University of Pavia

Department of Economics and Management

PhD Thesis in Economics and Management of Technology

Monetary Policy Transmission and Systemic Risk in the Eurozone

Author:
Laura Parisi

Supervisor:
Prof. Paolo Giudici

January 2017

It is better to be approximately right
than precisely wrong

C. Read,
"Logic: deductive and inductive" (1898)

Declaration by the Author

I declare that this thesis has been composed solely by myself and that it has not been submitted, in whole or in part, in any previous application for a degree. Except where states otherwise by reference or acknowledgment, the work presented is entirely my own.

Copyright statement

Non-technical Summary

In this thesis we will address both monetary and macro-prudential policy issues as the two main components of economic stability. In the first part of the thesis we will analyse the monetary policy transmission mechanism, trying to understand how the interest rate pass-through has changed over time and how it depends on country-specific macroeconomic fundamentals. In the second part of the thesis we will deal with systemic risk. Firstly by looking at how market-implied default probabilities change when macroeconomic variables such as the Debt/GDP ratio are included. Secondly, by analysing contagion effects and propagation mechanisms across countries and economic sectors. Finally, we will concentrate on the banking system at the micro-level, with the aim of understanding if the new bail-in regulation decreases systemic risk with respect to a bail-out.

From a methodological viewpoint, we will combine the cross-sectional and the time dimensions: in the first part of the thesis, by means of seemingly unrelated and dynamic hierarchical models; in the second part, by merging multivariate stochastic processes and partial correlation networks. All models will be applied to Eurozone countries.

We will show that the transmission mechanism of monetary rates on bank rates is much more effective in core countries rather than in peripheral ones; a strong heterogeneity can also be observed by analysing different kinds of loans. A further clustering effect between core and peripheral countries will be detected in terms of default probabilities, with the former behaving as importer and the latter more as exporter of systemic risk. The time-dimensional analysis will stress the differences between the financial and the sovereign debt crisis in terms of heterogenous consequences on countries; in addition, the sequence of these two events has determined an irreversible phase change leading to a new non-stable equilibrium, where instability derives from peripheral economies diverging from core ones. Finally, we will understand the advantages of a private bail-out resolution with respect to a bail-in, from both a single bank and a system perspective. Such advantage becomes larger as the dimension of the bank likely to fail increases, and as the correlation pattern becomes positively stronger.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

The recent years have witnessed a great amount of financial distress events that have become the main subject of economic study and research. After the stability that characterised the first 10 years of the European Economic and Monetary Union, the serious tensions that arose in international financial markets in august 2007 due to the U.S. sub-prime crisis, and the collapse of the America's fourth largest investment bank, Lehman Brothers, in September 2008, sparked a global financial crisis that affected the real sector and caused a rapid, synchronised deterioration in most major economies. Many European banks, in fact, had heavily invested in the U.S. mortgage market: in an attempt to stop some banks from failing, governments came to the rescue in many E.U. countries. The cost of bailing out so many banks, however, proved very high: in Ireland, it almost bankrupted the government until fellow European countries stepped in with financial assistance.

The European Union is thus an extremely challenging case study since it first faced a banking crisis, and then a sovereign debt crisis due to the propagation of the American sub-prime disaster to the European banking system and, in turn, to governments. Such evolution and the rising of so many issues in the financial system at the global level have introduced a number of questions and perspectives. Let us start with some questions: what financial stability or systemic risk is? How should it be identified and assessed? Are macro- or micro- prudential policy and monetary policy connected? And what should they have as an objective?

There is common consensus about the fact that the financial stability objective is to achieve a strength level in the provision of financial services over the entire business cycle: this must translate into a support to the economic growth, taking into account both the prevention of distress events and the mitigation of their consequences in case a crisis occurs. Consistently, the financial stability analysis is the study of potential sources of systemic risk that may come from the combination of vulnerabilities in the financial system (endogenous) and potential shocks propagating across different economic sectors, markets or countries. As an example, the CNB defines *"financial stability as a situation where the financial system operates with no serious failures or undesirable impacts on the present and future development of the economy as a whole, while showing a degree of resilience to shocks"*.[1]

But financial stability represents only only side of economic stability: since it deals with the financial side, it has to be combined with the monetary policy, which instead is more related to the real-economy side. A clear distinction between the two comes, for example, from V. Constâncio, who claims: *the financial cycle and the business cycle are not synchronised. [...] Monetary policy aims at ensuring price stability in the market for goods and services, and it should not be used to address pockets of instability in asset markets.*[2] In other words, while monetary policy needs to support real activities, financial stability policy has to address issues related to the asset market segment, paying attention to the rise of exuberances or imbalances.

1.1 Monetary Policy

Monetary policy consists in a number of actions a central bank or a regulatory committee can do in order to determine the size and the rate of growth of the money supply. Since they are the sole issuer of bank reserves, central banks can in fact set the conditions at which banks borrow from them, thus determining the conditions at which banks trade with each other and with their clients in the money market. The main objective of the monetary policy in Europe is to ensure price stability, since it is clearly stated in the Treaty on the Functioning of the European Union that this is the most important contribution in order to achieve a favourable economic environment and a high level of employment.

The study of monetary policy and its transmission mechanism to the real economy is a complex issue. To put it simply, there are two main types of monetary policy: expansionary and contractionary. A contractionary monetary policy aims at slowing the growth rate in the money supply in order to control inflation, and it usually consists, among others, in fixing high monetary rates. On the contrary, an expansionary monetary policy aims at increasing the money supply in order to lower unemployment, boost private lendings and consumer expenditure, and stimulate economic growth. One tool for reaching

[1]Source: CNB, 2004. Financial Stability Report.

[2]Source: V. Constâncio, 2015. Speech at the Warwick Economics Summit.

those objectives consists in fixing very low interest rates, and it applies to the recent time period when monetary rates are very close to zero or even negative in almost all developed economies, for helping western countries to recover after the financial crisis of 2008.

The theoretical literature on how the monetary policy can increase risk is quite mixed and inconclusive. Some researchers argue that an expansionary monetary policy, combined with banks moral hazard, may increase risk on the lending side. Other studies reach opposite conclusions, demonstrating that too low monetary rates may spur bank risk-taking. The relationships between a bank's risk, business cycle and monetary rates is not one of the objectives of this thesis. We are instead interested in understanding the monetary policy transmission mechanism, particularly interesting in the recent time period, characterised by very low monetary rates and crisis events.

In particular, the way monetary policy can impact the real economy is difficult to predict, since it involves long and uncertain time lags, as well as a great amount of agents. A change in money market interest rates, for example, can directly affect bank administered interest rates which, in turn, may modify both the supply and the demand sides for goods and labour. Such relationships, however, can be affected by external factors outside the control of the central bank, such as changes in the global or domestic economy. The way bank administered interest rates may depend on monetary rates or other exogenous factors is the main objective of Chapters 2 and 3. In particular, in *Predicting bank interests when monetary rates are close to zero* we analyse the relationship between interest rates on overnight deposits, aggregated at the Italian level, and monetary rates: we provide an alternative model to the traditionally used error correction model, and we apply it to derive banks interest risk, from both the income and the economic value perspective. Our results show that, differently from the past, the most important component in determining bank rates now is the autoregressive factor: such regime switching, however, seems to not strongly impact interest rate risk. In *Dynamic hierarchical models for monetary transmission* we analyse interest rates on lendings. More precisely, we provide a dynamic modelling (time-varying parameters) for describing how administered bank interest rates react in response to changes in money market rates, in a multi-country setting. By means of hierarchical equations, we take into account how such changes are affected by the macroeconomic fundamentals of each country. These models are applied to seven eurozone countries and three kinds of interest rates: loans to corporates up to 1 million euros (small-medium enterprises); loans to corporates over 1 million euros (big firms); loans to families for mortgages. to interest rates on different loans (to corporates and families). Our results show how the monetary policy and the specific situation of each country have differently impacted lendings, not only across countries but also across time.

1.2 Macroprudential Policy

As previously introduced, the financial stability analysis has to translate into financial stability policy, whose main element is macroprudential policy: as C. Borio said, in fact, *"paraphrasing Milton Friedman, we are all macroprudentialists now"*.[3] If the traditional microprudential regulation focuses on the resilience of individual financial institutions to exogenous events, macroprudential supervision analyses the stability of the system as a whole, by monitoring endogenous processes in which financial institutions are characterised by common behaviours and mutual interactions: in other words, the drivers of risk are endogenous and depend on the collective behaviour of financial institutions. This new approach means that regulatory arrangements have to be calibrated from a systemic perspective rather than focusing on the safety of individual institutions on a stand-alone basis.

While it is clear that the final objective of a macroprudential approach is to avoid risk and contain the consequences a crisis produces when it might occur, it is also important to understand the levels a macroprudential analysis should take into account. Systemic risk, in fact, has two dimensions: the time dimension, which reflects the evolution of the system over time; the cross-sectional dimension, which reflects the distribution of risk across different geographical areas or economic sectors at any given point in time. If it is true that these two dimensions have different sources of risk, being the former more linked to the business cycle while the latter mostly depends on mutual exposures or chained market co-movements, it is also true that they jointly evolve and can be hardly disentangled. The time-dependence can be addressed with the development of countercyclical buffers that help stabilising the system, so that each institution does not exceed a certain level of contribution to systemic risk even during distress events. On the other side, the cross-sectional dimension has to consider links between individual financial institutions, which can act as contagion channels through which shocks propagate. The combination of these two perspectives means that the absolute importance of a financial institution within an

[3]Source: C. Borio, 2010. Implementing a macroprudential framework: Blending boldness and realism. BIS Working Paper.

economic system depends not only by itself (its size, balance-sheet values, ...), but also by its interconnectedness with other financial institutions. Such systemic relevance can be interpreted as vulnerability or systemic importance: the former explains how much a financial institution is exposed to shocks coming from the others; the latter examines how much a financial institution can impact its neighbours when it is in distress. Moreover, systemic relevance can change over time, making the financial network extremely volatile.

So far we have identified two dimensions in the composition of systemic risk (time and cross-sectional), and we have concentrated on financial institutions. According to the cross-sectional approach, in fact, we have underlined how important are links within the financial network (horizontal perspective). But the cross-sectional dimension, and more precisely contagion, should take into account also a vertical perspective. The experience of the European debt crisis, in fact, has taught us that a distress event can originate within an economic sector (for example the financial one), it propagates within that economic sector but may also spread out to the real economy or other sectors. To sum up, different economic sectors are characterised by linkages within and across them, as well as all such linkages evolve over time.

This kind of study is exactly what we propose in Chapters 4 and 5. In *Sovereign Risk in the Euro Area: a multivariate stochastic process approach* we analyse the default probability of Eurozone countries in an interconnected framework, understanding how they depend on market data rather than on real economic indicators. If market data clearly divide Eurozone countries into core and peripheral ones, the inclusion of the GDP growth rate in the derivation of default probabilities partially changes that clustering structure, showing that the sovereign risk of some countries (especially France and Italy) is strongly affected not only by financial data, but also by the leverage ratio. In *CoRisk: measuring systemic risk through default probability contagion in the Eurozone* we consider again a sample of Eurozone countries, and we model each of them as a combination of three economic sectors: sovereign, corporate and banking. We build a time-varying correlation structure within and across them by means of multivariate stochastic processes and partial correlation networks in order to derive the multivariate nature of default probabilities, disentangling them into an idiosyncratic and a contagion component. Finally we propose two measures able to distinguish between vulnerability and systemic importance. Our results show that the sovereign crisis has increased systemic risks more than the financial one: the two events together have caused a phase transition difficult to reverse, as risk propagation does not act as a mean for balancing inequalities across countries but, on the contrary, weakens the weakest and strengthens the strongest. Finally, Chapter 6 is more related to macroprudential policy issues: it evolves the methodology introduced in the previous two papers but it concentrates on the banking system with the aim of understanding the effects of a bail-in rather than a bail-out scenario from a systemic risk perspective. More in detail, we simulate contagion effects through the network and we compare the long-run effects of a default (bail-in) and a private bail-out (private capital injection on voluntary basis) taking into account both the level of interconnectedness and the dimension of each bank: in addition, we analyse the results by looking at the bank and the system perspectives (micro- vs macro- prudential approach). The results show that the bail-out of a troubled bank is more convenient for the smaller, safer and highly correlated banks. From the system's viewpoint, the bail-out always reduces risk and, quite obviously, the failure of a big rather than a small bank considerably increases the total expected losses of the banking system in case of bail-in.

Chapter 2

Predicting bank interests when monetary rates are close to zero[1]

Applied Mathematics, 7(1), 1-12

Abstract

Monetary policies, either actual or perceived, cause changes in monetary interest rates. These changes impact the economy through financial institutions, which react to changes in monetary rates with changes in their administered rates, on both deposits and lendings. The dynamics of administered bank interest rates in response to changes in money market rates is essential to examine the impact of monetary policies on the economy. In this paper we examine the validity of the traditionally used error correction model in the recent time period, characterised by very low monetary rates, and we will concentrate on overnight deposits in Italy. We will provide a novel, more parsimonious, model and a predictive performance assessment methodology, which allows the comparison with the error correction model. Finally, we will propose a measure for interest rate risk, based on both the income or the economic value perspective. Our results show that, differently from before, the most important component in determining bank rates now is the autoregressive factor: such regime switching, however, seems to not strongly impact interest rate risk.

Keywords: Error correction model, Forecasting bank rates, Interest rate risk, Monetary policy transmission.

JEL: C15, C20, E47, G32.

2.1 Introduction

Monetary policies, such as variations in the official rate or liquidity injections, cause changes in monetary interest rates. These changes impact the economy mainly in an indirect way, through financial institutions, which react to changes in the monetary rates with changes in their administered rates, on both deposits and lendings.

The dynamics of administered bank interest rates in response to changes in money market rates is essential to examine the impact of monetary policies on the economy. This dynamics has been the subject of an extensive literature; the available studies differ,

[1] Joint work with I. Gianfrancesco (Banca Carige) and C. Giliberto (Banca Monte dei Paschi di Siena).

depending on the used models, the period under analysis and the geographical reference.

The relationship between market rates and administered rates is a complicated one and the evidence presented, thus far, is mixed and inconclusive. Hannan and Berger (1991), for example, examine the deposit rate setting behaviour of commercial banks in the United States and find that (a) banks in more concentrated markets exhibit greater rates rigidity; (b) larger banks exhibit less rates rigidity; and (c) deposit rates are more rigid upwards than downwards. Scholnick (1996), similarly, finds that deposit rates are more rigid when they are below their equilibrium level than when they are above; his finding on lending rate adjustment, however, is mixed. Heffernan (1997) examines how the lending and deposit rates of four banks and three building societies respond to changes in the base rate set by the Bank of England and finds that (a) there is very little evidence on the asymmetric nature of adjustments in both the deposit and lending rates, (b) there is no systematic difference in the administered rate pricing dynamics of banks and building societies, and (c) the adjustment speed for deposit rates is, on average, roughly the same as that for loan rates.

More recent papers on the same issue include: Ballester et al. (2009), Chong et al. (2006), Demirguc-Kunt and Huizinga (1999), Flannery et al. (1984), Maudos et al. (2004), Maudos et al. (2009). Among them, Chong et al. (2006), who applies and extends Engle and Granger (1987) error correction model has become a reference paper, in both the academic and the professional field.

The empirical evidence contained in all the previous papers can be summarised in the following points: (a) bank rates react with a partial and delayed change to changes in the monetary rates; (b) the speed and the degree to which they follow these changes present substantial differences between the various categories of banking products and between different countries.

The previous conclusions have been obtained for a relatively stable time period, previous to the emergence of the recent financial crisis.

After 2008, however, we have witnessed substantial changes. From a macroeconomic viewpoint, monetary interest rates are now, in most developed economies, close to zero, or negative; from a microeconomic viewpoint, bank management has changed substantially, for the compression of interest margins and for the increase in regulatory capital requirements. The effects of the previous changes on the transmission of monetary policies have not been yet fully investigated. In particular, the current state of close-to-zero interest rates is of particular relevance, as it has never been studied before.

When monetary rates are close to zero, the error correction model, albeit formally elegant, does not well capture the dynamic of administered rates, which appears strongly inertial.

The need of adapting the error correction model to the current situation is very relevant, not only from a macroeconomic point of view, but also from a microeconomic bank perspective and, in particular, in the measurement of interest rate risk, and in the related asset and liability management policies. We refer to Cocozza et al. (2015) for further details.

The aim of this paper is to broaden the error correction model of Chong et al. (2006), in a predictive performance comparison framework. Our results show that the error correction model performs quite well in a predictive sense. We also show that a more parsimonious model, described by only one equations, rather than two, is not inferior in terms of predictive performance, and, therefore, represents a valid alternative.

Our proposed methods are applied to data from the recent period (1999-2014), of a southern European country, with a traditional banking sector: Italy.

The paper is structured as follows. Section 2.2 describes the proposed models and, in particular: Section 2.2.1 describes the error correction model; Section 2.2.2 motivates and introduces the new proposed model; Section 2.2.3 provides the predictive performance environment used to compare the two models; Section 2.2.4 presents our proposal for the allocation of on demand deposits. Section 2.3 shows the empirical evidence obtained from the application of the models and, in particular: Section 2.3.1 describes the available data; Section 2.3.2 presents the estimation results obtained when the models are applied to such data; Section 2.3.3 compares the models in predictive performance; Section 2.3.4 presents the application to interest rate risk, by using both the income and the economic value perspectives. Finally, Section 2.4 concludes with some final remarks.

2.2 Methodology

2.2.1 The error correction model

In line with the contribution of Chong et al. (2006), the relationship between monetary rates and administered bank rates can be analysed with the use of the Error Correction Model (ECM), following the procedure proposed by Engle and Granger (1987). The model is based on two equations. A long-run relationship provides a measure of how a change in the monetary rate is reflected in the bank rate. A short-run equation, which includes an error correction term, analyses variations of the administered interest rates as a function of variations in the monetary rates.

Indeed, Chong et al. (2006) extended Engle and Granger by allowing the effect of the error correction term to depend on its sign. Their complete model can be formalised as follows:

$$\begin{cases} BR_t = k + \beta \cdot MR_t + \epsilon_t \\ \Delta BR_t = \alpha \cdot \Delta MR_t + \delta_1 (BR_{t-1} - \beta \cdot MR_{t-1} - k) + \\ \qquad + \delta_2 (BR_{t-1} - \beta \cdot MR_{t-1} - k) + u_t, \end{cases} \tag{2.2.1}$$

where

$$\begin{cases} \delta_1 = 0 & \text{if } BR_{t-1} - \beta \cdot MR_{t-1} - k < 0, \\ \delta_2 \neq 0 & \text{otherwise;} \end{cases}$$

$$\begin{cases} \delta_2 = 0 & \text{if } BR_{t-1} - \beta \cdot MR_{t-1} - k > 0, \\ \delta_1 \neq 0 & \text{otherwise.} \end{cases}$$

In equation (2.2.1) BR_t and MR_t represent, respectively, the bank administered rates and the monetary rates at time t; β is a regression coefficient that gives a measure of the extent of the monetary rate transmitted on bank rates in a long-term perspective: in the case of $\beta = 1$, the whole monetary rate is transmitted on the administered rate, while a value between 0 and 1 means that only a partial transmission mechanism occurs; k is a constant that synthetises all other factors that, in addition to the dynamics of monetary rates, may affect the transmission mechanism of the monetary policy on bank rates as, for example, the market power and the efficiency of a bank; ϵ is the error term of the long-run equation; δ_1 and δ_2 represent the adjustment speeds converge towards the equilibrium level; finally, u_t is the error term of the short-run equation.

2.2.2 The proposed model

The aim of this subsection is to propose a bank rate model that, while based on the ECM, is more parsimonious and, therefore, easier to interpret and manage. To achieve this aim we examine the main components of the error correction model, so to establish a statistical methodology for their simplification.

First, it is of interest to check whether the assumption of a double error correction coefficient, introduced by Chong et al. (2006), is justified and strictly necessary. To check this point the previous model can be compared, in a hypotheses testing framework, with the following nested model:

$$\begin{cases} BR_t = k + \beta \cdot MR_t + \epsilon_t \\ \Delta BR_t = \alpha \cdot \Delta MR_t + \delta(BR_{t-1} - \beta \cdot MR_{t-1} - k) + u_t. \end{cases} \tag{2.2.2}$$

Differently from equation (2.2.1), the model in (2.2.2) contains only one adjustment speed, so it does not admit the possibility of an asymmetric convergence of the administered interest rate to its equilibrium level.

