference. The willingness of having a theoretical model that represents reality as much as possible, drove me to push the incompleteness of information a lot of steps further. Besides the preferences profiles, the candidates do not know: a) which Institutions will offer a job during the game; b) when they will offer the job position; c) how many rounds there will be into the game; d) the composition of their own set; e) their own preference profile. This last feature comes out at first from a more realistic thinking that nobody holds a complete list of all Institutions. In a second time, this condition revealed to actually be also useful at the moment of the strategy choice. Candidates actions rely now on their *utility function* that helps them identifying when it is convenient to send an application and when it is not. By simply comparing their current level of utility to the level of utility they would have if matched to the institution that is offering the position, they decide if applying or not. The list of preferences is then built out during the game by the sequential decision at each round. The uncertainty about the job offer is condensed in a sole assumption: for all the candidates any round is considered as being the last round. Given this – strong - assumption, following a truncation strategy would be not rational. Indeed, it would imply to not apply to the institution that is offering a position at the current round, for waiting for something the candidate likes more. However, if any round is considered as being the last, nothing comes after except the end of the game. I concluded that in a

setting with scarce information – also referred as the *uncertainty setting* – for any candidate a truthfully strategy stochastically dominates any misreporting strategy. Then, the game reaches a *Bayesian Nash Equilibrium* for the strategy profile where both sides of the market play honestly by reporting their real preferences. Specifically, the strategy displayed by the candidate has been denoted as *implementing truth-telling strategy*, since they apply any time the current offer is better than what they are holding. The outcome at the BNE has been proved to be always stable and Institutional-Optimal. The mechanism demonstrated to be more robust than the original hypothesis. The huge lack of information then does not impede to the mechanism of well-performing, and actually drives candidate toward a sincere (maybe more preferable?) strategy and to a stable matching. The key assumptions on the mechanism – one-sided commitment, costless application and no restrictions on candidatures – keep being the fundamental basis of the results. The lack of information obviously plays an important role but not the one expected at the starting of this analysis. On the contrary, it found the causes of inefficiencies in managerial features more than in information problems.

Once again the results are applied to the Italian case-study and the policy advices are confirmed. The decentralized structure needs some timing controls and for applications to be costless it has to centralize at least the examination of candidates – as it happens in the international Academic job market for Economists – or to allow the use of some technologies – for example online interviews. The timing controls will impose to university mandatory deadlines for their hiring procedure, such as fixed time periods during which announcing the job positions, evaluating the candidates and sending the offers. Universities will still maintain their autonomy – since the procedure is carried out independently – but they will be timing-aligned in their recruitments. However, the Italian market hides more problems to be solves than the ones that are clearly visible. An example is offered by the committees' composition. Committees are locally build up by any University according to what stated by Law 240/2010, and they have the task of evaluating the applicants and decide the winner of the competition. The rules about the committee's composition are reported in the Regulation of the University, and

a little survey outlined that not all of them are really aligned to the legislative requirements. The lack of documentation – that should be available on the websites of each University and of the MIUR by law – highlights that the market is surely missing the requirements of *transparence and cleanness* contained into the European Charter for Researchers 2005. Without doubts even if the decentralized mechanism implemented is able to perfectly work under the right conditions, there is more to take into account when proposing a restyling. It is not possible to ignore that a decentralized structure leaves more space to *out-of-law actions* of agents – as pre-arrangements of matching – or to distortions in the Law implementation that are hard to identify, and to *maintain the control* on anything it happens during the different hiring procedure.

It came then spontaneous to provide a prototype of how this market would be if *centralized*. Indeed, despite the stability of the outcome when proposing a redesign of a matching procedure it is indispensable to consider any aspect. Then, given the current situation of the Italian case – that boasts low confidence and reputational level too – it seemed more opportune to present both easy restyling policies and huge reorganizational policies. The latter refers to the possibility of change the structure of the market and to adopt a centralize mechanism that implements the Deferred-Acceptance algorithm. This solution is more invasive than the suggestions of implementation of the current model, but for sure is more incisive. I showed the formal transformation of the market on a centralized basis and all its advantages, that – as already outlined – go beyond matching features. With the aim of creating a safety economic space for candidates – and driving an increase in the confidence levels – I proposed a Candidate-Proposing Deferred Acceptance algorithm. Even though recent studies demonstrated that usually applicants are truth-telling even when they are not on the proposing side (Ehlers and Massó 2007; Featherstone and Niederle 2016), I preferred the model to be strategy-proof for them. Moreover, since the Universities are Public Institutions and partially controlled by MIUR, it seemed easily to operate a truth-telling campaign on them respect to a non-controllable amount of candidates. Of course, the system worked as expected: it is efficient and the outcome is stable. However, still there

was something that was affecting the system. The controversial so called *meritocracy problem*, that gathers all that competitions won by candidates who did not *deserve* it. Specifically, candidates that up to the recruitment regulation were not able to win because they did not meet some of the requirements, or there was someone who satisfy them better. Understanding the reason why this happens was also the key of understanding how to integrate the problem into the matching setting, in order to find a way to solve it directly through the mechanism.

The third paper introduced for the first time the agency relation (Ross 1973a) in a matching problem. The roots of the meritocracy problem stay in the decision process implemented by Institutions for building up their preferences' lists: the regulations define the guidelines for the choice of the candidate but actually it is the committee who actively selects the winner of a competition. The misalignment of interests – the regulation address to deserve candidates while the committee to applicants they already know – explodes in a list of preferences that should represent Institutions' choices but instead reveals committees' personal desires. The, the Agency Problem entails a misrepresentation of the real Institutional preferences and the outcome of a Deferred Acceptance mechanism is stable up to Committee's choices. The inevitable domino effect (Roth 1984b) affects the whole final matching, and the worst consequences are for the candidates – who even deserving a job positions could end unassigned or assigned to an Institution they like less. Identifying the agency problem is then essential for ensuring the well-functioning of the mechanism, even it is a Deferred Acceptance algorithm – notice that it does not matter if the Institutions' side is the first on proposing or not. Since it could be a hidden feature of the market, I provided a categorization of two-sided matching markets on the basis of the decision rules adopted by their agents. It makes easier and faster to identify other real-world cases that could suffer of the same problem, and helps future researchers in being attentive in analyzing this kind of markets. Note that the formal – identified as the Institution – is not able to detect the misrepresentation of its preferences and it does not have the necessary information – held by the committee. The asymmetry of information between formal and representatives allow unfair actions to succeed. Since this asymmetry cannot

be fulfilled – due to the impersonal nature of the formal – the only possible way is to act on the range of freedom of committees' tasks. How to solve the agency problem vary by each specific case, this is why I proposed a range of solutions as starting point for other works, and a specific one for the Italian case. Then, I demonstrated that the meritocracy problem is really a cause of inefficiency of the market – and not just a supposed one – and that it is sufficient even also one misrepresentation for create a distortion of the final matching – opposed to who sustain that the corrupted contests are just a few and do not represent the true problem of the market. Moreover, I offer a more determined solution of what have been so far, whose implementation would surely annihilate unfair results.

The redesigning project on the hiring procedure for Assistant Professors in Italy ended then, with the final proposal of a centralized system, that implements a Candidate-Proposing Deferred Acceptance algorithm, and where the committees' selections are controlled through a computerized system.

The information is a fundamental element of matching markets and plays a plenty of roles according to the kind of market, but also according to where the point of focus is directed. Into the real case-study, for example, it does not create inefficiencies at a general model level; on a player level, it is the cause of the agency problem that drives the market to be unfair. The research works outlined the importance of taking into account the degree and the different forms of information that can coexist into the same mechanism, and to go beyond the traditional information boundaries. Moreover, there are still information roles to be explored – as the ones illustrated into the section on further studies.

Main Contributions to Literature

The dissertation presents a complete work of market design of the real-world hiring procedure for Assistant Professors in Italy. It has never been explored before, such that it enriches the case studies treated into the literature, bringing with it new features that were absent in previous cases. The dissertation covers all the main phases of a market re-design project, from the presentation and the analysis of the current model, passing by the reorganizational process, to the final proposal of a system that is able to solve almost all the inefficiencies it is affected by. It demonstrates how theories of different fields are used during the path for achieving the best solution possible: not only matching theories, but also behavioral economics, experimental economics (citing the results of others that sustain the theoretical work presented), game theory, corporate governance joined to analytical and problem solving skills. Since the papers treats both decentralized and centralized structures, the dissertation contributes to both branches of the literature on matching theories and market design.

Decentralized Mechanisms Literature. The first paper introduces a new kind of decentralized mechanism, never treated by other authors so far. It represents a quite usual hiring procedure scheme where the Recruiters' side is not going to offer the job positions all at the same time but over a finite number of periods – defined as rounds of the game. It could appear similar to the repetition of the same game by multiple times – already present in the literature (Damiano and Lam 2005) – but this is not the case. The main difference is given by the fact that the Recruiters' set is divided in sub-sets, and at each round a different sub-set opens the job positions. Candidates are allowed to send proposals only to the recruiters that belong to this sub-set. This means that Candidates do not achieve to send an application to all/some institutions they like only playing one round of the game. The dynamic game showed some similarities with the Deferred-Acceptance mechanism, and with the results for other decentralized mechanisms. Moreover, it incorporates some “contests' features” typical of the private job market and of public competitions – for example all candidates who

applied are ordered into a waiting list, if a candidate is not on the waiting list cannot be chosen since direct calls are not admitted. It also introduces a condition of “institutional acceptability” according to which no candidate who meets all the job requirements can be treated as an unacceptable one – and so rejected without reason. The condition has been designed on a Public Institutions setting, but can be easily adapted also to a private job market as an assumption of “costly hiring procedure”. In this case, companies prefer to hire someone as soon as possible by accepting an applicant who is currently available, instead of waiting an unknown number of rounds for a candidate they like more. The results about the SPE are not new in the literature, and the stability results are surely more interesting. In fact, the mechanism achieves stable results also for all agents truthfully revealing their preferences, and the outcome is Institutional-Optimal. The mechanism has been modeled into the context of Institutions-Candidates, but it can easily be applied to a context Firms-Candidates – as demonstrated – such as the mechanism resulted to be strongly generalizable – even if inspired by a specific case.

The second paper, firstly analyze a decentralized procedure into an incomplete information setting. The only results available until now are mostly confined to centralized model (Ehlers and Massó 2007; Roth and Rothblum 1999) and very little has been done so far in the decentralized domain (Pais et al. 2012). The study fulfills a dearth of the literature and poses itself as a starting point with the hope of triggering future research in this – highly interesting and unexplored – directions. While centralized systems are quite similar between them, decentralized matching can differ enough to do not guarantee that the results obtained in one of them would be valid also for others. This characteristic of the decentralized literature leaves a lot of space to further studies in the exploration of their functioning when preferences’ profiles are private information. The results obtained for this model retrace the ones presented by Roth and Rothblum (1999) and Ehlers and Massò (2007) for the Deferred-Acceptance algorithm. A confirmation of how much this mechanism is close to the centralized procedure even being a dynamic game. But the paper offers still more to the whole matching