Second, the error correction model contains one equation for the level of administered interest rates, and one for its variations. The two can be analysed separately, with the simple regression models:

$$BR_t = k + \beta \cdot MR_t + \epsilon_t \tag{2.2.3}$$

$$\Delta BR_t = k + \beta \cdot \Delta MR_t + u_t. \tag{2.2.4}$$

While model (2.2.3) explains the levels of banking rates in terms of the level of monetary ones, equation (2.2.4) is a model for the variations of bank rates in terms of the variations of monetary rates. These models, albeit very simple, should be considered in practical applications, and compared in predictive performance with the error correction model, to check whether the latter can be simplified.

We anticipate that the above models are too simple to lead to a good predictive performance. However, the idea of replacing the error correction model with a one-equation one is tempting and, therefore, we now propose a one equation model that can be a valid competitor of the ECM. To achieve this aim we first examine the economic rationales behind the relationship we would like to investigate.

From a microeconomic viewpoint, as deposits are saving tools in competition with other instruments (such as bonds), it seems quite reasonable to assume that banks decide on the administered rate looking primarily at its level. Starting from the level, one can always obtain its variation through differentiation. A second consideration concerns the determinants of administered bank levels. Again, it is reasonable to think that bank deposit rates depend on both the level and on the variation of monetary rates. A third assumption, particularly important when monetary rates are close to zero, is that the level of deposit rates depends on the previous level of the same quantity, to allow for a slow and partial reaction to monetary rate changes, given that deposit rates affect considerably the income of a bank.

A macroeconomic perspective confirms the previous assumption: in particular, that is correct to consider, as a response variable, the level of the administered rate and not its variations. This because the relevant response variable for an expansion/restriction effect on the economy is represented by the level of the rates; on the explanatory side, we can model administered rate levels as a function of changes in the monetary rates, but also of their levels, which remain important even when close to zero.

On the basis of the above economic rationales, our proposed model is the following:

$$BR_t = k + \beta \cdot MR_{t-1} + \gamma \cdot \Delta MR_t + \delta \cdot BR_{t-1} + \epsilon_t. \tag{2.2.5}$$

The proposed model can be equivalently written in terms of the variations of the administered rates:

$$\Delta BR_t = k + \beta \cdot MR_{t-1} + \gamma \cdot \Delta MR_t + (\delta - 1) \cdot BR_{t-1} + \epsilon'_t. \tag{2.2.6}$$

To improve interpretability, the proposed model can also be expressed in a differential form:

$$\frac{\mathrm{d}BR}{\mathrm{d}s} = \beta \cdot \left[\frac{\mathrm{d}MR}{\mathrm{d}s}\right]_{s=t} + \gamma \cdot \left[\frac{\mathrm{d}^2 MR}{\mathrm{d}s^2}\right]_{s=t} + \gamma \cdot \left[\frac{\mathrm{d}BR}{\mathrm{d}s}\right]_{s=t-1}. \tag{2.2.7}$$

The previous equation shows that the model can be interpreted as a "physical" description of the banking behaviour in terms of deposit interest rates through its differentiation: the derivative of the bank administered rate depends both on the speed and on the acceleration/deceleration of monetary rates, as well as on the derivative of the administered rate with respect to its level in the previous time.

Note that the proposed model can be directly compared with the ECM with one adjustment speed. Comparing equation (2.2.2) and equation (2.2.5) it is clear that our proposal is a particular case of the latter, with some constraints on the parameters. By using the notational index 1 for the coefficients of the one-speed ECM and the index 2 for the coefficients referred to the proposed model, such constraints are the following:

$$\begin{cases} -\delta_1 k_1 = k_2, \\ -\delta_1 \beta_1 = \beta_2, \\ \alpha_1 = \gamma_2, \\ \delta_1 + 1 = \delta_2. \end{cases} \tag{2.2.8}$$

Note, in particular, that the last equation in (2.2.8) implies that $(\delta - 1)$ represents the adjustment speed to which bank administered rates react to changes in the monetary rates, equivalently as the parameters δ_1 and δ_2 of Chong et al. (2006) Error Correction Model.

A full comparison of our model with the ECM cannot be easily carried out in a statistical testing framework, as the two models are, evidently, not nested; however, they can be compared in terms of predictive performance and, for this purpose, the next Subsection introduces an appropriate methodology.

A different comparison between the two models can be carried out by looking at their time dynamics. This is of particular interest in the context of interest rate risk modelling. For sake of simplicity we illustrate this comparison for the first three one-month rates and, then, for the general situation.

For the error correction model, we consider the case of $\delta_1 \neq 0$; the other case of $\delta_2 \neq 0$ can be obtained analogously, replacing δ_1 with δ_2. Then, assume that:

$$\begin{cases} \delta_1 \neq 0, \\ BR(0) = BR_0, \\ MR(0) = MR_0; \end{cases}$$

then, for the first month ahead:

$$\begin{aligned} BR_1 &= BR_0 + \Delta BR_1 = \\ &= BR_0(1+\delta_1) + \alpha \Delta MR_1 - \delta_1 \beta MR_0 - \delta_1 k. \end{aligned}$$

For the second and the third month ahead, instead, we obtain:

$$\begin{aligned} BR_2 &= BR_1 + \Delta BR_2 = \\ &= BR_0(1+\delta_1)^2 + \Delta MR_1(\alpha + \delta_1\alpha - \delta_1\beta) - \delta_1\beta MR_0(2+\delta_1) + \\ &\quad + \alpha \Delta MR_2 - 2\delta_1 k; \end{aligned}$$

$$BR_3 = BR_0(1+\delta_1)^3 + \Delta MR_1[\alpha + \delta_1(\alpha - \beta)(2+\delta_1)] +$$
$$- MR_0\delta_1\beta[(1+\delta_1)(2+\delta_1)+1] + \Delta MR_2(\alpha - \delta_1\beta) - \delta_1 k(3+2\delta_1).$$

For our proposed model, assuming the same initial values BR_0 and MR_0 for the bank and the monetary interest rates, we find the following equation for the first month ahead:

$$BR_1 = MR_0\beta + \Delta MR_1\gamma + BR_0\delta + k.$$

whereas for the second and the third month ahead we obtain:

$$BR_2 = MR_0\beta(1+\delta) + \Delta MR_1[\beta + \delta\gamma] + \Delta MR_2\gamma + BR_0\delta^2 + k(1+\delta);$$

$$BR_3 = MR_0\beta(1+\delta+\delta^2) + \Delta MR_1[\beta + \delta(\beta + \delta\gamma)] +$$
$$+ \Delta MR_2[\beta + \delta\gamma] + \Delta MR_3\gamma + BR_0\delta^3 + k\delta(1+\delta).$$

From the above calculations we can derive a general iterative formula for both models, in order to calculate bank interest rates at any time t (BR_t), as functions of the levels of bank rates at time $t-1$ (BR_{t-1}).

For the error correction model such iterative equation is:

$$BR_t = BR_{t-1}(1+\delta_1) - \delta_1\beta\left[MR_0 + \sum_{s=1}^{t-1}\Delta MR_s\right] + \alpha\Delta MR_t - \delta_1 k. \tag{2.2.9}$$

Similarly, for our proposed model we obtain:

$$BR_t = \delta BR_{t-1} + \beta\left[MR_0 + \sum_{s=1}^{t-1}\Delta MR_s\right] + \gamma\Delta MR_t + k. \tag{2.2.10}$$

2.2.3 Predictive performance assessment

While the assumption of a double error correction coefficient can be easily tested against a one error correction model, other simplifications of the ECM model require a more general set-up. This can be provided, for example, by a predictive performance framework that we are going to illustrate in this subsection. Doing so, we can enrich the error correction model with a validation procedure that is, to our knowledge, not yet available in the literature.

In order to predict bank rates, we need to estimate reasonable future values of the monetary rates. Consistently with the literature, we assume that their variation follows a random walk process.

More formally, assume that we want to predict the level of monetary rates for each of the next 12 months. Let $\widehat{\Delta MR}_i$ indicate the variation of the monetary rate in a given month. We then assume that $\widehat{\Delta MR}_i$ are independently and identically distributed Gaussian random variables, so that:

$$\begin{cases} \widehat{\Delta MR} \sim N(0, \sigma^2) \\ \widehat{MR}_i = \widehat{MR}_{i-1} + \widehat{\Delta MR}_i \quad \text{i = 1, ..., 12.} \end{cases} \tag{2.2.11}$$

Equation (2.2.11) describes a recursive procedure to obtain predictions of the monetary rates for a given year ahead, based on the random walk process assumption. We can then insert the predicted monetary rates as regressor values in the models of the previous Subsection and, thus, obtain predictions for the administered bank rates. In particular, for model (2.2.1) we obtain:

$$\begin{cases} \widehat{BR}_i = \widehat{BR}_{i-1} + \widehat{\Delta BR}_i, \\ \widehat{\Delta BR}_i = \alpha \cdot \widehat{\Delta MR}_i + \delta_1(\widehat{BR}_{i-1} - \beta \cdot \widehat{MR}_{i-1} - k) + \\ \qquad\qquad + \delta_2(\widehat{BR}_{i-1} - \beta \cdot \widehat{MR}_{i-1} - k) \end{cases}$$

where

$$\begin{cases} \delta_1 = 0 \quad \text{if } \widehat{BR}_{i-1} - \beta \cdot \widehat{MR}_{i-1} - k < 0, \\ \delta_2 \neq 0 \quad \text{otherwise;} \end{cases}$$

$$\begin{cases} \delta_2 = 0 \quad \text{if } \widehat{BR}_{i-1} - \beta \cdot \widehat{MR}_{i-1} - k > 0, \\ \delta_1 \neq 0 \quad \text{otherwise.} \end{cases}$$

For model (2.2.5) we obtain that:

$$\widehat{BR}_i = k + \beta \cdot \widehat{MR}_{i-1} + \gamma \cdot \widehat{\Delta MR}_i + \delta \cdot \widehat{BR}_{i-1}.$$

According to the standard cross-validation (backtesting) procedure, to evaluate the predictive performance of a model, we can compare, for a given time period, the predictions of monetary rates obtained with the previous equations with the actual values. To obtain a robust measurement we can indeed generate N scenarios of monetary rates, using (2.2.11), and obtain the corresponding bank rates, using either (2.2.1) or (2.2.5). On the basis of them we can calculate and approximate Monte Carlo expected values and variances of the predictions, as follows.

Let Y be a bank rate to be predicted at time i, with unknown density function $f_Y(y)$. The expected value of Y can then be approximated with

$$\widehat{\mathbb{E}(Y)} = \frac{1}{N}\sum_{k=1}^{N} y^{(k)}, \tag{2.2.12}$$

and its variance with

$$\widehat{var(Y)} = \frac{1}{N^2}\sum_{k=1}^{N}[y_i - E(\hat{Y})]^2. \tag{2.2.13}$$

In the next Section we will use (2.2.12) and (2.2.13) to compare model predictive performances. Before proceeding, we would like to remark that the random number generation at the basis of the Monte Carlo algorithm is pseudo-random, and depends on an initial seed. Different seeds may lead to different results so that models can not be compared equally. We have thus decided to use the same random seed for all models, so that the differences in performances are not biased by the Monte Carlo random mechanism.

2.2.4 On-demand deposits allocation

The allocation of on-demand (overnight) customer deposits to an appropriate maturity time is a significant criticality in interest rate risk modelling, as well as in asset and liability management of banks, given their particular characteristics. The latter include: (i) the absence of a contractual maturity, with the correlated ability of the depositor to withdraw the funds at any time; (ii) the stability of the masses in time, along with the diversification of counterparties that makes basically constant the total volumes; (iii) the partial and delayed reaction rate charged by banks on such balance sheet items as a result of changes in the monetary rate, and especially when such changes are positive.

Theoretically, on-demand deposits could be assigned a zero maturity. Doing so, however, the term structure of the liabilities of a bank, and especially of commercial ones, do not match the term structure of the assets which, especially on the lending side, is characterised by positions with different maturities. Asset and liability management becomes, therefore, based on an incorrect representation of the cash flows of a bank. Concerning the interest rate risk, which is the main focus of our paper, allocating on-demand deposits to a zero maturity biases risk measurement: an increase of monetary rates has a negative impact, lower than it should be, as the duration of liabilities is lower than the real one. Similarly, a decrease of monetary rates has a lower positive impact.

Having established that a zero maturity cannot be the right time allocation for on-demand deposits, it remains the issue of finding an appropriate allocation of deposits in different time maturity bands. On one hand, an allocation shifted towards short maturity reflects the contractual nature of these deposits, which are subject to withdrawal at any time; on the other hand, an allocation shifted towards long maturity reflects their stability as a major source of funding.

From an asset and liability management perspective, the correct procedure seems to allocate overnight deposits to their actual maturity, so to balance liabilities with assets of corresponding maturities. Such an actual maturity can be estimated statistically, by analysing the observed decay of a bank's deposits. This is the approach followed, in current practice, by many banks. Overnight deposits are split between a non core component, which remains at a zero maturity, and a core component, whose volumes in the different maturity bands are estimated by means of a moving average filter, such as Hodrick and Prescott's (1997).

From an interest rate risk perspective, it is important to consider what regulatory requirements prescribe. The Basel Committee on Banking Supervision does not give specific guidelines in its main documentation on interest rate risk modelling (BIS, 2004); it does so in the recent document on the Net Stable Funding Ratio (BIS, 2014), where it suggests a decay percentage of 5% or of 10% of the overnight deposits in the first year. National regulators are more prescriptive; for example, the Bank of Italy, which is relevant for our application, suggests to allocate 25% in the non-core component and to allocate the remainder in the next five years, with a 1/60 decay in each month.

Here we join the two perspective and propose an allocation model that, while consistent with the regulatory methodology on interest rate risk, also takes the asset and liability management view into account.

Moreover, we propose that the allocation of overnight deposits to different time maturity bands can be performed, once regulatory requirements are minimally satisfied, using allocation coefficients that are function of the administered rate changes variation in the same band.

More precisely, we propose to split overnight deposits in a non-core component (NCC), with maturity zero, and a core component (CC), with higher maturities. The latter is allocated proportionally to weight coefficients as follows. Let a time period be j, with initial time t_i and final time t_f. We will allocate in it a volume that is equal to CC times the following weight:

$$W_j \propto e^{(BR_{t_f} - BR_{t_i}) \cdot (t_f - t_i)}$$

where BR_{t_i} and BR_{t_f} are the bank rates that correspond, respectively, to the initial and final time points of the time band.

To obtain the correct values of the weights, they have been normalised by dividing them by the sum of all previous terms over the considered total interval.

The rationale behind our proposal is that, rather than using a constant allocation or a historical one, we use an allocation that is based on the possible evolution of interest rates, according to a forward-looking, rather than backward-looking perspective. In such a context, time periods with higher interest rates attract more volumes and, conversely, time bands with lower interest rates are less attractive.

In the following and thus in the related procedure, we will illustrate our proposal, without loss of generality, for the time bands included in the first year: for this reason $T = 1$ year.

The models proposed in the previous sections allow us to estimate, through a recursive procedure, bank interest rates at the various maturities, as follows:

$$(1 + BR_N)^N = \prod_{j=0}^{N-1} (1 + {}_jBR_1), \tag{2.2.14}$$

where ${}_jBR_1$ are the estimated values of bank administered interest rates for the next N months.

The calculation of the above formula, at the given time bands, requires the estimation of the forward rates ${}_jBR_1$, $j = 1, ..., N-1$. Such forward rates, as well as ${}_0BR_1$, can be estimated following the procedure described in the previous Section. In other words, for the ECM:

$$\begin{cases} {}_jBR_i = \widehat{BR}_{i-1} + \widehat{\Delta BR}_i, \\ \widehat{\Delta BR}_i = \alpha \cdot \widehat{\Delta MR}_i + \delta_1(\widehat{BR}_{i-1} - \beta \cdot \widehat{MR}_{i-1} - k) + \\ \qquad + \delta_2(\widehat{BR}_{i-1} - \beta \cdot \widehat{MR}_{i-1} - k) \end{cases}$$

where

$$\begin{cases} \delta_1 = 0 & \text{if } \widehat{BR}_{i-1} - \beta \cdot \widehat{MR}_{i-1} - k < 0, \\ \delta_2 \neq 0 & \text{otherwise;} \end{cases}$$

$$\begin{cases} \delta_2 = 0 & \text{if } \widehat{BR}_{i-1} - \beta \cdot \widehat{MR}_{i-1} - k > 0, \\ \delta_1 \neq 0 & \text{otherwise.} \end{cases} \tag{2.2.15}$$

while for the proposed model the following holds:

$${}_jBR_i = k + \beta \cdot \widehat{MR}_{i-1} + \gamma \cdot \widehat{\Delta MR}_i + \delta \cdot \widehat{BR}_{i-1}.$$

We finally remark that Esposito et al. (2013) emphasise the importance of assessing the sensitivity of interest rate risk to different allocation assumptions; our assumptions could be inserted among the latter.

2.3 Data analysis and results

2.3.1 Descriptive analysis

The recent financial crisis has had a major impact on the banking sector and, in particular, has led to a change in the relationship between monetary and administered rates and, therefore, to the transmission mechanisms of monetary policies. In the Eurozone, characterised by one monetary authority (the European Central Bank), that regulates still fragmented national markets, this effect is particularly evident: southern european countries, differently from what expected, have benefited very little from the drop of monetary rates that has followed the financial crisis.

To investigate the above issues we focus on a southern european country, Italy, for which the transmission of monetary impulses is particularly problematic, given the importance of the banking sector and the difficult economic situation.

Accordingly, we have collected monthly time series data on monetary rates and on aggregate bank deposits administered rates from the statistical database provided by the Bank of Italy, for the period ranging from January 1999 to December 2014.

For the purposes of our analysis, the monetary rate used in this paper is the 1-month Euribor. This choice has been based on the fact that this rate has a greater correlation with the administered bank rate with respect to the other monetary rates, such as the EONIA and the Euribor at 3 and 6 months, as can be seen in Table 2.6.1.

[Table 2.6.1]

Figure 2.6.1 represents the time series of the chosen monetary rates, along with that of the aggregate administered bank rates on deposits, for the considered time period.

[Figure 2.6.1]

From Figure 2.6.1 note that both the administered and the monetary rates rapidly decreased in 2008 and 2009, while in the last two years they have remained quite stable and close to zero. Moreover, the two curves seem to have the same shape between 1999 and 2008, while the relationship between the two radically changes in the following years, leading to overlaps and different behaviours. In other words, the correlation pattern between the bank administered rate and the monetary rate shows a very heterogeneous behaviour: before 2008 they seem to have a stable relationship; in 2008 they both dropped; after that time they look stable and close to zero, with a relationship that is indeed quite different from the one observed before the crisis.

To obtain further insights, in Figure 2.6.2 we present the histogram and the corresponding density estimate of the two rates.

[Figure 2.6.2]

Figure 2.6.2 reveals that bank administered interest rates are more concentrated around their mean value, while monetary rates are quite spread.

It is also interesting to compare the distributions of the variations of the two rates, represented in Figure 2.6.3.

[Figure 2.6.3]

From Figure 2.6.3 note that the variations of the administered bank rates are more concentrated around zero, while monetary rates seem to have broader variations. Indeed, the behaviour of ΔMR justifies the assumption of considering the variations of monetary interest rates as a random walk process, so that they can be modelled according to equation (2.2.11).

We have previously commented on the change in the relationship between the two rates, comparing the situation before and after 2009. This switching behaviour can be easily seen by looking at the correlation between the rates and their variations. Table 2.6.2 shows the correlations between the rates and between their variations in the two periods (1999-2008) and (2009-2014), before and after the financial crisis.

[Table 2.6.2]

From Table 2.6.2 note that the correlation between the levels of bank and monetary rates has decreased after 2009, while the correlation between the variations of the administered bank rates and those of the monetary rates has increased during the same period.

2.3.2 Model estimates

For the models proposed in Section 2.2.1 and 2.2.2 we now show the corresponding parameter estimates, considering the following four time series: (a) data from 1999 to 2007; (b) data from 1999 to 2008; (c) data from 2009 to 2013; (d) data from 1999 to 2013. This choice of data windows is consistent with the aim of investigating the switching behaviour in the correlation structure of interest rates, which has occurred during the years 2008 and 2009. On the basis of this windows selection we intend to obtain predictions for the years 2008, 2009 and, finally, for the last available year, 2014. Predictions that can be compared with the actual occurred value, thus giving a measure of model predictive performance.

We now show the parameter estimates for all the considered models, including the two simple univariate linear models, and the four periods we have chosen. For each estimate we also report the corresponding t-value, and the R^2 contribution of each model.

Table 2.6.3 shows the parameter estimates for the error correction model proposed by Chong et al. (2006) which, we recall, has two equations and, correspondingly, two R^2 measures.

[Table 2.6.3]

From Table 2.6.3 note that, for the error correction model with two adjustment speeds, the results confirm a radical change in the relationship between the variables during the period under analysis: remembering that the long-run equation models the levels of interest rates, while the short-run equation is a function of the variations of the rates, it is clear that in the last few years the levels of the rates have become less and less important, while their variations have gained exploratory capacity.

Table 2.6.4 shows the parameter estimates for the error correction model with one adjustment speed.

[Table 2.6.4]

From Table 2.6.4 note that the error correction model with only one adjustment speed shows results very similar to those reported in Table 2.6.3: in particular, it has similar R^2 values, meaning that this simplified version of the error correction model fits past data quite well and, therefore, it may suffice. As a further confirmation, it can be shown that the equality assumption $\delta_1 = \delta_2$ in Chong et al. (2006) model is rejected only in one of the four considered time windows.

Table 2.6.5 shows the parameter estimates for the simple linear model in terms of the levels of the bank interest rates.

[Table 2.6.5]

From Table 2.6.5 note that the estimates obtained with the univariate linear model for interest rates are similar to those obtained by using the long-run equation of the error correction model.

Table 2.6.6 shows the parameter estimates for the simple linear model in terms of variations of bank interest rates.

[Table 2.6.6]

From Table 2.6.6 it is clear that the univariate linear model for the variations of administered bank interest rates, calculated as a function of the variations of monetary rates, shows different results: first of all, the intercept term is not significant; secondly, R^2 values have an opposite trend with respect to those in Table 2.6.5, increasing during the last period. This result is a further confirmation of the changing regime after 2009.

Table 2.6.7 shows the parameter estimates for our proposed model.

[Table 2.6.7]

Table 2.6.7 shows that our new model presents an interesting behaviour. For the whole period 1999-2013 all the variables (apart from the intercept) are significant to describe the administered interest rates. But the situation changes if one concentrates on the first or on the second period: within the years 1999-2007 and 1999-2008 the variations of the monetary rates do not affect the level of bank rates; on the contrary, during the last period the only significant variable is the autoregressive component.

This is a clear evidence of the fact that, when rates are close to zero as in the last few years, administered interest rates are not affected by monetary rates, or by their variations, but, rather, they depend only on their past values.

2.3.3 Predictive performances

After having estimated the coefficients of the different models we then predict monthly administered bank interest rates and their variations for 2008, 2009 and 2014, using a range of monetary rates scenarios, simulated from a random walk process as previously described. In particular, for the 2014 prediction we performed the simulations by using the coefficients obtained both by considering the whole period (1999-2013) and the second part of the time range under examination (2009-2013). In Figure 2.6.4 a comparison between the predictions for 2014 (data from 1999 until 2013) obtained with the error correction model and our proposed model is shown.

[Figure 2.6.4]

As a measure of predictive performance we have calculated the root mean square errors of the predictions from all models. Here we present the prediction results in terms of variations of bank rates rather than on their levels. This because, in this case, all the predictions are more challenging, being the variations on a smaller scale.

In Table 2.6.8 the root mean square errors referred to the predicted variations of administered interest rates obtained with the error correction model and our proposed new model are reported.

[Table 2.6.8]

The first column of Table 2.6.8 refers to the prediction errors for the year 2008, obtained with the two selected models, and using coefficients estimated on data from 1999 to 2007. Similarly, the second and the third column report the root mean square errors for 2009 and 2014. We decided to compare predictions on these crucial years because they represent the breaking points before and after which the relationship between the rates radically changes. The objective is thus to verify whether the two models can adapt to such strong variations in the underlying economic system. Note that the last two columns both refer to estimations for 2014, but the first one uses coefficients estimated only the second period data, while the second one is based on estimations on the entire period 1999-2013.

From the analysis of Table 2.6.8 some interesting conclusions emerge: (a) both models predict quite well future variations of bank interest rates; (b) the error correction model works better on the whole period and, most interestingly, (c) our proposed model supplies great improvements for the crucial year 2009. This means that the new model is much more flexible than the Error Correction Model, and it is able to capture essential changes in the economy not only from an estimation fit point of view, as seen in the last subsection, but also in a predictive perspective.

2.3.4 Application to interest rate risk

Movements in interest rates can have a negative impact on both the income results and the economic value of a bank. This has given rise to two distinct but complementary perspectives in order to measure the exposure to the interest rate risk: the earnings perspective and the economic value perspective. In the first one the analysis is based on the impact of changes in interest rates on short-term profits of banks; in the second one, instead, attention is focused on the sensitivity of the assets of a bank to changes in interest rates.

In this application we confine our risk measurement to overnight deposits. For the evaluation of interest margin variations and net position changes we thus refer to the allocation of such deposits in time maturity periods as described in Section 3.4. Consistently with that procedure we consider a one year period ahead. Table 2.6.9 and Table 2.6.10 describe the allocation coefficients that result, respectively, from the ECM and the proposed model estimated bank interest rates.

[Tables 2.6.9, 2.6.10]

By comparing Table 2.6.9 with Table 2.6.10 note that allocation coefficients are quite stable across time periods: this is consistent with the fact that, within the predicted years (2008, 2009 or 2014), the monthly variations of interest rates are quite flat. The allocation weights are, therefore, essentially a function of the number of months in each time maturity period. If the allocation was done proportionally to the number of months, as suggested by some regulators, we would indeed get similar results.

We also remark that the ECM and the proposed model lead to very similar allocations, and this is a further evidence that the simpler model could be preferred as previously discussed.

Income Perspective

The measurement of the exposure to interest rate risk in the banking book from the perspective of income takes place over a short-term period (called gapping period); in operating practice, this is usually equal to 1 year. According to this approach we can use the repricing gap model, which calculates the expected change in the interest margin as the result of a change in monetary rates. The formula is the following:

$$\widehat{\Delta IM} = \widehat{\Delta MR}_j \cdot \sum_{\substack{j \\ (t_j \leq T)}} G'_{t_j} \cdot (T - t_j^*) = \widehat{\Delta MR}_j \cdot G_T^w, \tag{2.3.1}$$

where G'_{t_j} indicates marginal gaps ($= assets - liabilities$), $t_j^* = \frac{t_j + t_{j-1}}{2}$ represents the average time maturity; G_T^w indicates the cumulative gap.

In Table 2.6.11 we present, for each node in the term structure of interest rates, the impact on the interest margin of a positive change of 200 basis points (the Basel II level) in the level of monetary rates, when the ECM model is used to allocate volumes and it is assumed to consider core overnight deposits totalling to 100 euro.

[Table 2.6.11]

In Table 2.6.12 we present, for each node in the term structure of interest rates, the impact on the interest margin of a positive change of 200 basis points (the Basel II level) in the level of monetary rates, when the proposed model is used to allocate volumes.

[Table 2.6.12]

In the above tables we have considered the impact of an increase of monetary rates. The impact of a decrease is obviously opposite.

Comparing Table 2.6.11 with Table 2.6.12 note that, as could be expected from the corresponding volume allocation tables, there are not substantial differences between the two models and across the different time periods.

Economic Value Perspective

The measurement of the exposure to interest rate risk in the economic perspective is based on the Basel II regulation and it relies on the concepts of duration and modified duration.

Let F_t be the cash flow and t it's maturity; MR represents the interest rate at maturity, and NP is the net position. The duration D can be calculated as:

$$D = \sum_{t=1}^{T} t \cdot \frac{F_t}{\frac{(1+MR)^t}{NP}}, \tag{2.3.2}$$

while the modified duration follows the equation:

$$MD = \frac{D}{1+MR}. \tag{2.3.3}$$

Net position variations can be expressed by the formula:

$$\frac{\partial NP_i}{\partial MR_i} = -NP_i \cdot MD_i, \tag{2.3.4}$$

and, finally, variations of the economic value take the form:

$$\mathrm{d}\,EV = \sum_i \sum_j \mathrm{d}\,NP_{ij}. \tag{2.3.5}$$

The previous equations are referred to the general case: by remembering that net positions are defined as the difference between assets and liabilities, the sign in the second member of equation (2.3.4) becomes positive if we consider liabilities (overnight deposits). Moreover, equation (2.3.5) can be simplified by considering its discrete version:

$$\Delta EV = -\sum_i \sum_j NP_{ij} \cdot MD_{ij} \cdot \Delta MR_{ij}, \tag{2.3.6}$$

where i refers to the time slot, while j considers different currencies.

In Table 2.6.13 we present, for each node in the term structure of interest rates, the impact on the economic value of a positive change of 200 basis points (the Basel II level) in the level of the monetary rate, when the ECM model is used to allocate volumes and it is assumed to consider core overnight deposits totalling to 100 euro. We have considered the approximate duration suggested by the Basel Committee.

[Table 2.6.13]

In Table 2.6.14 we present, for each node in the term structure of interest rates, the impact on the economic value of a positive change of 200 basis points (the Basel II level) in the level of the monetary rate, when our proposed model is used to allocate volumes.

[Table 2.6.14]

Comparing Table 2.6.13 with Table 2.6.14 note that, for the economic capital as well, there are not substantial differences between the two models and across the different time periods.

A comparison between the variation in the interest margin and in the economic value shows that the main difference between the two is due to the different consideration of the time factor ($T - t^*$ for the former and the duration for the latter).

2.4 Conclusions

The main contribution of this paper consists in the understanding and improvement of the Error Correction Model, used in standard professional practice to model variations of the administered bank rates as a function of monetary rates. We add to the model a predictive methodology, that allows its validation, and propose a simpler to interpret one equation model, that can be seen as a special case of the ECM itself.

We have explained the implications of our proposals on data for the aggregate Italian banking sector that concerns the recent period, characterised by a substantial shift in the relationship between monetary and bank rates, with the former getting close to zero. Our results show that, differently from before, the most important component in determining bank rates now is the autoregressive factor: such regime switching, however, seems to not strongly impact interest rate risk.

2.5 References

Ballester, L., Ferrer, R., Gonzales, C., Soto, G.M., 2009. Determinants of interest rate exposure of the Spanish banking industry. Working Paper, University of Castilla-La Mancha.

Basel Committee on Banking Supervision, 2004. Principles for Management and Supervision of Interest Rate Risk.

Basel Committee on Banking Supervision, 2014. Net Stable Funding Ratio disclosure standards.

Chong, B. S., Liu, M., Shrestha, K., 2006. Monetary transmission via the administered interest rates channel. Journal of Banking and Finance 30(5), 1467-1484.

Cocozza, R., Curcio, D., Gianfrancesco, I., 2015. Non maturity deposits and bank's exposure to interest rate risk: issue arising from the Basel Regulatory framework. Journal of Risk 17(4), forthcoming.

Demirguc-Kint, A., Huizinga, H., 1999. Determinants of commercial bank interest margins and profitability: some international evidence. World Bank Economic Review 13(2), 379-408.

Engle, R. F., Granger, C. W. J., 1987. Co-integration and error correction: representation, estimation, and testing. Econometrica 55(2), 251-276.

Flannery, M.J., James, C.M., 1984. The effect of interest rate changes on the common stock returns of financial institutions. The Journal of Finance 39, 1141-1153.

Gambacorta, L., 2005. How do banks set interest rates?. European Economic Review 52(5), 792-819.

Gambacorta, L., Iannotti, S., 2007. Are there asymmetries in the response of bank interest rates to monetary shocks?. Applied Economics 39(19/21), 2503-2517.

Hannan, T., Berger, A., 1991. The rigidity of prices: Evidence from banking industry. American Economic Review 81, 938-945.

Heffernan, S.A., 1997. Modelling British interest rate adjustment: An error correction approach. Economica 64, 211-231.

Hodrick, R., Prescott, E.C., 1997. Postwar U.S. Business Cycles: An Empirical Investigation. Journal of Money, Credit, and Banking 29(1), 1-16.

Kleimeier, S., Sander, H., 2004. Convergence in euro-zone retail banking? What interest rate pass-through tells us about monetary policy transmission competition and integration. Journal of International Money and Finance 23(3), 461-492.

Kleimeier, S., Sander, H., 2006. Expected versus unexpected monetary policy impulses and interest rate pass-through in euro-zone retail banking markets. Journal of Banking and Finance 30(7), 1839-1870.

Maudos, J., Guevara, J.F., 2004. Factors explaining the interest margin in the banking sectors of the European Union. Journal of Banking and Finance 28(9), 2259-2281.

Maudos, J., Solis, L., 2009. The determinants of net interest income in the Mexican banking system: an integrated model. Journal of Banking and Finance 33(10), 1920-1931.

Resti, A., Sironi, A., 2007. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. John Wiley, New York.

Scholnick, B., 1996. Asymmetric adjustment of commercial bank interest rates: evidence from Malaysia and Singapore. Journal of International Money and Finance 15(3), 485-496.

Sierra, G. E., Yeager, T. J., 2004. What Does the Federal Reserve Economic Value Model Tell Us About Interest Rate Risk at U.S. Community Banks?. The Federal Reserve Bank of St. Louis Review 86(6).

2.6 Appendix A: Tables and Figures

Table 2.6.1: Correlation matrix between interest rates

	EONIA	Euribor (1m)	Euribor (3m)	Euribor (6m)	Bank Rate
EONIA	1.0000				
Euribor (1m)	0.9904	1.0000			
Euribor (3m)	0.9801	0.9951	1.0000		
Euribor (6m)	0.9701	0.9876	0.9972	1.0000	
Bank Rate	0.9488	0.9512	0.9453	0.9333	1.0000

Notes: Correlation matrix between the EONIA rate, the Euribor rates and the Bank administered rates. 1-month Euribor rates present the greatest correlation with the administered bank rates with respect to the other monetary rates.

Table 2.6.2: Correlation matrix between rates and their variations

	1999 - 2008	2009 - 2014	1999 - 2014
BR, MR	0.95	0.71	0.96
ΔBR, ΔMR	0.43	0.83	0.58

Notes: Correlation matrix between bank and monetary rates and their variations, in different time periods. The correlation coefficient between the levels of bank and monetary rates has decreased after 2009, while the correlation between the variations has increased during the same period.

Table 2.6.3: Parameter estimates, ECM with two adjustment speeds

	1999 - 2007		1999 - 2008		2009 - 2013		1999 - 2013	
	Coeff.	t	Coeff.	t	Coeff.	t	Coeff.	t
k	-0.133	-3.426	-0.100	-2.542	0.263	12.265	0.146	7.896
β	0.351	29.741	0.341	29.425	0.138	4.836	0.271	41.114
α	0.107	4.412	0.0909	4.863	0.096	4.430	0.126	7.455
δ_1	-0.286	-5.028	-0.288	-5.513	-0.348	-11.214	-0.175	-5.154
δ_2	-0.209	-4.194	-0.220	-4.680	-0.032	-0.813	-0.109	-3.008
R^2 long	0.893		0.880		0.287		0.905	
R^2 short	0.443		0.485		0.902		0.449	

Notes: Parameter estimates for the error correction model with two adjustment speeds. The table confirms that in the last few years the levels of the rates have become less and less important, while their variations have gained exploratory capacity.

Table 2.6.4: Parameter estimates, ECM with one adjustment speed

	1999 - 2007		1999 - 2008		2009 - 2013		1999 - 2013	
	Coeff.	t	Coeff.	t	Coeff.	t	Coeff.	t
k	-0.133	-3.426	-0.100	-2.542	0.263	12.265	0.146	7.896
β	0.351	29.741	0.341	29.425	0.138	4.836	0.271	41.114
α	0.111	4.620	0.096	5.300	0.145	5.316	0.131	7.903
δ	-0.242	-6.388	-0.250	-7.132	-0.235	-6.707	-0.144	-5.721
R^2 long	0.893		0.880		0.287		0.905	
R^2 short	0.437		0.480		0.822		0.444	

Notes: Parameter estimates for the error correction model with one adjustment speed. Since these results are very similar to the ones obtained with the ECM with two adjustment speeds, we can conclude that this simplified version of the error correction model fits past data quite well and, therefore, may suffice.

Table 2.6.5: Parameter estimates, linear model for the levels of bank rates

	1999 - 2007		1999 - 2008		2009 - 2013		1999 - 2013	
	Coeff.	t	Coeff.	t	Coeff.	t	Coeff.	t
k	-0.133	-3.426	-0.100	-2.542	0.263	12.265	0.146	7.896
β	0.351	29.741	0.341	29.425	0.138	4.836	0.271	41.114
R^2	0.893		0.880		0.287		0.905	

Notes: Parameter estimates for the linear model in terms of the levels of bank interest rates. The results seem to be similar to those obtained by using the long-run equation of the error correction model.

Table 2.6.6: Parameter estimates, linear model for the variations of bank rates

	1999 - 2007		1999 - 2008		2009 - 2013		1999 - 2013	
	Coeff.	t	Coeff.	t	Coeff.	t	Coeff.	t
β	0.149	5.444	0.131	6.344	0.278	11.28	0.162	9.592
R^2	0.219		0.254		0.683		0.341	

Notes: Parameter estimates for the linear model in terms of the variation of bank interest rates. The results show that (a) the intercept term is not significant, and (b) R^2 values have an opposite trend with respect to those in Table 2.6.5, increasing during the last period, thus confirming the regime switching occurred in 2009.

Table 2.6.7: Parameter estimates, proposed model

	1999 - 2007		1999 - 2008		2009 - 2013		1999 - 2013	
	Coeff.	t	Coeff.	t	Coeff.	t	Coeff.	t
k	-0.064	-4.394	-0.061	-4.561	0.077	8.498	-	-
β	0.100	9.369	0.098	10.992	-	-	0.042	6.205
γ	-	-	-	-	-	-	0.091	4.750
δ	0.743	25.695	0.746	30.454	0.731	24.544	0.869	40.836
R^2	0.986		0.987		0.974		0.998	

Notes: Parameter estimates for the proposed model. The results show that (a) for the whole period 1999-2013 all the variables (apart from the intercept) are significant; (b) within the years 1999-2007 and 1999-2008 the variations of the monetary rates do not affect the level of bank rates; (c) during the last period the only significant variable is the autoregressive component.

Table 2.6.8: Root mean square errors

Model	2008 (1999-2007)	2009 (1999-2008)	2014 (2009-2013)	2014 (1999-2013)
Error correction model	0.055	0.171	0.016	0.003
Proposed model	0.065	0.069	0.014	0.018

Notes: A comparison between the root mean square errors referred to the predictions of ΔBR. The first column refers to the prediction errors for the year 2008 obtained with the two selected models and using coefficients estimated on data from 1999 to 2007. The second and the third column report the root mean square errors for 2009 and 2014. The Table shows that (a) both models predict quite well future variations of bank interest rates; (b) the error correction model works better on the whole period; (c) our proposed model supplies great improvements for the crucial year 2009.

Table 2.6.9: Positioning coefficients, ECM

Maturity	1999-2007 (2008)	1999-2008 (2009)	2009-2013 (2014)	1999-2013 (2014)
Up to 1 month	0.0835	0.0793	0.0834	0.0834
From 1 to 3 months	0.1660	0.1644	0.1667	0.1666
From 3 to 6 months	0.2500	0.2516	0.2501	0.2511
From 6 months to 1 year	0.5005	0.5047	0.4998	0.4989

Notes: Positioning coefficients referred to the error correction model.

Table 2.6.10: Positioning coefficients, proposed model

Maturity	1999-2007 (2008)	1999-2008 (2009)	2009-2013 (2014)	1999-2013 (2014)
Up to 1 month	0.0842	0.0803	0.0833	0.0838
From 1 to 3 months	0.1666	0.1667	0.1667	0.1682
From 3 to 6 months	0.2499	0.2511	0.2500	0.2513
From 6 months to 1 year	0.4993	0.5019	0.5000	0.4967

Notes: Positioning coefficients referred to our proposed model. Compared to the results in Table 2.6.9, the allocation coefficients seem to be stable across time and similar across different models.

Table 2.6.11: Expected changes in interest margins, ECM

Maturity	$T - t_j^*$	1999-2007 (2008)	1999-2008 (2009)	2009-2013 (2014)	1999-2013 (2014)
Up to 1 month	0.9583	-0.1600	-0.1521	-0.1598	-0.1599
From 1 to 3 months	0.8333	-0.2768	-0.2740	-0.2778	-0.2777
From 3 to 6 months	0.6250	-0.3125	-0.3145	-0.3126	-0.3138
From 6 months to 1 year	0.2500	-0.2502	-0.2523	-0.2499	-0.2495
		-0.9995	-0.9929	-1.0001	-1.0009

Notes: Impact of a positive change in the level of monetary rates on interest margins obtained by using the error correction model to allocate volumes.