literature by introducing for the first time a strong incomplete information setting – i.e. not only preference profiles are unknown by agents. The model has been stressed by removing step by step more and more information, till assuming that agents do not have starting list of preferences but just utility functions. According to them, during the game they select their actions, and through them they build up the preference order. The aim was to bring the model in an information setting that was as close to real-world markets as possible. Inspired by the Italian case, the information available are just a few, and the degree of uncertainty is extremely high. This condition would already be against a possible implementation of truncation strategies – whose risk would be very high and unpredictable – however, I pose a key assumption for simplifying the analysis: this time all agents play as each round would be the last round. As any chances of displaying a truncation strategy vanish, the only alternative for the agents is to truthfully reveal their preferences. Then, I demonstrated that for the decentralized multiple-round mechanism into a strong incomplete information setting, there exists a Bayesian Nash Equilibrium. The BNE is denoted by a strategy profile where all agents – both institutions and candidates – truthfully reveal their preferences. It could seem quite obvious that as most of the information are removed, the better strategy is to reveal real preferences. However, it is the closest model to real information setting and it was never explored and analyzed before. It wants to pave the way to further theoretical studies for being more reflexive of real conditions, such that they can offer more realistic responses and would be rapidly implementable. In fact, under a complete information setting the best strategy for each candidate was to reporting only a truncation of her preferences. If I would suggest it in the real-world case, probably the applicant would end unmatched. Studying the model in its natural context, allowed me to identify the really best strategy for candidates – not the one who represents an equilibrium for a theoretical model in an unrealistic setting, but the one obtained in same conditions as in the real market. The whole dissertation shows the importance of theoretical models, formalizing them and studying them in “benchmark conditions”, but then highlights also the indispensability of bringing them – step by step – into more

real-world settings. Note that in this case the result heavily relies on the final assumption, such that there is still space for future investigation also on this same model. Unique in this kind of – original and innovative – information setting, it is surely the highest contribution of this dissertation.

Centralized Mechanisms literature. Part of the dissertation and the third papers both contributes to the centralized structures branch of the literature. The dissertation per se adds a new case-study on the implementation of the Deferred Acceptance algorithm – following the well-known lessons learned from previous authors – into a real market that is experiencing inefficiencies, proposing it as a definitive solution. The third paper introduces a new feature that could be part of a matching problem but never treated before: an agency relation that exists between two different actors that together represent a sole Recruiter into the game. It happens quite often specially in the public market where Institutions have recruitment regulations (i.e. the formal) and use their employees (i.e. the representatives) to implement them. The agency relation has no influence till it does not work properly, i.e. till there is a misalignment of interests between the formal and the representatives (Donaldson and Davis 1991; Ross 1973a). This misalignment is translated into a misrepresentation of the Recruiter's preferences into the mechanism, but the recruiter does not know it is misreporting. A common consequence is that undeserving candidates are selected for the job position, and for this reason it has been introduced the definition of *unfair market* – notice that other definition of fairness exists in the matching literature but are not related. Simply implementing a centralized structure and run the D-A algorithm, it is not always sufficient. In case of a failing agency relation, the final matching would continue to be unstable up to real recruiters' preferences and totally unfair. The consequences are also over the assignment, as the reputation and reliable levels of the market that triggers the thickness of the candidates' side. Then, I focused on a new threat of a matching mechanism that was never highlighted before, and whose hidden nature can misconduct the re-design work. Additionally, I integrate a theory from the Corporate Governance into the Matching field, underlining as hints from other fields could be useful too.

Further Studies

The research works contained into this dissertation are exceptional starting point for a great number of interesting – original and innovative – further studies.

Integrating the announcement-timing of job positions as a strategy of Recruiters. In all the model the announcement-timing of Institutions has been considered as given and random. In some real-world markets there could be rules for deciding the announcing order, however it is widely common that Institutions and Firms do not have constraint on when announcing a job position. The timing of entrance into the market could be itself a strategy for addressing the candidates they prefer the most. For example, into the DMR model, the strategy space of Institutions has been reduced by the introduction of the institutional acceptability condition. This could drive them to look for new way of strategizing into the game. Removing the assumption of one-sided commitment and taking into account the two-sided commitment, the issue acquires relevance – indeed, I demonstrated that under one-sided commitment the permutation of the Institutions' set does not affect the game. The hypotheses behind this research proposal are a) there could be a connection between the timing of the announcements on the international and national markets; b) Institutions that look for the best deserving candidates would go on the market as soon as possible, otherwise they will wait; c) we should observe a recruitment trend based on the academic value of candidates (the bests are hired first and so on). Moreover, in case an Institution would like to hire someone they know but who is not deserving the position – unfair action – I would expect to register its entrance into the market as later as possible. The reason behind this potential strategy is very easy: the most deserving candidates have probably already been hired so there is less competition – and their selected winner could be truly the better of the applicants or at least it would less stand out.

Unregulated Timing Structure. In real decentralized hiring procedure, the recruitment process of each institution finishes at a different time, but mostly they do not finish during the same round of the job announcement. Actually, the

competitions open and there is a timing-crossing between new announcements that come up into the market, and old competitions that last for a finite – and undefined – number of rounds of the game. Candidates are forced to take a decision while still waiting the conclusion of another process, what will be the undominated strategy? And, it could be a strategy for Institutions making the competitions last longer than the others? Coming back to the example before, assume the committee wants to hire a specific non-deserving candidate but the applicants' lists is quite long and there are a lot of candidates who are better than the selected one. If the competition lasts enough, the applicants' lists can result split in half after a certain number of rounds. In fact, in the meanwhile some of them will accept other offers and will withdraw.

Public Opinion as external *influencing* agent. One of the vary forms information could have into a market, is the Public Opinion, denoted as the general thinking and speaking about the market by people who are not involved into the game. The idea is to give a voice to the external agents and to identify a different Public Opinion for each side of the market. The issue falls more in the behavioral field, but it has policy implications on a redesign proposal. The hypothesis is that a negative public opinion can trigger the misrepresentation of the preferences' lists. A low level of reliability and confidence in the assignment system could drive the players to not follow the suggestions given by ministries and politicians and to try to strategize. At the end, candidate will be penalized by their actions, but as in a vicious-cycle the bad assignment received could be perceived as the confirm of the public opinion and not as the consequence of a wrong strategy. On the other hand, professors – following the universities' example – will be more preoccupied in recommending their “protect candidate” to colleagues who are potential members of committees. The distortions would be evident even in a centralized structure, since a central national committee cannot substitute the universities' commissions. The worst final scenario would be a thin market, where the only ones who participate are already sure to receive a call as soon as they are evaluated by the central committee. The project aim is to test this hypothesis and to show the potential consequences, but also the

potential solutions. Additionally, it would light up a more behavioral aspect of matching markets, outlining that considering mechanism and strategy could not be sufficient in some real-world case, and that there is more to explore for ensuring the real success of its redesign.

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Appendix

The space of the Appendix is reserved for an in-depth analysis of the data about the recruitment procedure for researchers in the Italian Academic market. This information are necessary for a carrying on a correct and detailed project of market design where the deep knowledge of the market represents the key success factor.

A1. Comparison between Recruitment Regulations of single random selected Universities and the Law in force (Law 240/2010)

The Regulations of a sample of random selected universities:

- University of Torino (announcement 19/12/2011, Dep. of Political Sciences): the committees are made up of three members selected by the requesting department and chosen among full or associate professors, foreign universities' ones are included, and they must have been working in the specific scientific-sector selected. Two of them must work in external universities. If necessary the committee's members could be expert in similar scientific-sectors. Incompatibility and conflict of interests' rules are considered during the choice of the committee's members.
- University of Genova (Decree n. 328, 20/02/2013): the committee is nominated by the requesting department; the three members' scientific value must be proved. One of them is nominated president and has to be a full or associate professor, two of them must come from external universities (also foreigner ones). The committee nomination is published on the website of the University.
- University of Sassari (Decree n. 1414, 27/05/2013): the committee is composed of three members: a senior professor (full or associate) as president of the committee and other two professors whose at least one from an external university. The committee nominated is published on the website of the University.

- University of Venezia (Decree n. 99, 08/02/2013): the members of the committee are chosen among senior professors, first proposed by the department and then confirmed by the dean. Two members have to be of external universities, foreign ones are included. The committee's members must have been working in the scientific area during the last 5 years and they need to respect the criteria of incompatibility and conflict of interests'.
- University of Reggio Calabria (Decree n. 227, 31/07/2013): the committee is nominated with a decree of the dean and it will be published on the website of the University. University of Reggio Calabria doesn't specify the composition of the committee in the announcement neither inserts a reference to a regulation for the recruitment of researchers with fixed term contracts. However, searching in the website of the university it is possible to find the regulation (Decree n. 353, 21/12/2011) : the committee will be made up by three professors, at least one external, at least one member of the scientific-sector that offers the job positions and at least one who is a full professor. The members of the committee are chosen by the Department that makes the request.
- University of Cagliari (Decree n. 858, 05/06/2013): the committee is composed by three professors, in particular two senior professors of the Department that offers the job positions and one professor external to the university.
- University of Milano (Decree n. 5109, 25/07/2013): the committee is made up by three members, one is chosen among the senior professors of the department and the other two members have to be of external universities, even foreign, and have to be randomly selected from the appropriate lists.

The Universities of Milan, Turin, Genoa and Venice respect the rules imposed by the Law 240/2010 to the letter, on the contrary, the others present an out-of-the-rules committee's structure. This aspect could be regulated by MIUR through direct and strong control of what redacted by the Universities.

A2. Art. 24 Law 240/2010 Recruitment of Assistant Professors

Art. 24.

(Ricercatori a tempo determinato)

1. Nell'ambito delle risorse disponibili per la programmazione, al fine di svolgere attività di ricerca, di didattica, di didattica integrativa e di servizio agli studenti, le università possono stipulare contratti di lavoro subordinato a tempo determinato. Il contratto stabilisce, sulla base dei regolamenti di ateneo, le modalità di svolgimento delle attività di didattica, di didattica integrativa e di servizio agli studenti nonché delle attività di ricerca.

2. I destinatari sono scelti mediante procedure pubbliche di selezione disciplinate dalle università con regolamento ai sensi della legge 9 maggio 1989, n. 168, nel rispetto dei principi enunciati dalla Carta europea dei ricercatori, di cui alla raccomandazione della Commissione delle Comunità europee n. 251 dell'11 marzo 2005, e specificamente dei seguenti criteri:

a) pubblicità dei bandi sul sito dell'ateneo e su quelli del Ministero e dell'Unione europea; specificazione del settore concorsuale e di un eventuale profilo esclusivamente tramite indicazione di uno o più settori scientifico-disciplinari; informazioni dettagliate sulle specifiche funzioni, sui diritti e i doveri e sul relativo trattamento economico e previdenziale; previsione di modalità di trasmissione telematica delle candidature nonché, per quanto possibile, dei titoli e delle pubblicazioni;

b) ammissione alle procedure dei possessori del titolo di dottore di ricerca o titolo equivalente, ovvero, per i settori interessati, del diploma di specializzazione medica, nonché di eventuali ulteriori requisiti definiti nel regolamento di ateneo, con esclusione dei soggetti già assunti a tempo indeterminato come professori universitari di prima o di seconda fascia o come ricercatori, ancorché cessati dal servizio;