Table 2.6.12: Expected changes in interest margins, proposed model

Maturity	$T - t_j^*$	1999-2007 (2008)	1999-2008 (2009)	2009-2013 (2014)	1999-2013 (2014)
Up to 1 month	0.9583	-0.1614	-0.1540	-0.1597	-0.1605
From 1 to 3 months	0.8333	-0.2776	-0.2778	-0.2778	-0.2804
From 3 to 6 months	0.6250	-0.3124	-0.3139	-0.3125	-0.3141
From 6 months to 1 year	0.2500	-0.2496	-0.2509	-0.2500	-0.2483
		-1.0010	-0.9966	-1.0000	-1.0033

Notes: Impact of a positive change in the level of monetary rates on interest margins obtained by using our proposed model to allocate volumes.

Table 2.6.13: Expected changes in economic values, ECM

Maturity	Duration	1999-2007 (2008)	1999-2008 (2009)	2009-2013 (2014)	1999-2013 (2014)
Up to 1 month	0.0400	-0.0067	-0.0063	-0.0067	-0.0067
From 1 to 3 months	0.1600	-0.0532	-0.0526	-0.0533	-0.0533
From 3 to 6 months	0.3600	-0.1800	-0.1811	-0.1801	-0.1808
From 6 months to 1 year	0.7100	-0.7106	-0.7166	-0.7097	-0.7084
		-0.9505	-0.9566	-0.9498	0.9492

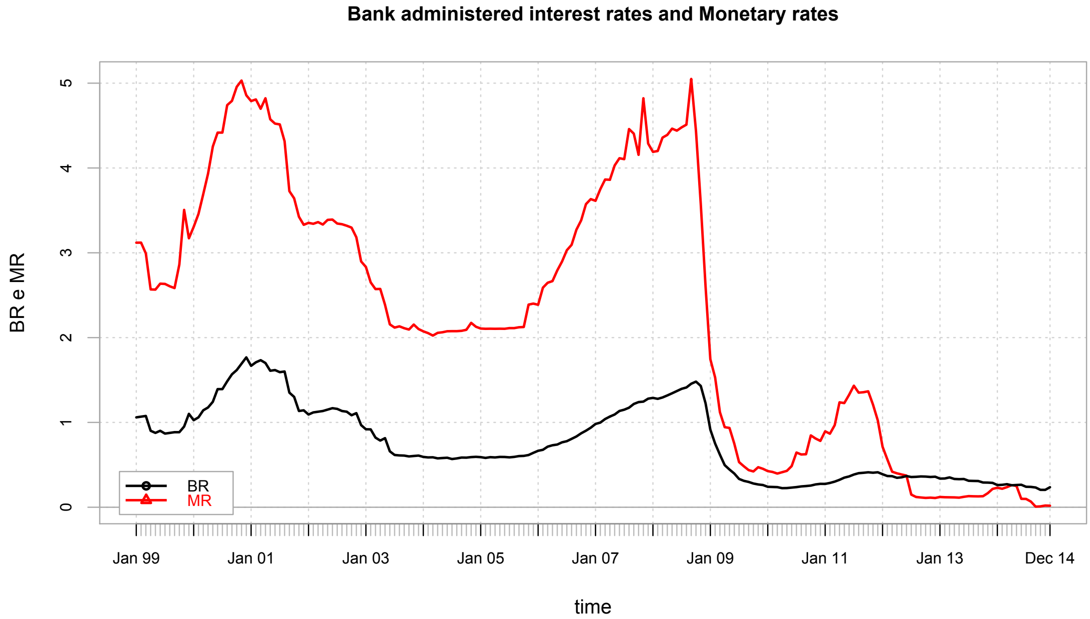
Notes: Impact of a positive change in the level of monetary rates on the economic value obtained by using the error correction model to allocate volumes.

Table 2.6.14: Expected changes in economic values, proposed model.

Maturity	Duration	1999-2007 (2008)	1999-2008 (2009)	2009-2013 (2014)	1999-2013 (2014)
Up to 1 month	0.0400	-0.0067	-0.0064	-0.0067	-0.0067
From 1 to 3 months	0.1600	-0.0533	-0.0533	-0.0533	-0.0538
From 3 to 6 months	0.3600	-0.1799	-0.1808	-0.1800	-0.1809
From 6 months to 1 year	0.7100	-0.7090	-0.7126	-0.7100	-0.7053
		-0.9489	-0.9531	-0.9500	-0.9468

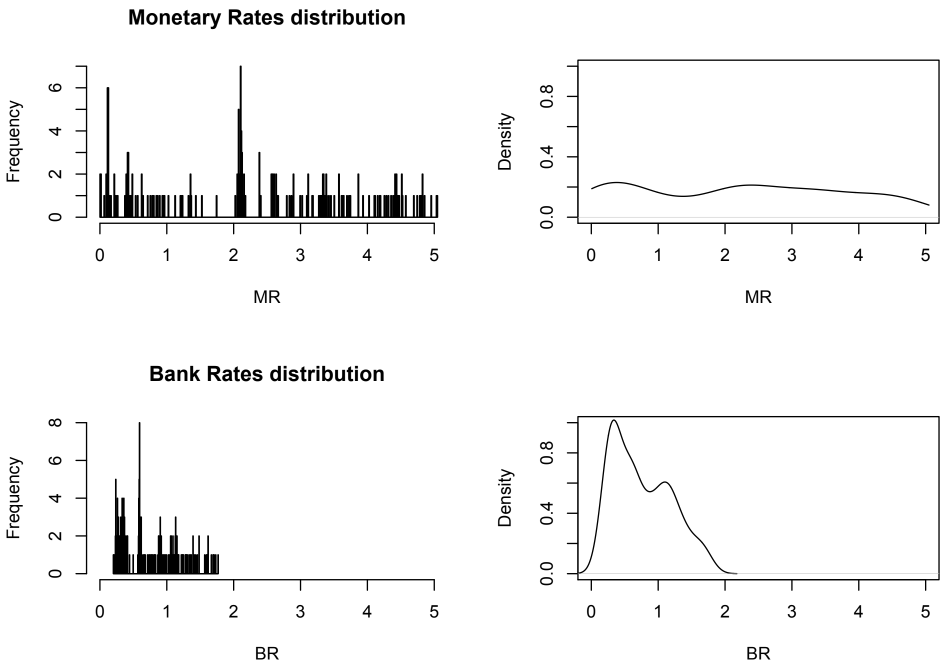
Notes: Impact of a positive change in the level of monetary rates on the economic value obtained by using our proposed model to allocate volumes.

Figure 2.6.1: Monetary and administered bank rates



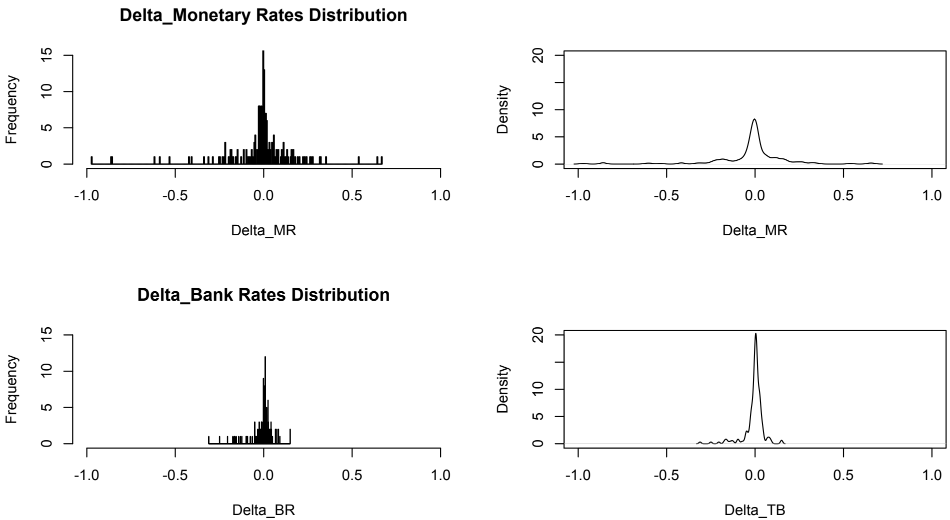
Notes: Time series of the observed 1-month Euribor rates and the aggregate administered bank rates on deposits. The two curves seem to have the same behaviour between 1999 and 2008, while the relationship between the two radically changes in the following years.

Figure 2.6.2: Distribution of monetary and administered bank rates



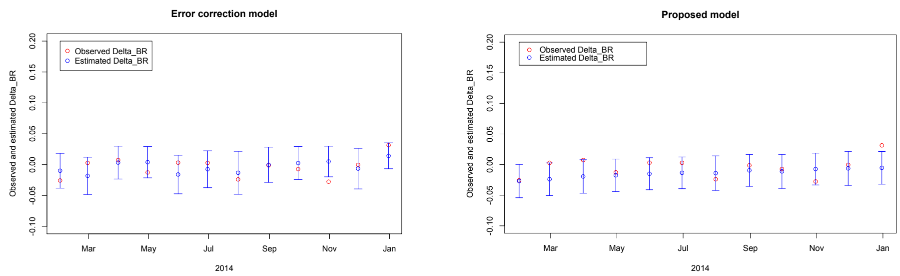
Notes: Histogram and density estimate of the monetary and bank administered interest rates. The latter are more concentrated around their mean value, while the former are quite spread.

Figure 2.6.3: Distribution of the variations of monetary and administered bank rates



Notes: Histogram and density estimate of the variations of monetary and bank administered interest rates. The latter are more concentrated around zero, while the former seem to have broader variations.

Figure 2.6.4: Estimated variations of administered bank rates



Notes: The estimated variations of administered bank interest rates for 2014, obtained with the error correction model (left) and with our proposed model (right) by using coefficients calculated on the whole period 1999 - 2013. The estimated values and their confidence intervals (blues) are compared with the real data (red).

Chapter 3

Dynamic hierarchical models for monetary transmission

Statistical Analysis and Data Mining, forthcoming

Abstract

Monetary policies, either actual or perceived, cause changes in monetary interest rates. These changes impact the economy through financial institutions, which react to changes in the monetary rates with changes in their administered rates, on both deposits and lendings. In this paper we provide a dynamic modelling for describing how administered bank interest rates react in response to changes in money market rates, in a multi-country setting: in addition, by means of hierarchical equations, we take into account how such changes are affected by the macroeconomic fundamentals of each country. The paper applies the proposed models to interest rates on different loans (to corporates and families) in seven European economies, showing how the monetary policy and the specific situation of each country differently impact lendings, not only across countries but also across time.

Keywords: Forecasting bank interest rates, Dynamic time series models, Hierarchical models, Monetary policy transmission.

JEL: C13, C32, C53, E43, E52.

3.1 Introduction

Monetary policies, such as variations in the official rate or liquidity injections, cause changes in monetary interest rates. These changes mainly impact the economy in an indirect way through financial institutions, which react to changes in the monetary rates with changes in their administered rates, on both deposits and lendings. The monetary transmission mechanism, which describes the effects of the monetary policy on the real economy through banks intermediation, has a central role in policy research since it is crucial in determining the monetary policy itself, especially in a monetary union such as the Eurozone. Because of the heterogeneity across different Eurozone countries, the monetary policy may have different effects on member states, especially after the financial and the sovereign crisis. In addition, the regime change in the monetary policy strategy that has been adopted since the end of 2008 may has changed the structure of the relationship between monetary rates and bank rates, differently affecting the real economy. The analysis of all these sources of heterogeneity (across countries and time) in the transmission mechanism from monetary rates to bank rates, conditioned by the economic peculiarities of each country, is the objective of this study.

The dynamics of administered bank interest rates in response to changes in money market rates is essential to examine the impact of monetary policies on the economy, especially in the recent time period characterised by monetary rates at the zero lower bound. This dynamics has been the subject of an extensive literature; the available studies differ, depending on the proposed models, the period under analysis and the geographical reference.

Most papers concentrate on the relationship between monetary rates and loans volumes, in order to understand how the monetary policy can affect banks' decisions of allowing for more or less loans: see, for example, Gertler and Gilchrist (1994), Bernanke and Gertler (1995), Kashyap and Stein (1995), Kashyap and Stein (2000), Ehrmann et al. (2001), Den Haan et al. (2007), Kleimeier and Sander (2004) and Kleimeier and Sander (2006). If it is true that these papers explain the effects of the monetary policy on real economies by understanding the changes in loans levels, they however do not address the issue of interest margins from the banks perspective, by looking at interest rates.

An alternative stream of research compares the different impact of monetary rates on deposit or loan rates. Hannan and Berger, for example, examine the deposit rate setting behaviour of commercial banks in the United States and find that (a) banks in more concentrated markets exhibit greater rates rigidity; (b) larger banks exhibit less rates rigidity; and (c) deposit rates are more rigid upwards than downwards. Similarly, Scholnick (1996) finds that deposit rates are more rigid when they are below their equilibrium level than when they are above; his finding on lending rate adjustment, however, is mixed. Heffernan (1997) examines how the lending and deposit rates of four banks and three building societies respond to changes in the base rate set by the Bank of England and finds that (a) there is very little evidence on the asymmetric nature of adjustments in both the deposit and lending rates, (b) there is no systematic difference in the administered rate pricing dynamics of banks and building societies, and (c) the adjustment speed for deposit rates is, on average, roughly the same as that for loan rates.

The empirical evidence contained in more recent papers on the same issue, such as Ballester et al. (2009), Chong et al. (2006), Demirguc-Kint and Huizinga (1999), Flannery and James (1984), Maudos and Guevara (2004), Maudos and Solis (2009), can be summarised in the following points: (a) bank rates react with a partial and delayed change to changes in the monetary rates; (b) the speed and the degree to which they follow these changes present substantial differences between the various categories of banking products and between different countries.

All the previous conclusions have been obtained for a relatively stable time period, previous to the emergence of the recent financial crisis. Moreover, most of them do not take into account the heterogenous transmission of monetary rates due to the different macroeconomic conditions across countries.

Regarding the peculiarities of the recent time-period, from a macroeconomic viewpoint monetary interest rates have become close to zero, or even negative, in most developed economies. In addition, from a microeconomic viewpoint the bank management has radically changed after the financial crisis, because of the compression of interest margins and the increase in regulatory capital requirements. The effects of the previous changes on the transmission of monetary policies have not been yet fully investigated. In particular, the current state of close-to-zero interest rates is of particular relevance, and, to our knowledge, Parisi et al. (2016) is the only paper that has concentrated on this topic, in a classical linear regression framework.

Regarding the heterogeneous cross-country conditions in the Eurozone, the sovereign debt crisis has increased the divergence of financial fundamentals in the European countries, making the transmission of the monetary impulse less uniform (see, e.g., Draghi). In particular, Neri (2014) has demonstrated that sovereign debt tensions have had a substantial impact in the transmission mechanism of monetary rates in peripheral countries, while such a change can not be detected in core ones. However, the analysis proposed by Neri (2014) only concentrates on the impact of sovereign bond interest rates on bank administered rates within a static framework, without considering time-varying relationships as well as different macroeconomic indicators.

The aim of this paper is to broaden the existing literature in order to overcome the above described limitations, introducing a hierarchical time dynamics able (a) to capture the evolving relationship between bank rates and monetary rates, taking into account correlation effects between different kinds of loans within each country, as well as cross-country correlations driven by the common monetary policy; (b) to understand how country-specific macroeconomic factors may have differently affected the monetary transmission across European countries and how such relationships have changed over time; (c) to provide better results in terms of both in-sample tests and out-of-sample predictions.

The proposed methods are applied to data from the recent period (2003-2014), for core (France, Germany, the Netherlands) and peripheral (Ireland, Italy, Portugal, Spain) countries.

The effect of monetary policies is studied for three categories of loans: (a) loans to non-financial corporates up to 1 Mln euros; (b) loans to non-financial corporates over 1 Mln euros; (c) loans to households for mortgages.

The results show that the reactions to monetary policy differ according to both the lending type and the reference country. In particular, large corporate loans are the most affected by monetary rates, whereas small-medium corporate loans and household loans depend less on monetary rates and more on country-specific macroeconomic factors, such as interest rates on deposits and GDP variations. Moreover, in core countries, such as Germany and the Netherlands, bank rates depend almost exclusively on monetary rates and, therefore, the transmission of monetary policy is expected to be effective. In peripheral countries, instead, all lending rates depend on bank risk, corporate risk and, more recently, sovereign risk, as reflected by deposit rates, GDP variations and sovereign bond rates. Hence, in these countries, the transmission of the monetary policy appears to be more problematic. In general, this paper shows that the financial crisis and the sovereign crisis have differently affected the time dynamics of bank interest rates on loans, with the former leading to homogenous reactions in all countries and the latter causing a clustering effect between peripheral and core countries.

The paper is structured as follows. Section 3.2 describes the proposed models and, in particular: Section 3.2.1 describes the theoretical framework; Section 3.2.2 introduces the new proposed models, with Subsection 3.2.2 concentrating on seemingly unrelated dynamic models and Subsection 3.2.2 describing hierarchical dynamic processes. Section 3.2.3 provides the parameters estimation techniques and the predictive performance environment used to compare the models. Section 3.3 shows the empirical evidence obtained from the application of the models and, in particular: Section 3.3.1 describes the available data; Section 3.3.2 presents the results obtained with the seemingly unrelated dynamic model; Section 3.3.3 presents the results obtained with the hierarchical dynamic model; Section 3.3.4 shows predictive performances and compares the models. Finally, Section 3.4 concludes with some final remarks.

3.2 Methodology

3.2.1 Theoretical Framework

The direct relationship between monetary rates and administered bank rates can be analysed by means of the Error Correction Model (ECM), following the procedure proposed by Engle and Granger (1987). The model is based on two equations. A long-run relationship, which provides a measure of how a change in the monetary rate is reflected in the bank rate. A short-run equation, which includes an error correction term, which evaluates variations of the administered interest rates as a function of variations in the monetary rates.

Parisi et al. (2016) analysed and extended the ECM (proposed, among others, by Chong et al. (2006) for the analysis of the monetary transmission mechanism), by deriving an alternative one-equation model. More precisely, they assumed that bank interest rates depend on their previous level, to allow for a slow and partial reaction of bank rates to monetary rates changes. Thus, they modelled bank administered interest rates as a function of monetary rates, their variations and the previous level of bank rates. Their complete model, that in the next Sections will be called the *Parisi et al. model*, can be formalised as follows:

$$BR_t = k + \beta \cdot MR_{t-1} + \gamma \cdot \Delta MR_t + \delta \cdot BR_{t-1} + \epsilon_t. \tag{3.2.1}$$

In equation (3.2.1) BR_t and MR_t represent, respectively, the bank administered rates and the monetary rates at time t; β is a regression coefficient that gives a measure of the extent of the monetary rate transmitted on bank rates in a long-term perspective; γ is the coefficient that explains the influence of the variations of monetary rates on bank rates levels; δ weights the autoregressive component BR_{t-1}; k is a constant that summarises all the other factors that, in addition to the dynamics described by the regressors, may affect the transmission mechanism of the monetary policy on bank rates as, for example, the market power and the efficiency of a bank; finally, ϵ_t is the error term.

Neri (2014) proposed a model that, differently from Parisi et al. (2016), explains bank rate dynamics as a function of monetary rates and exogenous variables. More precisely, he established a relationship between lending rates ($BR^k_{j,t}$, with j = country, k =type of loan) and the spread between the yields on government bonds and the 10-year swap rate

$(R^{10}_{j,t-1} - R^{10,swap}_{t-1})$, as follows

$$BR^k_{j,t} = \overline{BR}^k_j + \alpha_1 D^{crisis}_t + \alpha_2 D^{2008}_t + \alpha_3 R^{ov}_{t-1} + \alpha_4 (MR^{3m}_{t-1} - MR^{ov}_{t-1}) + \alpha_5 (R^{10}_{j,t-1} - R^{10,swap}_{t-1}) + \alpha_6 Y^k_{t-1} + \alpha_7 BR^k_{j,t-1} + \epsilon_t, \quad (3.2.2)$$

where D^{crisis}_t and D^{2008}_t are dummy variables that become one during Lehman Brothers and 2008 crisis, $MR^{3m}_{t-1} - MR^{ov}_{t-1}$ represents the spread between three-months Euribor and the EONIA, and Y^k_{t-1} is a confidence indicator. A similar approach has been followed also by De Santis and Surico (2013).

While both (3.2.1) and (3.2.2) concentrate on the changes in interest rates on loans due to monetary rates or macroeconomic factors, they still do not account for time-dependent parameters and cross-country correlations and, therefore, they need to introduce time-windows and dummy variables in order to consider changes in time.

In this paper, we overcome this issue by introducing dynamic models, which not only allow to derive time-varying relationships without imposing *ex-post* dummies, but also enable the prediction of future values. Time-varying parameters models (TVP) have recently been the subject of an extensive literature: we refer, for example, to the TVP-VAR model proposed by Primiceri (2005), the Markov-switching VAR of Sims and Zha (2006), the TVP-VAR with stochastic volatilities by Clark and Ravazzolo (2015) and the stochastic correlation MS-VAR of Casarin et al. (2016).

In addition, we also merge together the models (3.2.1) and (3.2.2) by considering a dynamic hierarchical process, composed by two stages. The first one models bank rates on loans as a time-varying function of monetary rates by using seemingly unrelated regressions; in such a way we can understand how the monetary transmission has changed over time in different geographical ares, starting from the assumption that interest rates on different loans or in different countries are correlated since they are all affected by the same monetary policy. The second one explains the residual components obtained with the first stage by means of multiple dynamic models; the result is a further dependence structure between bank rates and country-specific macroeconomic indicators, again in a time-varying framework.

3.2.2 The proposed models

Seemingly Unrelated Dynamic Linear Models

We have anticipated that the relationship between bank rates and monetary rates has radically changed during the last years, reaching a situation of almost-zero monetary rates. In order to better analyse how bank rates react to changes in monetary rates, and to understand how this reaction changes over time, simple linear models can be enriched with a dynamic structure.

Dynamic linear models are a particular class of state-space models, in which the regression coefficients are allowed to vary over time: for an introduction, see, e.g., Petris et al. (2009). More formally, since we consider N different countries and three different kinds of loans, let us introduces two sets, $V = \{1, ..., N\}$ and $W = \{1, 2, 3\}$, and let the index i be $i \in W \times V$. The dynamic equations can thus be expressed as follows:

$$\begin{cases} BR_t^i = k_t^i + \beta_t^i MR_t + \epsilon_t^i, & \epsilon_t^i \sim N(0, \sigma^2_{\epsilon^i}) \\ k_t^i = k_{t-1}^i + \omega_{k,t}^i, & \omega_{k,t}^i \sim N(0, \sigma^2_{\omega^i,k^i}) \\ \beta_t^i = \beta_{t-1}^i + \omega_{\beta,t}^i, & \omega_{\beta,t}^i \sim N(0, \sigma^2_{\omega^i,\beta^i}), \end{cases} \tag{3.2.3}$$

where the quantities $(\epsilon_t^i, \omega_{k,t}^i, \omega_{\beta,t}^i)$, for $t = 1, ..., T$, are assumed to be independent from each other.

Equation (3.2.3) can be rewritten in a compact form: by using the following substitutions,

$$\Theta_t^i = \begin{bmatrix} k_t^i \\ \beta_t^i \end{bmatrix}, \quad F_t = \begin{bmatrix} 1 \\ MR_t \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad W_t^i = \begin{bmatrix} \omega_{k,t}^i \\ \omega_{\beta,t}^i \end{bmatrix},$$

(3.2.3) becomes:

$$\begin{cases} BR_t^i = F_t^T \Theta_t^i + \epsilon_t^i, \\ \Theta_t^i = G\Theta_{t-1}^i + W_t^i. \end{cases} \tag{3.2.4}$$

The previous model consists of $N \times 3$ independent equations, each of them referred to a different kind of loan in a specific country. Such independence hypothesis can be relaxed, since we assume that, within each country, interest rates on different kinds of loans similarly react to changes in monetary rates. In order to take into account the hypothesis that, within each of the N countries but not across them, BR_t^j (with $j \in W$) follow the same type of dynamics, we introduce seemingly unrelated dynamic regressions.

More formally, let us consider the state vector $\Theta_t^h = (k_t^{1,h}, k_t^{2,h}, k_t^{3,h}, \beta_t^{1,h}, \beta_t^{2,h}, \beta_t^{3,h})^T$, with $h \in V$ and $\{1, 2, 3\} \in W$. Similarly to equation (3.2.3), the jth component of bank rates, within each country, follows a dynamic linear model:

$$\begin{cases} BR_t^j = F_t\theta_t^j + \epsilon_t^j, \\ \theta_t^j = G\theta_{t-1}^j + \omega_t^i, \end{cases} \tag{3.2.5}$$

where $\theta_t^j = (k_t^j, \beta_t^j)^T$ for $j = 1,2,3$ and h fixed. The seemingly unrelated dynamic regression for bank rates, for each country $h = 1, ..., N$, can thus be derived as:

$$\begin{cases} BR_t^h = (F_t^T \otimes I_3)\Theta_t^h + \epsilon_t^h, & \epsilon_t^h \sim N(0,V) \\ \Theta_t^h = (G \otimes I_3)\Theta_{t-1}^h + w_t^h, & w_t^h \sim N(0,W), \end{cases} \tag{3.2.6}$$

where, again, $F_t = \begin{bmatrix} 1 \\ MR_t \end{bmatrix}$ and $G = I_2$. Furthermore, since we want the levels k_t and the slopes β_t to evolve independently, we consider W as a block-diagonal matrix, whose two blocks (W_k, W_β) can be obtained through (3.2.3).

Hierarchical Dynamic Models

Dynamic linear models are very adaptive to past data, since the time-dependent parameter k_t, which describe the dynamic levels of bank rates, moves to capture most of the variability unexplained by the regressors, thus becoming, *de facto*, a latent explanatory variable (factor). It is interesting to explain such latent variable k_t through a set of regressors, again in a dynamic framework. The model proposed in (3.2.6) can be enriched with a further stage, thus becoming a hierarchical dynamic model, specified by the following:

$$\begin{cases} BR_t^h = (F_{BR,t}^T \otimes I_3)\Theta_t^h + \epsilon_t^h, & \epsilon_t^h \sim N(0,V_{BR}) \\ \Theta_t^h = (F_{\theta,t}^h)^T\Gamma_t^{j,h} + v_t^{j,h}, & v_t^{j,h} \sim N(0,V_\theta) \\ \Gamma_t^{j,h} = G_t\Gamma_{t-1}^{j,h} + w_t^{j,h}, & w_t^{j,h} \sim N(0,W), \end{cases} \tag{3.2.7}$$

where

$$F_{BR,t} = \begin{bmatrix} 1 \\ MR_t \end{bmatrix}, \quad F_{\theta,t}^h = \begin{bmatrix} 1 \\ X_{1,t}^h \\ \vdots \\ X_{p,t}^h \end{bmatrix}, \quad \Gamma_t^{j,h} = \begin{bmatrix} \alpha^{j,h} \\ \gamma_{1,t}^{j,h} \\ \vdots \\ \gamma_{p,t}^{j,h} \end{bmatrix}, \quad G = I_{p+1}, \tag{3.2.8}$$

being the vector $X = (X_{1,t}^h, ..., X_{p,t}^h)$ an ensemble of country-specific economic variables. From equation (3.2.7) it is clear that the state vector Θ_t^h, derived through the seemingly unrelated dynamic regression, is itself modelled by a multiple dynamic linear model. This allows to understand which are the main economic country-specific components able to describe the dynamics of the bank rates not explained by the monetary rates. It can also be used to understand the elasticity of the slope coefficient that relates monetary to bank rates.

3.2.3 Parameter Estimation and Model Adequacy

All the parameters included in both state vectors Θ_t^h and Γ_t^h have to be estimated through a two-stages process, according to the hierarchical structure introduced in (3.2.7).

As previously described, the first stage consists of a double step: a dynamic model, which provides the estimated parameters $\hat{\Theta}$ and their variances, through which we can obtain the final estimation Θ. According to Petris et al. (2009), we can define the following:

$$\begin{cases} f_t = \mathbb{E}(BR_t|\mathcal{D}_{t-1}), \\ Q_t = \mathrm{Var}(BR_t|\mathcal{D}_{t-1}), \end{cases} \tag{3.2.9}$$

where $\mathcal{D}_{t-1}$ denotes the information provided by the first $t-1$ observations $BR_1, ..., BR_{t-1}$. In such a way the log-likelihood can be derived, as a function of the unknown parameter vector Θ:

$$\ell(K) = -\frac{1}{2}\sum_{t=1}^{T} \log|Q_t| - \frac{1}{2}\sum_{t=1}^{T}(BR_t - f_t)^T Q_t^{-1}(BR_t - f_t), \tag{3.2.10}$$

where f_t and Q_t both depend implicitly on the vector Θ. The previous expression can be numerically maximised in order to obtain the maximum likelihood estimator of the unknown parameters' vector Θ;

$$\Theta = \arg\max_{\Theta} \log \ell(\Theta). \tag{3.2.11}$$

Moreover, denoting by H the Hessian matrix of $-\ell(\Theta)$, the inverse H^{-1} provides an estimate of the variance of the estimator. Once the parameters k_t and β_t have been estimated, together with their errors $\omega_{k,t}, \omega_{\beta,t}$, future values for k and β can be calculated. Nite that the difference in the estimation between dynamic models and seemingly unrelated dynamic regressions stands in $\mathcal{D}_{t-1}$: while in the first case it includes identity matrices for V and W, in the second one it includes the estimated variances for BR_t (in V) and for the parameters k_t and β_t (in W) obtained with the first step.

We remark that similar estimation procedure can be applied to parameter estimation of the second stage of the hierarchical process (multiple dynamic models).

In order to predict future values of the parameters in the state vector Θ_t^i, we can extract ω_k^i and ω_β^i from their distributions (obtained on past data) for the next twelve months of the year to be predicted ($\hat{\omega}_{k,q}^i$, $\hat{\omega}_{\beta,q}^i$; $q = 1, ..., 12$, $i = 1, ..., N$), thus estimating the parameters as follows:

$$\begin{cases} \hat{k}_q^i = \hat{k}_{q-1}^i + \hat{\omega}_{k,q}^i, \\ \hat{\beta}_q^i = \hat{\beta}_{q-1}^i + \hat{\omega}_{\beta,q}^i. \end{cases} \tag{3.2.12}$$

Future values for the parameters included in Γ_t^i can be estimated through the same procedure.

Finally, in order to predict bank rates, we need to estimate reasonable future values of monetary rates. Consistently with the literature, we assume that their variations follow a random walk process.

More formally, assume that we want to predict the level of monetary rates for each of the next 12 months. Let $\widehat{\Delta MR}_q$ indicates the variation of the monetary rate in a given month. We then assume that all the $\widehat{\Delta MR}_q$ are independently and identically distributed Gaussian random variables, so that:

$$\begin{cases} \widehat{\Delta MR} \sim N(0, \sigma^2) \\ \widehat{MR}_q = \widehat{MR}_{q-1} + \widehat{\Delta MR}_q \quad \mathrm{q} = 1, ..., 12. \end{cases} \tag{3.2.13}$$

Equation (3.2.13) describes a recursive procedure to obtain predictions of the monetary rates for a given year ahead, based on the random walk process assumption. We can then insert the predicted monetary rates as regressor values in the models of the previous Subsection and, thus, obtain predictions for the administered bank rates. Bank rates are thus predicted as:

$$\widehat{BR}_q^i = \hat{k}_q^i + \hat{\beta}_q^i \widehat{MR}_q. \tag{3.2.14}$$

The estimation of the state vector Θ_t^i, based on the second stage of the hierarchical process in (3.2.7), can be achieved by similarly applying equation (3.2.12) for estimating $\hat{\Gamma}_t^i$, and thus deriving future values for Θ_q^i as follows:

$$\widehat{\Theta}_q^i = \hat{\alpha}_q^i + \sum_{z=1}^{p} \hat{\gamma}_{q,z}^i \widehat{X}_{q,z}^h, \tag{3.2.15}$$

where as $\widehat{X}_{q,z}^{h}$ we will consider the forecasts of that economic variable made by official institutions (such as the European Central Bank and the International Monetary Fund).

It is important to test whether the addition of the hierarchical two-stages model improves the one-stage model. In order to compare them on the same playing field, we will run two kinds of test: an in-sample test, based on the MANOVA Wilk's Λ, and an out-of-sample test, based on Root Mean Square Errors ($RMSE$) and on Mean Absolute Percentage Errors ($MAPE$). We use $MAPE$, together with $RMSE$, since it is less sensitive to occasional large errors and it allows for comparisons between different measure scales.

3.3 Data analysis and results

3.3.1 Descriptive analysis

The recent financial crisis has had a major impact on the banking sector and, in particular, has led to a change in the relationship between monetary and administered rates. In the Eurozone, characterised by one monetary authority (the European Central Bank), that regulates still fragmented national markets, this effect is particularly evident: southern european countries, differently from what expected, have benefited very little from the drop of monetary rates that has followed the financial crisis.

To investigate the above issues we focus on seven european countries: France, Germany, Ireland, Italy, the Netherlands, Portugal and Spain.

Accordingly, we have collected from the ECB public database, monthly time series data on monetary rates and on aggregate bank administered rates on lendings, divided into three categories: (a) loans to non-financial corporates up to 1 Mln euros, shown in Figure 3.6.1; (b) loans to non-financial corporates over 1 Mln euros, shown in Figure 3.6.2: (c) loans to households for mortgages, shown in Figure 3.6.3. For the purposes of our analysis, the monetary rate used in this paper is the 3-months Euribor. Note that such disaggregation of interest rates on loans to non-financial corporates can roughly be interpreted by saying that the first category represents loans to small-medium enterprises, and the second one represents loans to large corporates.

[Fig. 3.6.1]

From Figure 3.6.1 interesting behaviours emerge: firstly, interest rates on loans in all countries, together with monetary rates, have strongly increased during the financial crisis of 2008, while dropping the following year. Secondly, the gap between such two rates has increased, since monetary rates are now very close to zero while interest rates on loans have remained quite high, even if they show a decreasing trend in 2014. Finally, interest rates on loans in core countries (France, Germany and the Netherlands) are very similar to each other, while the situation is more heterogenous in peripheral countries, with Portugal presenting the highest values during the entire period and Ireland being characterised by a strong volatility and an independent time-evolution after 2010.

[Fig. 3.6.2]

Figure 3.6.2 reports interest rates on loans to non-financial corporates over 1 Mln euros. Firstly, it is interesting to observe that, in absolute values, such interest rates are much lower than in the previous case, consistently with the assumption that loans to small-medium enterprises are riskier than loans to large corporates. Secondly, also this kind of interest rates has strongly increased during the financial crisis, decreasing in 2009. Finally, differently from before, all countries seem to have similar behaviours, strongly volatile in the short term but, on long run average, almost stable: again, Portugal presents a peculiar time-evolution, especially after 2010, characterised by very high and unstable values.

[Fig. 3.6.3]

Figure 3.6.3 reports interest rates on loans to households for mortgages. As for loans to non-financial corporates over 1 Mln euros, this category of interest rates presents lower values with respect to loans to small-medium enterprises. However, the main peculiarity of this last rates consists in their completely heterogeneous behaviours after 2009: in the latest years, in fact, a common evolution pattern between different countries can not be detected, as well as, in each country, such time-evolutions are characterised by continuous changes. In particular, Italy, Portugal and Spain suffered an increase in interest rates during their sovereign crisis of 2012, while such increase is anticipated in Germany (2011) and hardly visible in France and the Netherlands; moreover, in France, Germany, Italy and the Netherlands interest rates on loans to households started decreasing in 2014, while such a trend can not be identified in the remaining countries.

From the above Figures it is clear that the relationship between bank rates and monetary rates considerably changes over time for all types of lendings. Indeed, further analysis shows that, for small-medium corporate loans, the relationship is almost linear in all countries before the financial crisis of 2008; it remains so only in Germany and the Netherlands after the crisis. For large corporate loans, instead, core countries present a linear relationship during the entire period, while peripheral ones show a non-linear relation after the financial crisis. Family loans follow the pattern of small-medium corporate loans, with the Netherlands behaving as France and peripheral countries.

In order to better understand how interest rates on different kinds of loans and in different countries are correlated to each other and with 3-months Euribor, Table 3.6.1 reports correlation coefficients between them. In Table 3.6.1 *corp1* refers to interest rates on loans to non-financial corporates up to 1 Mln euros, *corp2* refers to interest rates on loans to non-financial corporates over 1 Mln euros and *fam* refers to interest rates on loans to households for mortgages.

[Table 3.6.1]

Table 3.6.1 shows interesting results. Firstly, by looking at the diagonal blocks, one can notice that, within each country, correlations between different kinds of loans are all positive and very high, meaning that a seemingly unrelated dynamic model has to be preferred with respect to a simple dynamic model in which different lending type rates are considered independently from each other. Secondly, by looking at the cross correlations between different countries, such relationships remain positive but lower: this suggests that, to avoid further complexity, lending rates of different countries can be assumed to be independent. Finally, by looking at the correlations between interest rates on loans and the 3-months Euribor, one can conclude that all interest rates on loans are positively and strongly related to monetary rates: again, Portugal and *corp1* in Spain represent the only two exceptions, being characterised by very low correlations.

Finally, in order to better understand our data and to better interpret the results presented in the next Section, we have performed a volatility analysis based on the EWMA (Exponentially Weighted Moving Average) filter. Such results are shown in Figure 3.6.4.

[Fig. 3.6.4]

Figure 3.6.4 firstly shows that the volatility of interest rates of all kinds of loans substantially increased during the financial crisis in all countries. Such result can be explained by looking at Figures 3.6.1, 3.6.2 and 3.6.3, which show a radical drop in interest rates (both monetary and bank administered ones) in 2008-2009. Furthermore, interest rates on loans to corporates over 1 Mln euros have the highest volatility in all countries with the exception of Portugal, consistently with Figure 3.6.2. Finally, Germany shows different patterns, especially during the last years: its volatility, in fact, has started increasing in the last years, more precisely during the sovereign crisis of 2012: to a lesser extent, France presents an analogous situation.

3.3.2 Seemingly Unrelated Dynamic Regression

For the model proposed in Section 3.2.2, we now show the corresponding estimation of the regression coefficients. As previously described, we first compute the dynamic model, deriving the coefficients $\hat{k}_t^i$ and $\hat{\beta}_t^i$ obtained considering each regression model independently from the others, for each $i \in V \times W$. After that we calculate, for each country, the variance-covariance matrices between such coefficients, obtained for the three kinds of loans: $W_k^h = \mathrm{Var}(\hat{k}^{1,h}, \hat{k}^{2,h}, \hat{k}^{3,h})$ and $W_\beta^h = \mathrm{Var}(\hat{\beta}^{1,h}, \hat{\beta}^{2,h}, \hat{\beta}^{3,h})$, for $h \in V$ and $\{1, 2, 3\} \in W$. These results, together with the variance-covariance matrix of the dependent variable BR_t^h, $V^h = \mathrm{Var}(BR^{1,h}, BR^{2,h}, BR^{3,h})$, are used to derive the regression coefficients of the seemingly unrelated dynamic model described in (3.2.6).

The time-evolutions of the two estimated regression coefficients are presented in Figures 3.6.5, 3.6.6 and 3.6.7. More precisely, Figure 3.6.5 refers to interest rates on loans to non-financial corporates up to 1 Mln euros, Figure 3.6.6 refers to interest rates on loans to non-financial corporates over 1 Mln euros, Figure 3.6.7 refers to interest rates on loans to households for mortgages.

[Fig. 3.6.5]

Figure 3.6.5 shows on the left the evolution of the intercepts k_t^h and, on the right, the evolution of the coefficients β_t^h, which explain the time-varying elasticity of the monetary rates in the explanation of bank interest rates on loans to non-financial corporates up to 1 Mln euros. From the analysis of the curves, important conclusions emerge. First, the contribution of monetary rates in the determination of bank rates has remained almost constant in all countries, with a weak peak at the end of 2008, coinciding with the financial crisis and the strong increase in 3-months Euribor. If one considers that, in the recent years, monetary rates are very close to zero, this result tells us that, in absolute value, bank rates are no more explained by monetary ones. Second, all the intercepts, which represent what monetary rates are not able to explain in the variability of bank rates, are strongly time-varying and heterogenous across different countries. In particular, before the crisis all the coefficients k_t^h move together, but the situation changes after 2009: in the last period, in fact, it is clear that France, Germany and the Netherlands (core countries) on one side, as well as Italy and Spain on the other, behave quite similarly; Portugal is characterised by the highest values during the whole period; Ireland follows a completely different trend.

[Fig. 3.6.6]

Figure 3.6.6 shows on the left the evolution of the intercepts k_t^h and, on the right, the evolution of the coefficients β_t^h, referred to bank interest rates on loans to non-financial corporates over 1 Mln euros. From the analysis of these results an important difference with respect to the previous case can be detected: in all countries, the contribution of monetary rates in the determination of bank rates on loans to large corporates is much higher than the one referred to interest rates on loans to small-medium enterprises, as the residual component, expressed by k_t^h, is much lower. Moreover, the time evolution of β_t^h is constant in almost all countries, with the exception of France and, to a lesser extent, of Italy. In addition, a common pattern between the time evolution of the intercepts k_t^h can not be identified after 2009, neither for core nor peripheral countries.