c) valutazione preliminare dei candidati, con motivato giudizio analitico sui titoli, sul curriculum e sulla produzione scientifica, ivi compresa la tesi di dottorato, secondo criteri e parametri, riconosciuti anche in ambito internazionale, individuati con decreto del Ministro, sentiti l'ANVUR e il CUN; a seguito della valutazione preliminare, ammissione dei candidati comparativamente più meritevoli, in misura compresa tra il 10 e il 20 per cento del numero degli stessi e comunque non inferiore a sei unità, alla discussione pubblica con la commissione dei titoli e della produzione scientifica; i candidati sono tutti ammessi alla discussione qualora il loro numero sia pari o inferiore a sei; attribuzione di un punteggio ai titoli e a ciascuna delle pubblicazioni presentate dai candidati ammessi alla discussione, a seguito della stessa; possibilità di prevedere un numero massimo, comunque non inferiore a dodici, delle pubblicazioni che ciascun candidato può presentare. Sono esclusi esami scritti e orali, ad eccezione di una prova orale volta ad accertare l'adeguata conoscenza di una lingua straniera; l'ateneo può specificare nel bando la lingua straniera di cui è richiesta la conoscenza in relazione al profilo plurilingue dell'ateneo stesso ovvero alle esigenze didattiche dei corsi di studio in lingua estera; la prova orale avviene contestualmente alla discussione dei titoli e delle pubblicazioni. Nelle more dell'emanazione del decreto di cui al primo periodo, si applicano i parametri e criteri di cui al decreto del Ministro adottato in attuazione dell'articolo 1, comma 7, del decreto-legge 10 novembre 2008, n. 180, convertito, con modificazioni, dalla legge 9 gennaio 2009, n. 1;

d) formulazione della proposta di chiamata da parte del dipartimento con voto favorevole della maggioranza assoluta dei professori di prima e di seconda fascia e approvazione della stessa con delibera del consiglio di amministrazione.

A3. The European Charter for Researchers 2005

Commission Recommendation of 11 March 2005 on the European Charter for researchers and on a Code of Conduct for the Recruitment of Researchers.

«The European Charter for Researchers is a set of general principles and requirements which specifies the roles, responsibilities and entitlements of researchers as well as of employers and/or funders of researchers. The aim of the Charter is to ensure that the nature of the relationship between researchers and employers or funders is conducive to successful performance in generating, transferring, sharing and disseminating knowledge and technological development, and to the career development of researchers. The Charter also recognizes the value of all forms of mobility as a means for enhancing the professional development of researchers.»

«The code of conduct for the recruitment of researchers consists of a set of general principles and requirements that should be followed by employers and/or funders when appointing or recruiting researchers. These principles and requirements should ensure observance of values such as transparency of the recruitment process and equal treatment of all applicants, in particular with regard to the development of an attractive, open and sustainable European labor market for researchers, and are complementary to those outlined in the European Charter for Researchers. Institutions and employers adhering to the Code of Conduct will openly demonstrate their commitment to act in a responsible and respectable way and to provide fair framework conditions to researchers, with a clear intention to contribute to the advancement of the European Research Area.»

In succession a list of some general requirements and principles for the Code of Conduct:

«Recruitment_Employers and/or funders should establish recruitment procedures which are open, efficient, transparent, supportive and internationally comparable, as well as tailored to the type of positions advertised. Advertisements should give

a broad description of knowledge and competencies required, and should not be so specialized as to discourage suitable applicants.»

«Selection_Selection committees should bring together diverse expertise and competences and should have an adequate gender balance and, where appropriate and feasible, include members from different sectors (public and private) and disciplines, including from other countries and with relevant experience to assess the candidate. Whenever possible, a wide range of selection practices should be used, such as external expert assessment and face-to-face interviews. Members of selection panels should be adequately trained.»

«Transparency_Candidates should be informed, prior to the selection, about the recruitment process and the selection criteria, the number of available positions and the career development prospects. They should also be informed after the selection process about their strengths and weaknesses of their applications.»

A4. Data from the survey

– based on 150 answers. The survey was send during the year 2014 to current researchers of random selected Departments at 6 different randomly selected universities: the University of Ancona (Politecnico delle Marche), Modena and Reggio Emilia, Reggio Calabria (Mediterranea), Napoli (Parthenope), Catania, Bergamo. The answers have been gathered anonymously, somebody reported the University and Department they belong to, but not all of them.

Survey Data (based on 150 responses)

N. of competitions per person	Average 2.25
Recruitment System Evaluation	<i>Likert scale 1 to 7</i>
General Efficiency	2.76 (average)
Amount of Information	3.44
Timing	3.66
Evaluation of Scientific Research	3.36
Evaluation of Oral Test	3.36
Factors with Positive Impact	<i>Multiple Choice</i>
Geographic Position	68.70%
Reputation	22%
Research Projects	28.70%
Infrastructures	7%
Services	6%
External Collaborations	10%
Previous Experiences at that University	63%
Meritocratic Evaluation	15%
Absence of Internal Candidates	15%

Factors with Negative Impact

Multiple Choice

Geographic Position	25%
Reputation	12%
Research Projects	16%
Infrastructures	16%
Services	14%
External Collaborations	17%
Lack of Previous Experiences at that University	13%
Non-Meritocratic Evaluation	27%
Presence of Internal Candidates	32.70%

Have you ever renounced to participate to a contest for a job position you liked?

YES 12%
NO 88%

How do you evaluate the Meritocracy Level you experienced during the recruitment procedure?

Likert Scale 1 to 5

2.8 (average)

Would you prefer a decentralized or a centralized recruitment procedure?