[Fig. 3.6.7]

Figure 3.6.7 shows on the left the evolution of the intercepts k_t^h and, on the right, the evolution of the coefficients β_t^h, referred to bank interest rates on loans to households for mortgages. Similarly to the previous two kinds of loans, the coefficients β_t^h look almost constant in all countries and during the whole period, with the exception of France. By looking at the coefficients k_t^h, the situation is extremely heterogeneous across different countries, both before and after the financial crisis.

To summarise, from the seemingly unrelated dynamic model, proposed in (3.2.6) and analysed in this Section, the most important conclusions are as follows. (a) The contribution of monetary rates in the composition of bank rates is almost constant in time, in all countries and for the three kinds of loans, with the exception of France. (b) Interest rates on loans to large corporates are much more explained by monetary rates with respect to the other two kinds of loans, because of their higher values for β_t^h and of their lower values for k_t^h. (c) The time evolution of the intercepts k_t^h is extremely heterogenous, especially after 2009 and for interest rates on loans to households: by combining such result with the volatility analysis proposed in Figure 3.6.4, this means that country-specific effects have to be considered in the explanation of bank rates. More precisely, this result means that the extreme increase in the volatility after the financial crisis has not been endogenous, but determined by other macro-economic factors. (d) For small-medium enterprises, we observe a divergence effect between core and peripheral countries in the behaviour of k_t^h after the crisis, again consistently with Figure 3.6.4.

3.3.3 Hierarchical Dynamic Models

In this Subsection we apply the hierarchical procedure proposed in (3.2.7) to the intercept coefficients k_t^h previously obtained through the seemingly unrelated dynamic model, to take into account country-specific macroeconomic effects in the explanation of the time-varying parameters.

We propose the following three regressors: interest rates on 10-years government bonds, aggregate interest rates on deposits and annual variations of GDP. This choice is due to both practical and economical motivations. Firstly, they all are publicly available data, so this analysis can be replicated. Secondly, interest rates on government bonds can be considered as a proxy for sovereign risk; interest rates on deposits, together with interest rates on loans, define the income of the banking sector in each country; annual variations of GDP describe, at the country level, the overall risk of the corporate sector. Consistently with interest rates on loans, all the variables described so far have monthly frequencies and have been collected from the ECB database, from January 2003 until December 2014. We remark that GDP variations are quarterly published, so we have applied linear interpolation to obtain monthly data.

In order to better understand how these explanatory regressors have evolved in time, Figure 3.6.8 shows interest rates on 10-years government bonds, Figure 3.6.8 shows aggregate interest rates on deposits, and Figure 3.6.8 shows annual GDP growth rates, in the seven considered european countries.

[Fig. 3.6.8]

Figure 3.6.8 shows a great differentiation between european countries after the financial crisis of 2008. In the first period, in fact, interest rates on government bonds are extremely similar to each other, following the same dynamics. After 2008 the situation completely changes: on one hand, core countries are clearly characterised by similar values and by a long-run average decreasing trend; on the other hand, rates on government bonds in peripheral countries diverge, with Ireland and Portugal presenting the highest peaks during their sovereign crisis (respectively in 2011 and 2012).

[Fig. 3.6.9]

Figure 3.6.9 shows a more heterogeneous behaviour with respect to Figure 3.6.8. Interest rates on deposits strongly increased in 2008 in all countries and decreased afterwards, together with the drop of monetary rates. In the recent years, such rates increased again during the sovereign crisis of 2012, starting a further decrease in 2013. It is important to observe the peculiar case of France, whose bank rates on deposits have not been subject to great changes during the entire period, remaining, on average, almost constant.

[Fig. 3.6.10]

Figure 3.6.10 represents the economic growth evolutions of the seven european countries and shows many similarities across them. All GDP growth rates, in fact, dropped during both the financial crisis and the sovereign crisis: among them, Germany and France suffered less the sovereign crisis of 2012, while the situation is reversed for Italy, Portugal and Spain. Finally, it is interesting to observe the strong GDP growth in Ireland, started in 2013 after the important package of reforms introduced by the Irish government in order to recover from their dramatic crisis.

In order to understand how the exogenous variables previously described are related to each other, to interest rates on loans and to monetary rates, Table 3.6.2 shows the correlation coefficients for all the countries.

[Table 3.6.2]

Table 3.6.2 concentrates on correlations between interest rates on loans and exogenous country-specific variables, showing interesting results. Firstly, one can notice that there is a clear difference between core and peripheral countries. France, Germany and the Netherlands are characterised by high and positive coefficients between rates on government bonds and interest rates on loans: this is consistent with the fact that, if bond rates increase, sovereign risks increase and, consequently, loan rates increase. Such correlations, however, are very low or negative in Ireland, Italy, Portugal and Spain. Secondly, the GDP growth rate is negatively related to interest rates on loans in all countries, consistently with the fact that when the economy improves, the demand for loans increases and, consequently, interest rates on loans decrease.

After having described the variables involved in the second step of the hierarchical model, we now show the corresponding estimation of the regression coefficients for the model proposed in (3.2.7), with

$$F_{\theta,t}^{h} = \begin{bmatrix} 1 \\ X_{1,t}^{h} \\ \vdots \\ X_{p,t}^{h} \end{bmatrix} = \begin{bmatrix} 1 \\ Bond_t^h \\ Deposit_t^h \\ \Delta GDP_t^h \end{bmatrix}, \quad \Gamma_t^{j,h} = \begin{bmatrix} \alpha^{j,h} \\ \gamma_{1,t}^{j,h} \\ \gamma_{2,t}^{j,h} \\ \gamma_{3,t}^{j,h} \\ \gamma_{4,t}^{j,h} \end{bmatrix}, \tag{3.3.1}$$

and where Θ_t^h, the dependent variable, is the vector of the regression coefficients derived in Section 3.3.2. We remark that, since the coefficients β_t^h previously obtained are almost constant, this second step of the hierarchical procedure will be applied only to the intercepts k_t^h, so that Θ_t^h becomes $\Theta_t^h = (k_t^{1,h}, k_t^{2,h}, k_t^{3,h})$, with $h \in V$ and $\{1,2,3\} \in W$.

The time-evolutions of the four regression coefficients are presented in Figures 3.6.11, 3.6.12 and 3.6.13. More precisely, Figure 3.6.11 refers to interest rates on loans to non-financial corporates up to 1 Mln euros, Figure 3.6.12 refers to interest rates on loans to non-financial corporates over 1 Mln euros, Figure 3.6.13 refers to interest rates on loans to households for mortgages. We remark that, in each Figure, only significant regressors have been plotted: this implies, for example, that Germany does not appear in the following graphs since its Θ_t^{ger} does not significantly depend on exogenous variables.

[Fig. 3.6.11]

Figure 3.6.11 shows the time dynamics of the four components of $\Gamma_t^{j,h}$, thus representing how these components are important in describing the time evolution of bank rates on loans that has not been fully captured by monetary rates. Firstly, one can notice that the coefficients α are now almost constant, meaning that they can be interpreted as the fixed, long-run average components of bank rates, not explained by other exogenous variables. Secondly, interest rates on government bonds seem to have a significant effect only in France, Ireland and the Netherlands during the entire period, and in all countries after 2012. The contribution of interest rates on deposits is particularly significant in Italy and Spain during the whole years, while, since 2012, it started decreasing in France and increasing in Ireland and Spain. Finally, the contribution of the GDP growth rate became more significant after the financial crisis of 2008, not only for peripheral countries but also for France.

[Fig. 3.6.12]

Figure 3.6.12 shows the time dynamics of the components of $\Gamma_t^{j,h}$ for loans to non-financial corporates over 1 Mln euros. By looking at the residual parts α_t, two conclusions emerge: firstly, in all countries the estimated coefficients are almost constant; secondly, they are significantly lower than the ones obtained in the previous case. This is a further confirm of the fact that interest rates on loans to large corporates are much more explained by monetary rates, as well as they have a lower long-run average value since this kind of lending activity is riskier. As in the previous case, the contribution due to interest rates on deposits is very high in peripheral countries, especially in Portugal, meaning that, in such economies, banks adjust interest rates on loans to large corporates (assets) according to the ones on deposits (liabilities), or vice-versa, thus equilibrating their interest margins. The contribution of the GDP growth rates is almost zero in France and Spain, while it is strongly decreasing in Italy and Portugal.

[Fig. 3.6.13]

Figure 3.6.13 shows the time dynamics of the components of $\Gamma_t^{j,h}$ for loans to households for mortgages. Differently from loans to corporates, in this case the residual components α_t are constant only in Portugal and Spain: they show a weak decreasing evolution in Italy, and a strong increase in France, Ireland and the Netherlands, meaning that, in these countries, a further level of explanation should be investigated. The contribution of interest rates on government bond is significant in all countries with the exception of France, even if shows very heterogenous behaviours, not only across different geographical areas but also across time. The most interesting result regards the contribution of interest rates on deposits: γ_2, in fact, presents high values and similar evolutions in France and the Netherlands, while in Ireland it started increasing after the financial crisis, showing an even faster increase after the reform of the banking system of 2013. Italy, Portugal and Spain, on the other hand, are characterised by low values. Finally, the contribution of GDP growth rates shows two distinct patterns: on one side, France and Italy are characterised by a monotonic decreasing trend, while Portugal and Spain have similar, more stable behaviours.

As previously observed, France presents a peculiar situation since its coefficients β_t, derived with the first seemingly unrelated dynamic model, are time-varying for loans to big firms and households. By computing the multiple dynamic regression described in this Section also on such two coefficients, the results show that these stickiness parameters, which explain the contribution of monetary rates in the transmission mechanism to bank rates, significantly and negatively depends on interest rates on deposits.

To summarise, from the dynamic hierarchical model proposed in (3.2.7) and analysed in this Section, the most important conclusions are as follows. Germany lending interest rates depend only on monetary rates, and this is consistent with its role as the pivotal country of the Euro Area. The behaviour of the Netherlands is similar to that of Germany for the corporate sectors but, for households, there is an additional effect of sovereign bond rates and of bank deposit rates, which are the result of the bank search for a profit margin. The behaviour of France is mixed between core and peripheral countries. On one hand, large corporate rates are affected only by monetary rates, as in Germany and the Netherlands. On the other hand, household rates are affected by deposit rates and GDP variations, as it occurs in peripheral countries. Furthermore, small corporate rates are affected by sovereign bond rates, deposit rates and GDP variations, as in peripheral countries. Italy, Portugal and Spain, besides the previously mentioned ones, also show a dependence of large corporate rates on deposit rates and GDP variations. This is consistent with the bank search for a profit margin and with the pricing that affects corporate risk. Last, the behaviour of Ireland is close to that of peripheral countries in the pre-crisis period, and to that of core countries in the latest years.

3.3.4 Model Adequacy and robustness

In order to analyse the precision and the robustness of the model proposed in the previous Section, we will conduct both in-sample and out-of-sample tests: in such a way we can check for precision and robustness. More in detail, we will analyse the improvement due to the addition of country-specific macroeconomic factors (second stage of the hierarchical process) by performing Wilk's tests and predictions.

In order to test whether the hierarchical model has improved the explanation of bank interest rates on loans with respect to the one-stage model, we have performed a Wilk's Λ as an in-sample test. Table 3.6.3 reports such coefficients Λ referred, respectively, to the first-stage of the hierarchical process (λ_1) and to the results obtained with the complete hierarchical model (λ_2), for the three kinds of loans.

[Table 3.6.3]

Table 3.6.3 clearly shows that the hierarchical model strongly increases the performance of the overall model, dropping its unexplained variance. This means that the macroeconomic and country-specific factors introduced in the second stage of our model are essential in order to better the precision based on in-sample tests. Note that Portugal and Ireland are characterised by the highest values, thus confirming our previous results.

Concerning robustness, after having estimated the coefficients of the seemingly unrelated dynamic model and of the multiple dynamic regression, we are able to predict monthly administered bank interest rates for 2014, by means of out-sample-tests. In order to compute such predictions, we firstly estimate the parameters of the two dynamic models according to (3.2.12); secondly, we use a range of monetary rates scenarios, simulated from a random walk process as described in (3.2.13), in order to obtain future values for monetary rates; thirdly, for the regressors that appear in the multiple dynamic model, we calculate an average of the official forecasting measures made by financial institutions (ECB and IMF).

Figure 3.6.14 reports the predicted interest rates on loans in the different countries made by using the hierarchical model (3.2.7) (points and confidence bars): such results are compared with the observed values (solid lines).

[Fig. 3.6.14]

From Figure 3.6.14 one can firstly notice that the hierarchical dynamic procedure proposed in this work predicts quite well future values of interest rates on loans, in almost all countries. France, Germany, the Netherlands and Spain are the ones with the highest predictive performance for the three kinds of loans. Ireland presents some problems in the case of loans to households. This is due to the fact that, as shown in Figure 3.6.13, the intercept α of the multiple regression model has started strongly increasing in 2013 (after the bank reforms approved by the government). In Portugal the same problem regards loans to small-medium enterprises, because of the strong and unpredictable decreasing contribution of interest rates on bonds and interest rates on deposits in 2014, and loans to large corporates, because of the strong increase contribution of interest rates on deposits in the last year.

Table 3.6.3 shows the improvement in the precision of the model after the inclusion of the second stage of the hierarchical process based on in-sample tests: we now want to perform such comparison based on the predictive performance derived from out-sample-tests. To this aim, Table 3.6.4 reports the measures $RMSE$ and $MAPE$ referred, respectively, to the first stage of the hierarchical model ($RMSE_1$, $MAPE_1$) and to the complete, double-stages hierarchical model ($RMSE_2$, $MAPE_2$): in addition, the percentage error reductions obtained through the addition of the second stage are reported ($\Delta_{RMSE}\%$ and $\Delta_{MAPE}\%$).

[Table 3.6.4]

Table 3.6.4 provides a further confirm of the performance improvement introduced in the methodological framework with the hierarchical model, explaining the different importance of the macroeconomic factors across countries and time. Furthermore, by looking at the root mean square errors, it seems that France, the Netherlands and Spain are characterised by the lowest errors, consistently with Figure 3.6.14. However, by concentrating on percentage measures ($MAPE$), the situation changes, showing more homogeneous results, with both Italy and Portugal significantly affected by worse predictions. The greatest improvements in the model can be detected for loans to non-financial corporates up to 1 Mln euros in France and Spain, loans to households in Ireland and loans to non-financial corporates over 1 Mln euros in Italy.

By comparing these results with Figure 3.6.11, we can conclude that, in France, the contribution of interest rates on government bonds in the explanation of bank rates on loans to small-medium enterprises is crucial. Similarly, in Spain the most important components for determining interest rates on loans to small-medium enterprises are interest rates on deposits and the GDP growth rate. By looking at Figure 3.6.12, in Ireland interest rates on loans to households are mostly dependent on interest rates on deposits, while in Italy the explanation of interest rates on loans to large corporates mostly relies on the GDP growth rate, especially after the financial crisis of 2009.

Finally, we propose a comparison, in terms of RMSE, between the dynamic hierarchical model proposed in this paper and the commonly used panel VAR methodology[1]. The results are shown in Table 3.6.5.

[Table 3.6.5]

Table 3.6.5 clearly shows that the dynamic hierarchical model proposed in this paper strongly increases the predictive performance with respect to a panel VAR methodology, in almost all countries and for the three kinds of loans. The reasons for such an improve can be explained by considering that (a) the seemingly unrelated equations consider cross-country correlations between bank rates and the Euribor, while the panel VAR ignores them, and (b) dynamic linear models allow a time-dependent estimation of the parameters.

[1]We used the model proposed in Goodhart and Hofmann (2008).

3.4 Conclusions

The paper has two main methodological contributions. Firstly, it introduces a dynamic model able to capture the evolving relationship between bank and monetary rates, taking into account correlations between different kinds of loans within each country. Secondly, it proposes a hierarchical model, to understand how country-specific macroeconomic factors may have different effects on the monetary transmission. Both models perform quite well from a predictive viewpoint.

The application of the methodology shows that the proposed models are able to explain the heterogenous transmission mechanism of monetary rates according to specific temporal and geographical contexts. Moreover, it shows that the financial crisis and the sovereign crisis have differently affected the time dynamics of bank interest rates on loans, with the former leading to homogenous reactions in all countries and the latter causing a clustering effect between peripheral and core countries.

The effect of monetary rates on bank rates is different according to different lending types: large corporate loans are the most affected, whereas small-medium corporate loans and household loans depend less on monetary rates and more on country-specific macroeconomic factors, such as interest rates on deposits and GDP variations. This dependency can be explained, respectively, as the consequence of the bank need for returns, mostly determined by interest spreads, and as the impact of corporate and family risk.

The dependence of bank rates on monetary rates considerably varies also across countries. In core countries, such as Germany and the Netherlands, bank rates depend almost exclusively on monetary rates and, therefore, the transmission of monetary policy is expected to be effective. In peripheral countries, instead, all lending rates depend on bank risk, corporate risk and, more recently, sovereign risk, as reflected by deposit rates, GDP variations and sovereign bond rates. Hence, in these countries, the transmission of the monetary policy appears to be more problematic.

3.5 References

Ballester, L., Ferrer, R., Gonzales, C., Soto, G.M., 2009. Determinants of interest rate exposure of the Spanish banking industry. Working Paper, University of Castilla-La Mancha.

Bernanke, B.S., Gertler, M., 1995. Inside the black box: the credit channel of monetary policy transmission. NBER Working Paper No. 5146.

Borio, C., Chu, H., 2012. Capital regulation, risk-taking and monetary policy: A missing link in the transmission mechanism? Journal of Financial Stability 8(4), 236-251.

Casarin, R., Sartore, D., Tronzano, M., 2016. A Bayesian Markov-switching correlation model for contagion analysis on exchange rate markets. Journal of Business and Economic Statistics, to appear.

Chong, B. S., Liu, M., Shrestha, K., 2006. Monetary transmission via the administered interest rates channel. Journal of Banking and Finance 30(5), 1467-1484.

Clark, T.E., Ravazzolo, F., 2015. Macroeconomic forecasting performance under alternative specifications of time-varying volatility. Journal of Applied Econometrics 30(4), 551-575.

Demirguc-Kint, A., Huizinga, H., 1999. Determinants of commercial bank interest margins and profitability: some international evidence. World Bank Economic Review 13(2), 379-408.

Den Haan, W.J., Sumner, S., Yamashiro, G.M., 2007. Bank loan portfolios and the monetary transmission mechanism. Journal of Monetary Economics 54, 904-924.

De Santis, R.A., Surico, P., 2013. Bank lending and monetary transmission in the euro area. Economic Policy 28(75), 423-457.

Draghi, M., 2012. The monetary policy of the European Central Bank and its transmission in the euro area. Speech at Bocconi University, Milan.

Ehrmann, M., Gambacorta, L., Martinez-Pages, J., Sevestre, P., Worms, A., 2001. Financial systems and the role of banks in monetary policy transmission in the Euro Area. ECB Working Paper No. 105.

Engle, R. F., Granger, C. W. J., 1987. Co-integration and error correction: representation, estimation, and testing. Econometrica 55(2), 251-276.

Flannery, M.J., James, C.M., 1984. The effect of interest rate changes on the common stock returns of financial institutions. The Journal of Finance 39, 1141-1153.

Gertler, M., Gilchrist, S., 1994. Monetary policy, business cycles, and the behavior of small manufacturing firms. The Quarterly Journal of Economics 109(2), 309-340.

Goodhart, C., Hofmann, B., 2008. House prices, money, credit and the macroeconomy. ECB Working Paper No. 888.

Hannan, T., Berger, A., 1991. The rigidity of prices: Evidence from banking industry. American Economic Review 81, 938-945.

Heffernan, S.A., 1997. Modelling British interest rate adjustment: An error correction approach. Economica 64, 211-231.

Kashyap, A., K., Stein, J., C., 1995. The impact of monetary policy on bank balance sheets. Working Paper, Carnegie-Rochester Conference Series on Public Policy.

Kashyap, A., K., Stein, J., C., 2000. What do a million observations on banks say about the transmission of monetary policy. American Economic Review 90(**3**), 407-428.

Kleimeier, S., Sander, H., 2004. Convergence in euro-zone retail banking? What interest rate pass-through tells us about monetary policy transmission competition and integration. Journal of International Money and Finance 23(3), 461-492.