CENTRALIZED 41%
DECENTRALIZED 59%

Extr. Neg	1
Negative	2
Almost Acc.	3
Acceptable	4
More than Acc.	5
Positive	6
Extr. Positive	7

Figure 8. Likert Scale 1 to 7

Extr. Neg	1
Negative	2
Acceptable	3
Positive	4
Extr. Positive	5

Figure 7. Likert Scale 1 to 5

- Each subject was asked to evaluate the level of meritocracy experienced in their own experience. The reported value depended on their personal perception.
- The 43% of the interviewed would prefer a centralized system. These data highlight, at the same time, that some academics already recognize the necessity of changing the structure and that a redesign of the system would be not so rejected by the academic market.
- **Some of the comments left by people who answered YES at the question “Have you ever renounced..” :**

“they suggested me to not participate”

“because of a non-meritocratic evaluation”

“do not hurt the internal candidate and put in embarrassment members of the committee that I personally know”

“contests with a pre-determined ending given the presence of the internal candidate”

“presence of internal candidates”

“internal policies”

“scarcity of competitions”

“Understanding of game mechanisms”

- Other comments left on the specific space after the question of the evaluation of the system:

“the competitions are just a few and most of the time it is not possible to deal with them because they are already assigned”

“The competition in Italy do not follow any meritocracy, they are assigned for nepotism or influence peddling”

“there is still lack of transparency on the final choices”

“it is needed to eliminate the Barons”

- In total, the survey was build up by 10 questions: 3 yes/no questions; 2 multiple choice questions; 2 Likert scale (one from 1 to 7; and one from 1 to 5); 1 open questions; 2 control questions (number of competitions completed for obtaining the job, name of the University and of the Department of belonging). The survey tool used was a free online survey tool called SurveyMonkey.

A5. Analysis of the winners of the period 2004-2011 of competitions announced by Economic Departments

The analysis has been focused on the competitions of Economics SSD announced by Economics Departments of 5 randomly selected universities: University of Pavia, University of Cagliari, University Politecnica delle Marche (Ancona), University of Bergamo, University Mediterranea (Reggio Calabria). They have been randomly selected into area sub sets, such that they can represent the entireness of the national population.

Data are reported in this paragraph together with some examples of how the they have been (manually) extrapolated.

The MIUR website presents a specific page for the academic recruitment, and there is a browser that facilitate the research of winners of contests per faculty and per university. Once the filter are imposed, the results appear in the following form:



For having access to contest's documentation it is sufficient to click on the object highlighted by the black circle. The basic information are provided for all the competitions (e.g. number of total applications,...), however, the richness of the documentation changes by contest.

Figure 9. Example of Data _ University of Pavia

Since the amount of information varies by competition, only the basic data have been gathered per contest, as reported into the following tables.

University of Pavia - Faculty of Economics

Session	Competition	SSD	Opening Date	Closing Date	# of Appl.	Winner
IV-2004	no					
I-2005	no					
II-2005	no					
III-2005	no					
IV-2005	no					
I-2006	no					
II-2006	no					
III-2006	no					
I-2007	yes	SECS P/07	03/04/2007	27/02/2008	2	Michela Pellicelli
		SECS P/10	03/04/2007	28/03/2008	2	Alberto Francesconi
		SECS S/01	03/04/2007	27/02/2008	17	Matteo Ruggiero
II-2007	yes	SECS P/01	07/03/2008	03/12/2008	37	Lorenza Rossi
		SECS P/07	07/03/2008	03/12/2008	1	Francesco Sotti
		SECS P/08	07/03/2008	03/12/2008	5	Francesco Velo
I-2008	no					
I-2008	no					
I straordinaria	no					
I-2008	no					
II straordinaria	no					
II-2008	yes	SECS P/01	30/10/2009	27/12/2010	83	Fabio Tramontana

		SECS P/08	30/10/2009	19/11/2010	9	Stefano Denicolai
II-2008						
I straordinaria	no					
I-2009						
straordinaria	no					
I-2010	yes	SECS P/02	09/07/2010	17/10/2011	61	Marcella Giovanna Maria Nicolini
		SECS P/07	27/07/2010	15/09/2011	12	Maria Chiara Demartini
I-2010						
I straordinaria	no					
II-2010	no					
2011	yes	SECS S/03	15/07/2011	25/11/2011	?*	Paola Cerchiello
		SECS P/09	30/09/2011	13/12/2011	?*	Emanuel Bagna

*the data are accessible only by password.

University of Cagliari - Faculty of Economics						
Session	Competition	SSD	Opening Date	Closing Date	# of Appl.	Winner
IV-2004	no					
I-2005	no					
II-2005	yes	SECS P/01	15/04/2005	06/07/2006	13	Stefano Matta
		SECS P /07	14/01/2005	19/04/2006	10	Milena Serra
		SECS P/07	15/04/2005	30/03/2006	15	Isabella Fadda
		SECS P/08	14/01/2005	30/03/2006	5	Giuseppe Argiolas
III-2005	no					
IV-2005	no					
I-2006	no					
II-2006	yes	SECS P/02	15/04/2005	16/07/2007	15	Luigi Mundula
		SECS P/08	15/04/2005	18/04/2008	7	Michela Floris
		SECS P/11	04/04/2006	24/04/2007	6	Fabrizio Crespi
III-2006	no					
I-2007	no					
II-2007	yes	SECS S/01	07/03/2008	02/02/2009	11	Isabella Sulis

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I-2008	yes	SECS S/06	07/03/2008	11/01/2011	5	Filippo Petroni
I-2008 I straordinaria	no					
I-2008 II straordinaria	no					
II-2008	yes	SECS P/01	01/09/2009	11/01/2011	29	Andrea Isoni
		SECS P/07	01/09/2009	11/01/2011	12	Patrizio Monfardini
II-2008 I straordinaria	no					
I-2009 I straordinaria	no					
I-2010	no					
I-2010 I straordinaria	no					
II-2010 2011	no yes					
		13/B1	13/07/2011	13/12/2011	4	Silvia Gaia
		13/D1	13/07/2011	12/12/2011	1	Massimo Cannas

University "Mediterranea" of Reggio Calabria – Fac. of Law and Economics

Session	Competition	SSD	Opening Date	Closing Date	# of Appl.	Winner
		SECS P/03		06/02/2004		Calogero Guccio
		SECS P/03		31/12/2004		Michela Mantovani
IV-2004	no					
I-2005	no					
II-2005	no					
III-2005	no					
IV-2005	no					
I-2006	no					
II-2006	no					
III-2006	no					
I-2007	yes	SECS P/03	16/03/2007	26/06/2008	8	Domenico D'Amico
		SECS P/06	16/03/2007	28/10/2008	10	Massimo Finocchiaro
		SECS P/07	16/03/2007	26/06/2008	6	Castro Carlo Vermiglio
II-2007	no					
I-2008 I straordinaria	no					

I-2008	
II straordinaria	no
II-2008	no
	no
II-2008	
I straordinaria	no
2009 straordinaria	no
I-2010	no
I-2010	
I straordinaria	no
II-2010	no
I-2011	no

University of Bergamo – Faculty of Economics

Session	Competition	SSD	Opening Date	Closing Date	# of Appl.	Winner
IV-2004 & I-2005	yes	SECS P/08	10/12/2004	21/12/2005	2	Mara Bergamaschi
		SECS P/11	02/11/2004	17/10/2005	4	Lucia Gibilaro
		SECS S/01	02/11/2004	16/10/2006	4	Valeria Caviezel
II-2005	yes	SECS P/03	02/11/2004	15/02/2006	13	Pietro Vertova
III-2005	yes	SECS P/01	11/10/2005	12/07/2006	14	Paolo Buonanno
		SECS P/01	31/05/2005	28/02/2006	20	Stefano Lucarelli
		SECS P/06	11/10/2005	24/07/2006	12	Alberto Prandini
		SECS P/07	31/05/2005	12/07/2006	1	Daniele Gervasio
		SECS S/06	11/10/2005	02/08/2006	11	Francesca Maggioni
IV-2005	no					Daniela
I-2006	yes	SECS P/08	24/02/2006	30/05/2007	3	Giuseppina Andreini
II-2006	no					
III-2006	yes	SECS P/07	22/12/2006	19/06/2007	3	Silvana Signori
		SECS S/03	22/12/2006	13/09/2007	3	Daniele Toninelli
I-2007	yes	SECS P/10	06/04/2007	12/06/2008	1	Stefano Basaglia
II-2007	yes	SECS P/02	18/03/2008	10/12/2008	15	Mara Grasseni

			SECS P/07	18/03/2008	27/11/2008	1	Massimo Contrafatto
			SECS P/08	14/12/2007	17/11/2008	3	Gianpaolo Baronchelli
			SECS S/01	14/12/2007	03/12/2008	13	Michela Cameletti
I-2008	no						
I-2008							
I straordinari	no						
a							
I-2008							
II straordinari	no						
a							
II-2008	yes		SECS P/07	23/10/2009	06/04/2011	4	Mariafrancesc a Sicilia
			SECS P/08	23/10/2009	06/04/2011	22	Cristina Bettinelli
			SECS P/10	23/10/2009	15/02/2011	14	Edoardo Ezio Dalla Torre
II-2008							
I straordinari	no						
a							
2009							
straordinari	no						
a							
I-2010	no						
I-2010							
I straordinari	no						
a							
II-2010	no						
I-2011	no						

The red frame highlights a duration of almost two years for the completion of the hiring procedure.

University Politecnica delle Marche (Ancona) – Fac. of Economics

Session	Competition	SSD	Opening Date	Closing Date	# of Appl.	Winner
IV-2004 e I- 2005	yes	SECS P/07	09/01/2004	28/02/2005	3	Simone Poli
		SECS P/01	01/10/2004	28/10/2005	10	Alessia Lo Turco
		SECS P/06	01/10/2004	27/02/2006	4	Nicola Matteucci
		SECS S/06	01/10/2004	27/02/2006	14	Sebastiano Silla

II-2005	yes	SECS P/01	08/04/2005	28/09/2006	17	Matteo Guido Richiardi David Bartolini Federica Pascucci Camilla Mazzoli Augusto Ciuffetti
		SECS P/03	08/04/2005	28/09/2006	8	
		SECS P/08	08/04/2005	28/09/2006	2	
		SECS P/11	04/01/2005	28/09/2006	3	
		SECS P/12	08/04/2005	27/02/2006	4	
III-2005	no					
IV-2005	no					
I-2006	no					
II-2006	yes	SECS P/08	14/04/2006	20/09/2007	2	Silvio Cardinali Barbara Zagaglia Andrea Filippo Presbitero
		SECS S/04	14/04/2006	27/02/2007	3	
III-2006	yes	SECS P/02	18/08/2006	26/10/2007	22	
I-2007	no					
II-2007	yes	SECS P/01	08/01/2008	23/12/2008	29	Alberto Russo Giulio Palomba Marco Giuliani Oscar Domenichelli Roberto Giulianelli
		SECS P/05	08/01/2008	30/10/2008	7	
		SECS P/07	08/01/2008	30/10/2008	3	
		SECS P/09	08/01/2008	23/12/2008	4	
		SECS P/12	08/01/2008	30/10/2008	5	
I-2008	no					
I-2008						
I straordinaria	no					
I-2008						
II straordinaria	no					
II-2008	yes	SECS P/08	13/10/2009	30/11/2010	7	Valerio Temperini Chiara Gigliarano Serena Brianzoni
		SECS S/01	13/10/2009	17/10/2010	24	
		SECS S/06	13/10/2009	25/02/2011	13	
II-2008						
I straordinaria	no					
2009 straordinaria	no					
I-2010	no					