Kleimeier, S., Sander, H., 2006. Expected versus unexpected monetary policy impulses and interest rate pass-through in euro-zone retail banking markets. Journal of Banking and Finance 30(7), 1839-1870.

Maudos, J., Guevara, J.F., 2004. Factors explaining the interest margin in the banking sectors of the European Union. Journal of Banking and Finance 28(9), 2259-2281.

Maudos, J., Solis, L., 2009. The determinants of net interest income in the Mexican banking system: an integrated model. Journal of Banking and Finance 33(10), 1920-1931.

Neri, S., 2014. The impact of the sovereign debt crisis on bank lending rates in the Euro-area. Bancaria 5, 1-34.

Parisi, L., Gianfrancesco, I., Giliberto, C., Giudici, P., 2015. Predicting bank interests when monetary rates are close to zero. Applied Mathematics 7(1), 1-12.

Petris, G., Petrone, S., Campagnoli, P., 2009. Dynamic Linear Models with R. Springer, New York.

Primiceri, G.E., 2005. Time varying structural vector autoregressions and monetary policy. The Review of Economic Studies 72(3), 821-852.

Scholnick, B., 1996. Asymmetric adjustment of commercial bank interest rates: evidence from Malaysia and Singapore. Journal of International Money and Finance 15, 485-496.

Sims, C.A., Zha, T., 2006. Were there regime switches in monetary policy? The American Economic Review 96(1), 54-81.

3.6 Appendix B: Tables and Figures

Table 3.6.1: Correlation coefficients between rates

		France			Germany			Ireland			Italy			Netherl.			Portugal			Spain		
		corp1	corp2	fam	corp1	corp2	fam	corp1	corp2	fam	corp1	corp2	fam	corp1	corp2	fam	corp1	corp2	fam	corp1	corp2	fam
Fra	corp1	1.00																				
	corp2	0.93	1.00																			
	fam	0.93	0.78	1.00																		
Ger	corp1	0.95	0.95	0.84	1.00																	
	corp2	0.96	0.96	0.85	0.99	1.00																
	fam	0.89	0.79	0.88	0.92	0.91	1.00															
Ire	corp1	0.71	0.84	0.52	0.70	0.72	0.43	1.00														
	corp2	0.91	0.96	0.75	0.95	0.95	0.80	0.82	1.00													
	fam	0.81	0.92	0.63	0.83	0.85	0.61	0.92	0.91	1.00												
Ita	corp1	0.75	0.84	0.61	0.71	0.72	0.46	0.88	0.81	0.86	1.00											
	corp2	0.84	0.93	0.68	0.85	0.86	0.62	0.88	0.91	0.92	0.95	1.00										
	fam	0.84	0.89	0.71	0.80	0.82	0.59	0.87	0.88	0.89	0.96	0.96	1.00									
Net	corp1	0.92	0.94	0.80	0.89	0.92	0.77	0.82	0.90	0.88	0.76	0.86	0.84	1.00								
	corp2	0.91	0.98	0.75	0.96	0.96	0.80	0.82	0.97	0.91	0.81	0.93	0.88	0.91	1.00							
	fam	0.81	0.69	0.87	0.71	0.75	0.75	0.49	0.60	0.56	0.47	0.55	0.58	0.82	0.63	1.00						
Por	corp1	0.60	0.66	0.52	0.50	0.52	0.28	0.74	0.54	0.64	0.84	0.78	0.78	0.64	0.59	0.53	1.00					
	corp2	0.37	0.48	0.27	0.26	0.30	0.00	0.69	0.36	0.54	0.72	0.63	0.63	0.49	0.41	0.36	0.91	1.00				
	fam	0.71	0.82	0.56	0.68	0.69	0.42	0.88	0.75	0.82	0.92	0.90	0.87	0.74	0.78	0.49	0.92	0.81	1.00			
Spa	corp1	0.46	0.58	0.36	0.36	0.40	0.08	0.81	0.50	0.68	0.85	0.73	0.77	0.61	0.51	0.41	0.87	0.90	0.81	1.00		
	corp2	0.84	0.95	0.66	0.85	0.87	0.62	0.93	0.93	0.95	0.91	0.96	0.93	0.91	0.94	0.59	0.73	0.61	0.87	0.74	1.00	
	fam	0.90	0.95	0.77	0.86	0.88	0.66	0.91	0.92	0.93	0.92	0.95	0.96	0.92	0.92	0.67	0.76	0.62	0.88	0.74	0.96	1.00
Euribor		0.92	0.96	0.77	0.99	0.98	0.87	0.74	0.97	0.87	0.74	0.88	0.82	0.88	0.98	0.62	0.49	0.27	0.69	0.38	0.88	0.87

Notes: Correlation coefficients between interest rates on loans and 3-months Euribor. corp1 refers to interest rates on loans to non-financial corporates up to 1 Mln euros, corp2 refers to interest rates on loans to non-financial corporates over 1 Mln euros and fam refers to interest rates on loans to households for mortgages. The Table shows that (a) within each country, correlations between different kinds of loans are all positive and very high; (b) across different countries, such relationships remain positive but lower; (c) almost all interest rates on loans are positively and strongly related to monetary rates.

Table 3.6.2: Correlation coefficients between country-specific regressors and rates

		Bond	Deposit	Δ GDP
	loans corp1	0.824	0.893	-0.228
Fra	loans corp2	0.716	0.771	-0.146
	loans fam	0.833	0.891	-0.306
	loans corp1	0.854	0.947	-0.159
Ger	loans corp2	0.833	0.952	-0.173
	loans fam	0.965	0.926	-0.160
	loans corp1	-0.068	0.694	-0.300
Ire	loans corp2	-0.185	0.615	-0.169
	loans fam	-0.158	0.682	-0.261
	loans corp1	0.357	0.836	-0.404
Ita	loans corp2	0.296	0.685	-0.263
	loans fam	0.310	0.763	-0.366
	loans corp1	0.705	0.853	0.088
Net	loans corp2	0.714	0.743	0.339
	loans fam	0.710	0.757	-0.216
	loans corp1	0.494	0.935	-0.567
Por	loans corp2	0.544	0.908	-0.584
	loans fam	0.254	0.857	-0.314
	loans corp1	0.394	0.719	-0.410
Spa	loans corp2	0.075	0.699	0.090
	loans fam	0.075	0.778	0.013

Notes: Correlation coefficients between country-specific regressors (interest rates on 10-years government bonds, interest rates on deposits and GDP variations) and interest rates on loans to non-financial corporates up to 1 Mln euros (loans corp1), loans to non-financial corporates over 1 Mln euros (loans corp2) and loans to households for mortgages (loans fam). The results show a clear distinction between core and peripheral countries regarding interest rates on government bonds, while the GDP growth rate is negatively related to interest rates on loans in all countries.

Table 3.6.3: Λ Wilk's test

Loans	Λ	France	Germany	Ireland	Italy	Netherlands	Portugal	Spain
corp1	λ_1	0.010	0.002	0.117	0.087	0.014	0.200	0.018
	λ_2	$9.58 \cdot 10^{-9}$	0.002	$8.62 \cdot 10^{-7}$	$2.02 \cdot 10^{-8}$	$3.53 \cdot 10^{-9}$	$7.98 \cdot 10^{-7}$	$5.85 \cdot 10^{-10}$
corp2	λ_1	0.025	0.011	0.238	0.115	0.023	0.296	0.031
	λ_2	$8.41 \cdot 10^{-7}$	0.011	$7.54 \cdot 10^{-6}$	$1.22 \cdot 10^{-6}$	$3.66 \cdot 10^{-8}$	$9.74 \cdot 10^{-5}$	$6.49 \cdot 10^{-7}$
fam	λ_1	0.009	0.001	0.056	0.042	0.009	0.131	0.011
	λ_2	$4.66 \cdot 10^{-10}$	0.001	$9.78 \cdot 10^{-9}$	$1.05 \cdot 10^{-8}$	$2.94 \cdot 10^{-8}$	$1.21 \cdot 10^{-7}$	$3.45 \cdot 10^{-10}$

Notes: Λ Wilk's test, obtained by using only the first stage (λ_1) or both the stages (λ_2) of the hierarchical process, for the three kinds of loans. The results show that the hierarchical model strongly increases the performance of the overall model, dropping its unexplained variance, meaning that the macroeconomic and country-specific factors are essential in order to better the precision based on in-sample tests.

Table 3.6.4: $RMSE$, $MAPE$ and the percentage improvement of the model

Country	Loans	$RMSE_1$	$RMSE_2$	$\Delta_{RMSE}\%$	$MAPE_1$	$MAPE_2$	$\Delta_{MAPE}\%$
Fra	corp 1	0.0387	0.0155	-60.0%	0.0873	0.0416	-52.4%
	corp 2	0.0552	0.0309	-43.9%	0.1161	0.0813	-30.0%
	fam	0.0835	0.0339	-59.4%	0.0759	0.0437	-42.4%
Ger	corp 1	0.0529	0.0529	0.0%	0.0655	0.0655	0.0%
	corp 2	0.0554	0.0554	0.0%	0.1175	0.1175	0.0%
	fam	0.1856	0.1856	0.0%	0.1501	0.1501	0.0%
Ire	corp 1	0.2271	0.1761	-22.4%	0.0860	0.0688	-20.0%
	corp 2	0.3417	0.0758	-77.8%	0.1706	0.0751	-56.0%
	fam	0.7018	0.0476	-93.2%	0.2340	0.0659	-71.8%
Ita	corp 1	0.3495	0.0917	-73.8%	0.1284	0.0651	-49.3%
	corp 2	0.2869	0.0459	-84.0%	0.1916	0.0724	-62.2%
	fam	0.1968	0.0719	-63.5%	0.1134	0.0626	-44.8%
Net	corp 1	0.0832	0.0224	-73.0%	0.0673	0.0360	-46.5%
	corp 2	0.0727	0.0458	-37.0%	0.1117	0.0848	-24.1%
	fam	0.0827	0.0293	-64.6%	0.0745	0.0418	-43.8%
Por	corp 1	0.4774	0.2206	-53.8%	0.1065	0.0719	-32.5%
	corp 2	0.5649	0.2589	-54.2%	0.1657	0.1003	-39.4%
	fam	0.0557	0.0251	-54.9%	0.0554	0.0540	-2.5%
Spa	corp 1	0.2690	0.0480	-82.2%	0.0953	0.0412	-56.8%
	corp 2	0.0901	0.0445	-50.6%	0.0737	0.0690	-6.4%
	fam	0.0454	0.0186	-58.9%	0.0532	0.0301	-43.5%

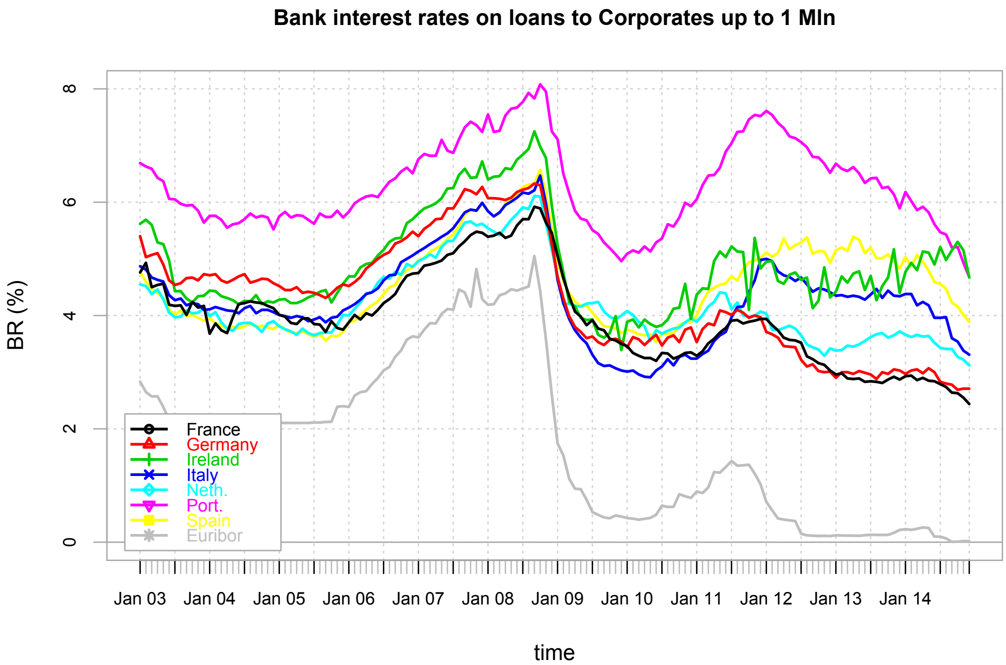
Notes: $RMSE$ and $MAPE$ referred to the first stage of the hierarchical model ($RMSE_1$, $MAPE_1$) and to the complete, double-stages hierarchical model ($RMSE_2$, $MAPE_2$); $\Delta_{RMSE}\%$ and $\Delta_{MAPE}\%$ measure the percentage error reductions obtained through the addition of the second stage. The table is a further confirm of the performance improvement introduced in the methodological framework with the hierarchical model.

Table 3.6.5: $RMSE$ and $MAPE$ comparison: hierarchical dynamic model vs Panel VAR

		Dynamic Hierarchical		Panel VAR	
Country	Loans	RMSE	MAPE	RMSE	MAPE
Fra	corp 1	0.0155	0.0416	0.0178	0.0478
	corp 2	0.0309	0.0813	0.0355	0.0935
	fam	0.0339	0.0437	0.0305	0.0393
Ger	corp 1	0.0529	0.0655	0.0688	0.0852
	corp 2	0.0554	0.1175	0.0526	0.1116
	fam	0.1856	0.1501	0.2413	0.1951
Ire	corp 1	0.1761	0.0688	0.2201	0.0860
	corp 2	0.0758	0.0751	0.0948	0.0939
	fam	0.0476	0.0659	0.0595	0.0824
Ita	corp 1	0.0917	0.0651	0.1146	0.0814
	corp 2	0.0459	0.0724	0.0574	0.0905
	fam	0.0719	0.0626	0.0899	0.0783
Net	corp 1	0.0224	0.036	0.0325	0.0522
	corp 2	0.0458	0.0848	0.0664	0.1230
	fam	0.0293	0.0418	0.0425	0.0606
Por	corp 1	0.2206	0.0719	0.3309	0.1079
	corp 2	0.2589	0.1003	0.3884	0.1505
	fam	0.0251	0.054	0.0377	0.0810
Spa	corp 1	0.048	0.0412	0.0576	0.0494
	corp 2	0.0445	0.069	0.0534	0.0828
	fam	0.0186	0.0301	0.0149	0.0241

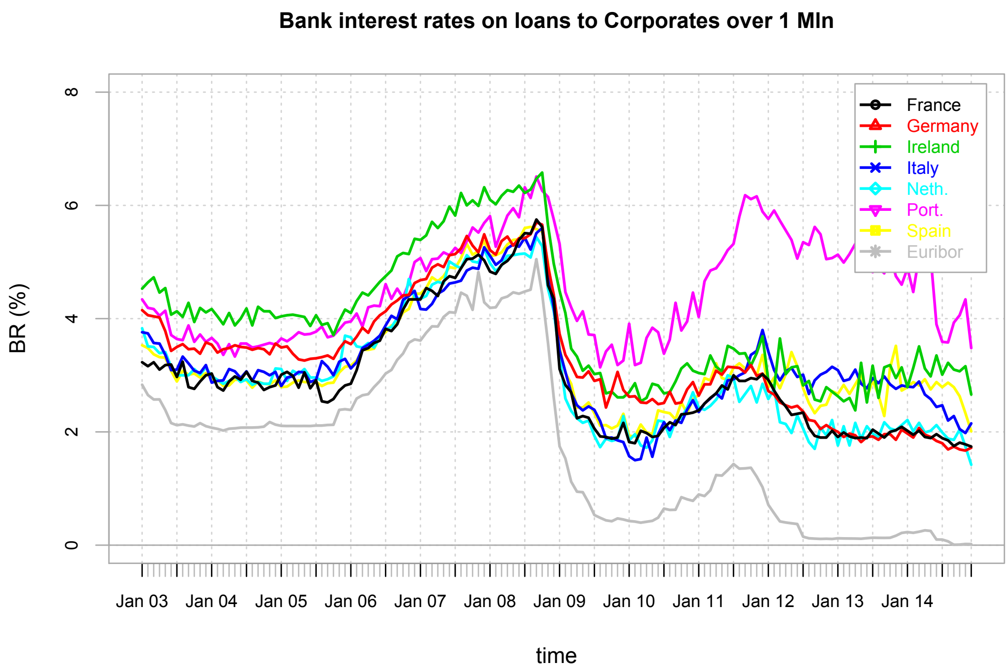
Notes: $RMSE$ and $MAPE$ obtained by using the hierarchical dynamic model or the Panel VAR methodology. The results show that the the former strongly increases the predictive performance with respect to the latter, in almost all countries and for the three kinds of loans.

Figure 3.6.1: Loans corporates 1



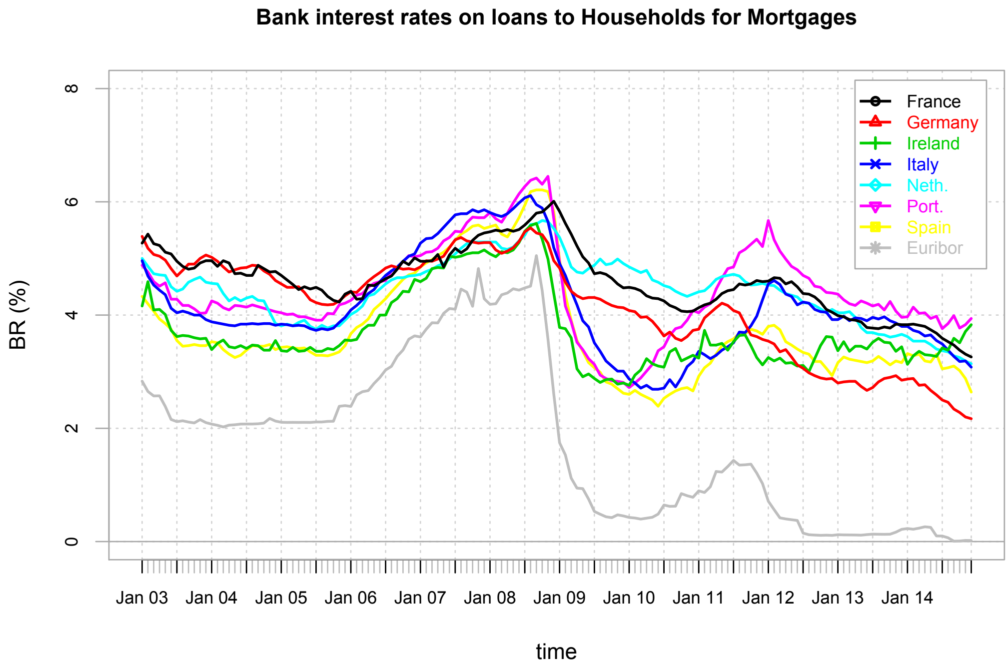
Notes: Interest rates on loans to non-financial corporates up to 1 Mln euros and 3-months Euribor, from January 2003 until December 2014. Bank rates on loans in core countries (France, Germany and the Netherlands) are very similar to each other, while the situation is more heterogeneous in peripheral countries, with Portugal presenting the highest values during the entire period and Ireland being characterised by a strong volatility and an independent time-evolution after 2010.