I-2010 I straordinaria	no						
II-2010	yes	SECS P/02	21/12/2010	29/10/2012	41	Matteo Picchio	
		SECS P/07	21/12/2010	29/10/2012	12	Marco Gatti	
I-2011	no						

Then, the focus of the analysis was shifted on the winners of the competitions. First of all, I considered interesting – and useful – to check whether the winners had a previous work relationship with the same Institutions that hired them¹⁹¹. In the affirmative case, they could be considered the “internal candidates” of the competition.

# of Appl.	Winner	University of Pavia			
		PhD	Research Fellowship	Teaching Activity	Post-Doc
2	Michela Pellicelli	x			
2	Alberto Francesconi	x	x	x	
17	Matteo Ruggiero				
37	Lorenza Rossi				
1	Francesco Sotti	x	x	x	
5	Francesco Velo	x	x		
83	Fabio Tramontana				
9	Stefano Denicolai	x			
61	Marcella Giovanna Maria Nicolini				
12	Maria Chiara Demartini	x			

The table shows that 6 out of 10 winners had a previous relationship with the university, mostly all of them achieved their PhD degree at the hiring institution. As above-mentioned, these winners could be considered the internal candidates of the competitions. Their presence, then, could have discouraged others to apply. It

¹⁹¹ Actually, the operation was extended to all the ones who were Assistant Professors during the time of this analysis – carried out at the end of 2014 and starting of 2015.

is not possible to sustain this hypothesis since the information is not sufficient, and because the number of applications could be affected also by other factors. However, it is interesting to notice that the competitions with the higher numbers of applications are the ones won by “external candidates” in the case of the University of Pavia. Unfortunately, this relation is not confirmed by the cases of the other Universities, so it is clear that a deeper analysis is needed.

University of Cagliari

# of Appl.	Winner	PhD	Research Fellowship	Teaching Activity	Post-Doc
13	Stefano Matta	x			
10	Milena Serra				
15	Isabella Fadda	x			
5	Giuseppe Argiolas	x			
15	Luigi Mundula			x	
7	Michela Floris	x			
6	Fabrizio Crespi				
11	Isabella Sulis		x		
5	Filippo Petroni				
29	Andrea Isoni				
12	Patrizio Monfardini				
4	Silvia Gaia		x		
1	Massimo Cannas			x	

University “Mediterranea” of Reggio Calabria

# of Appl.	Winner	PhD	Research Fellowship	Teaching Activity	Post-Doc
	Calogero Guccio				
	Michela Mantovani				
8	Domenico D'Amico				
10	Massimo Finocchiaro Castro				
6	Carlo Vermiglio				

University of Bergamo

# of Appl.	Winner	PhD	Research Fellowship	Teaching Activity	Post-Doc
2	Mara Bergamaschi			x	
4	Lucia Gibilaro			x	
4	Valeria Caviezel			x	

13	Pietro Vertova				
14	Paolo Buonanno			x	
20	Stefano Lucarelli				
12	Alberto Prandini				
1	Daniele Gervasio			x	
11	Francesca Maggioni	x	x		
3	Daniela Giuseppina Andreini			x	
3	Silvana Signori			x	
3	Daniele Toninelli	x	x	x	
1	Stefano Basaglia				
15	Mara Grasseni	x	x	x	
1	Massimo Contrafatto			x	x
3	Gianpaolo Baronchelli	x		x	
13	Michela Cameletti		x		
4	Mariafrancesca Sicilia				
22	Cristina Bettinelli	x			
14	Edoardo Ezio Dalla Torre				

University “Politecnica delle Marche” of Ancona

# of Appl.	Winner	PhD	Research Fellowship	Teaching Activity	Post- Doc
3	Simone Poli		x	x	
10	Alessia Lo Turco	x			
4	Nicola Matteucci	x			
14	Sebastiano Silla	x			
17	Matteo Guido Richiardi				
8	David Bartolini				
2	Federica Pascucci	x	x	x	
3	Camilla Mazzoli	x			
4	Augusto Ciuffetti			x	
2	Silvio Cardinali	x			
3	Barbara Zagaglia	x	x		
22	Andrea Filippo Presbitero	x	x		
29	Alberto Russo				
7	Giulio Palomba	x		x	
3	Marco Giuliani		x		

4	Oscar Domenichelli	x		x
5	Roberto Giulianelli		x	
7	Valerio Temperini	x	x	
24	Chiara Gigliarano		x	
13	Serena Brianzoni			
41	Matteo Picchio	x		
12	Marco Gatti	x	x	

The following table presents a recap on the number of winners who had a previous relation with the hiring universities.

University of Pavia	6 out of 10
University of Cagliari	8 out of 13
University “Mediterranea” of Reggio Calabria	0 out of 5
University of Bergamo	14 out of 20
University “Politecnica delle Marche” of Ancona	17 out of 20

The analysis on winners was then shifted toward their scientific work. The idea was to gather data on the number of publications before and after the competition. At first, the idea was to compare data of winners with data of competitors, unfortunately data on other competitors of the contests are not available for all of them (even if they should be published online by Law). Since the dataset is large and do not enrich the work at this state of the art, it is not included in the dissertation.