Figure 3.6.2: Loans corporates 2



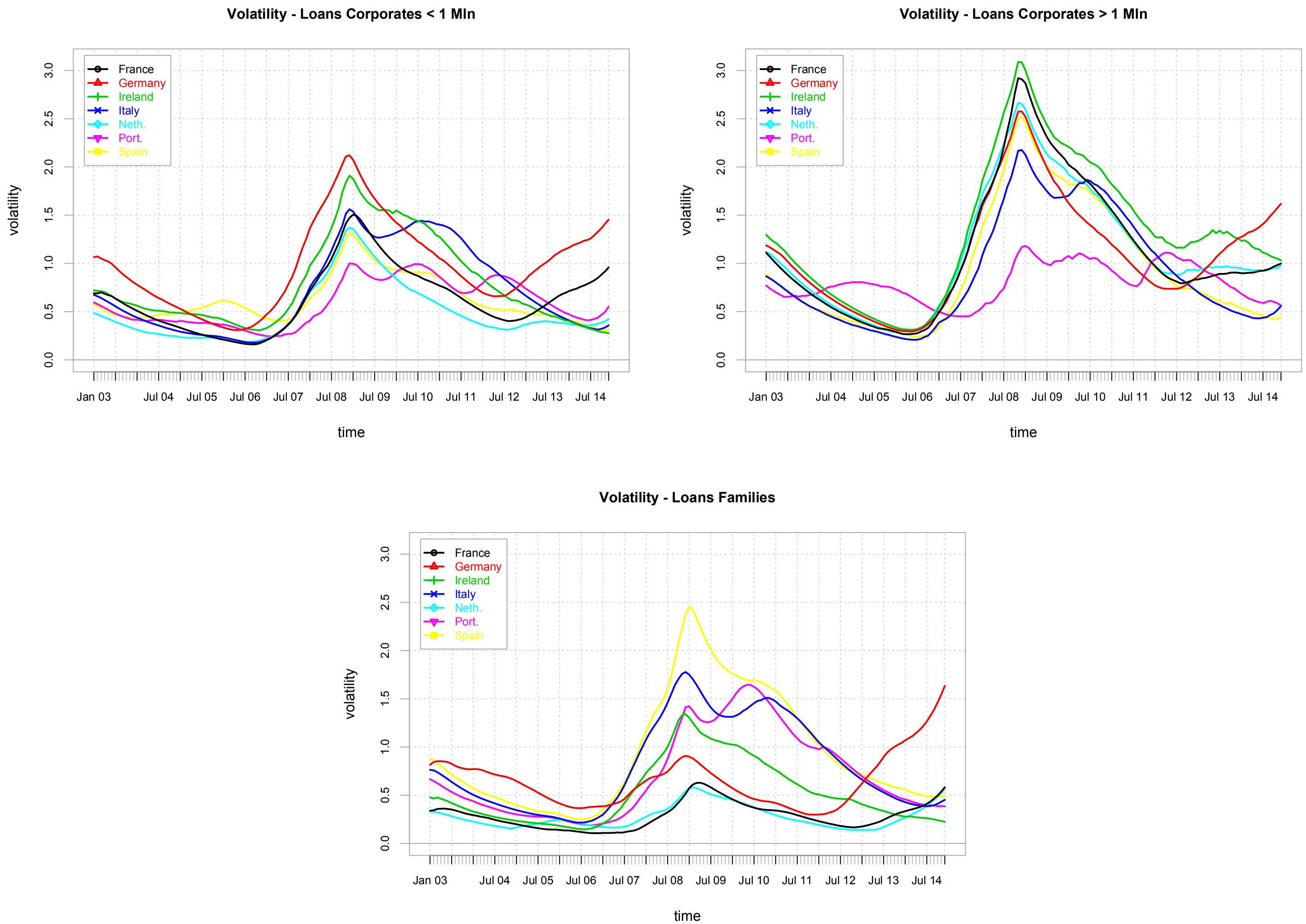
Notes: Interest rates on loans to non-financial corporates over 1 Mln euros and 3-months Euribor, from January 2003 until December 2014. All countries seem to have similar behaviours, strongly volatile in the short term but, on long run average, almost stable (with the exception of Portugal).

Figure 3.6.3: Loans households



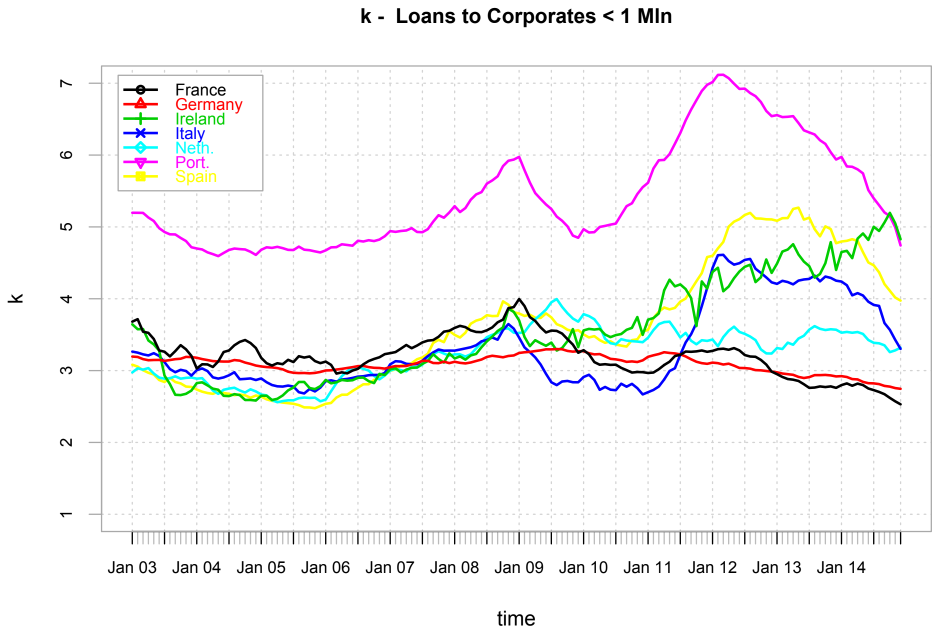
Notes: Interest rates on loans to households for mortgages and 3-months Euribor, from January 2003 until December 2014. Data show extremely heterogeneous behaviours after 2009: Italy, Portugal and Spain suffered an increase in interest rates during their sovereign crisis of 2012, while such increase is anticipated in Germany (2011) and hardly visible in France and the Netherlands.

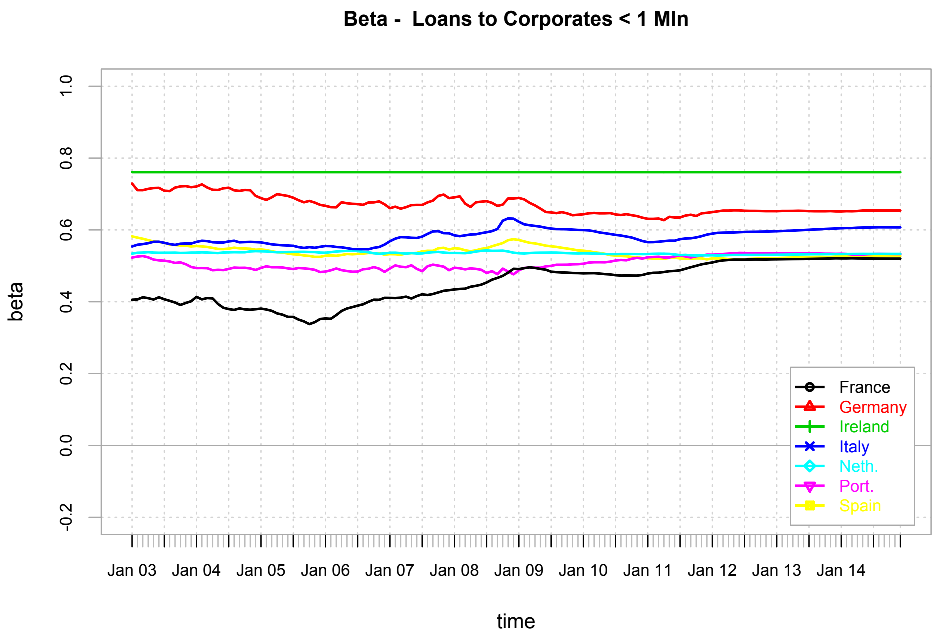
Figure 3.6.4: Volatility



Notes: Time-dependent volatility of interest rates on loans to non-financial corporates up to 1 Mln euros, loans to non-financial corporates over 1 Mln euros and loans to households for mortgages, obtained through the EWMA filter. The volatility of interest rates of all kinds of loans substantially increased during the financial crisis in all countries.

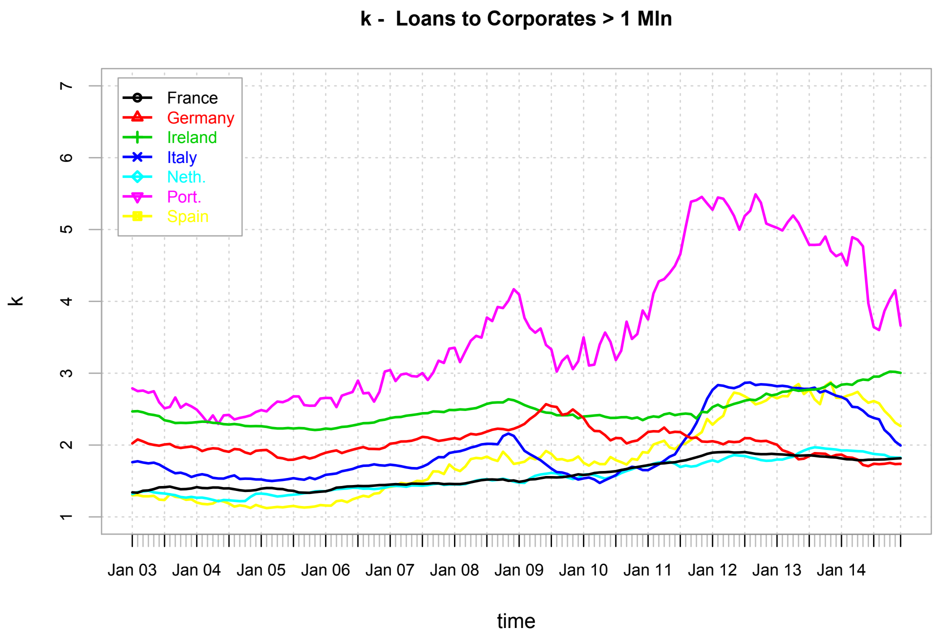
Figure 3.6.5: Corporates 1: k and β coefficients

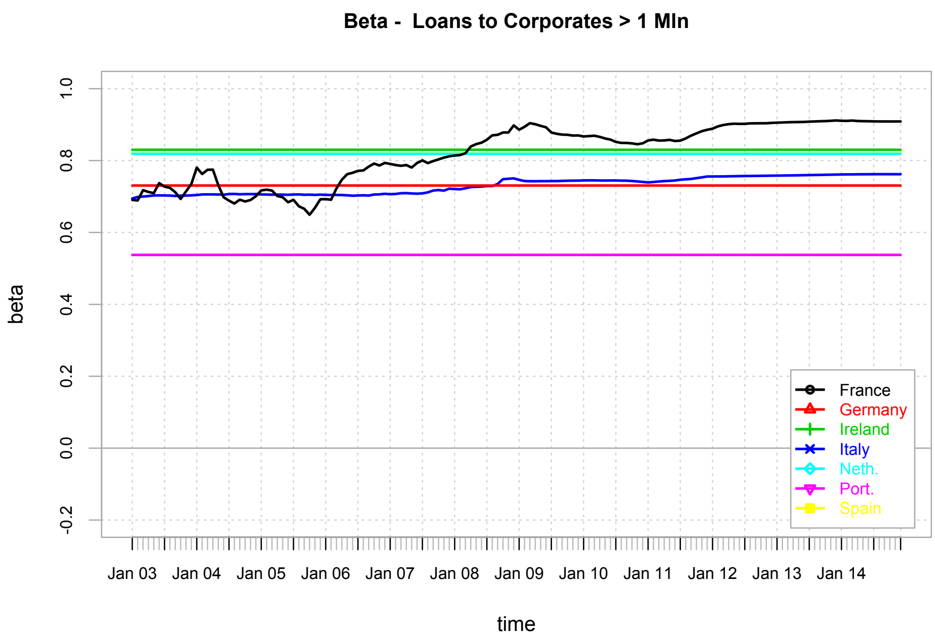




Notes: Regression coefficients obtained through the seemingly unrelated dynamic model for interest rates on loans to non-financial corporates up to 1 Mln euros, from January 2003 until December 2014. The results show that (a) the contribution of monetary rates (bottom) has remained almost constant in all countries; (b) all the intercepts (top) are strongly time-varying and heterogenous across different countries.

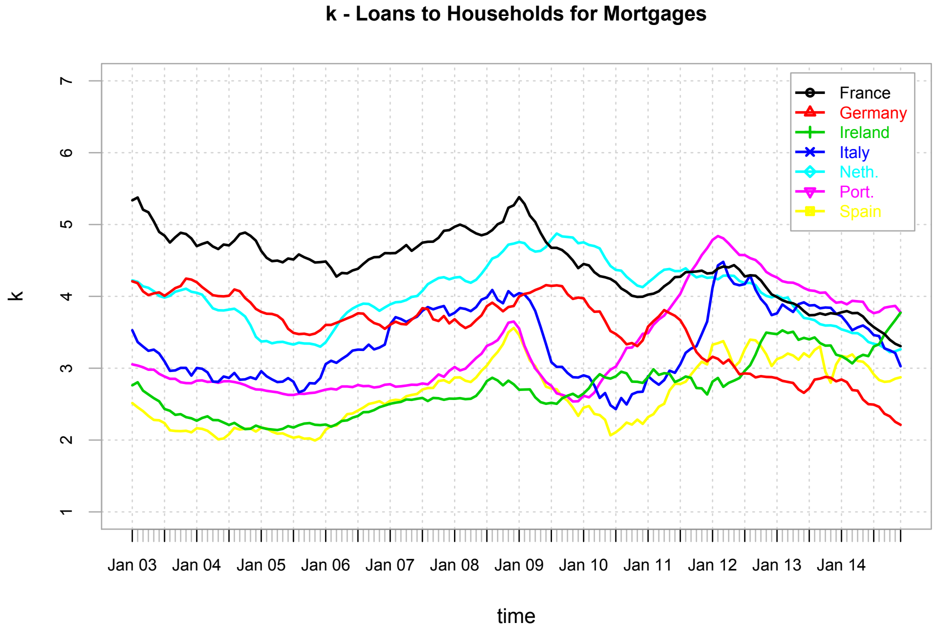
Figure 3.6.6: Corporates 2: k and β coefficients

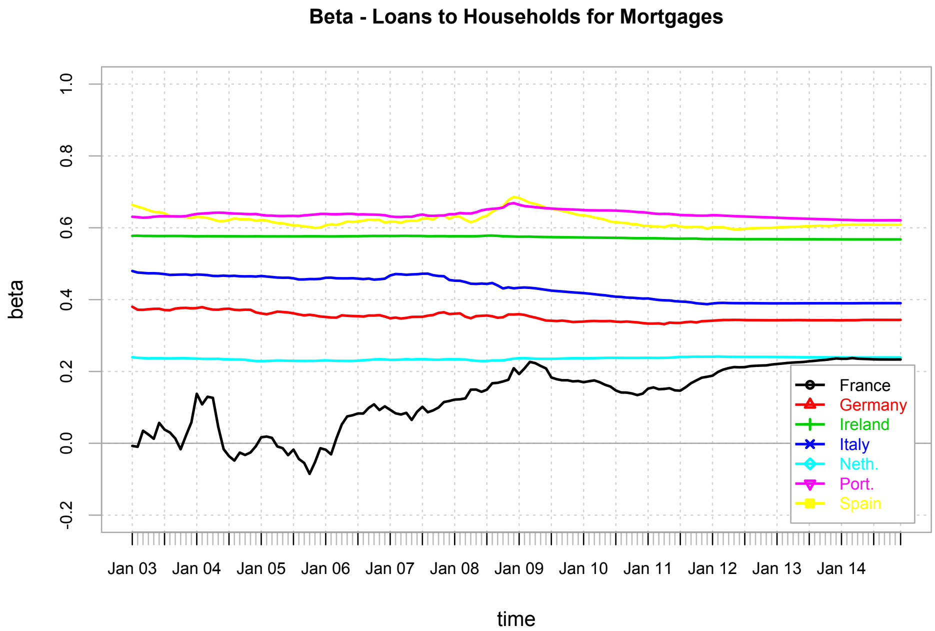




Notes: Regression coefficients obtained through the seemingly unrelated dynamic model for interest rates on loans to non-financial corporates over 1 Mln euros, from January 2003 until December 2014. In all countries, the contribution of monetary rates in the determination of bank rates on loans to large corporates is much higher than the one referred to interest rates on loans to small-medium enterprises.

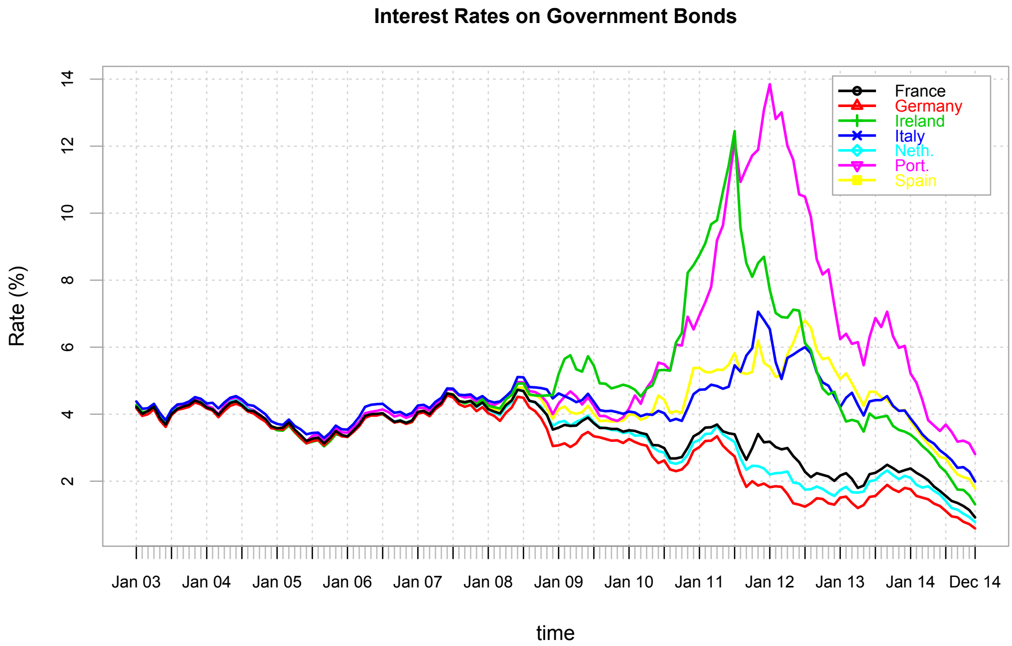
Figure 3.6.7: Households: k and β coefficients





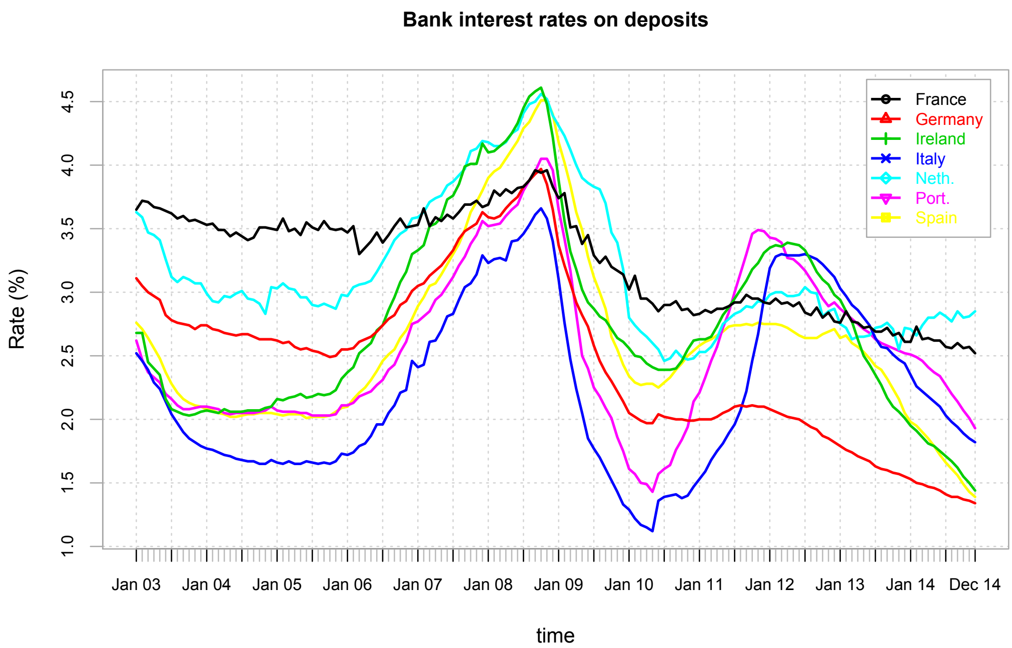
Notes: Regression coefficients obtained through the seemingly unrelated dynamic model for interest rates on loans to households for mortgages, from January 2003 until December 2014. The results show that (a) the coefficients β_t^h *(bottom) look almost constant in all countries and during the whole period (with the exception of France); (b) by looking at the coefficients* k_t^h *(top), the situation is extremely heterogeneous across countries, both before and after the financial crisis.*

Figure 3.6.8: Interest rates on 10-years government bonds



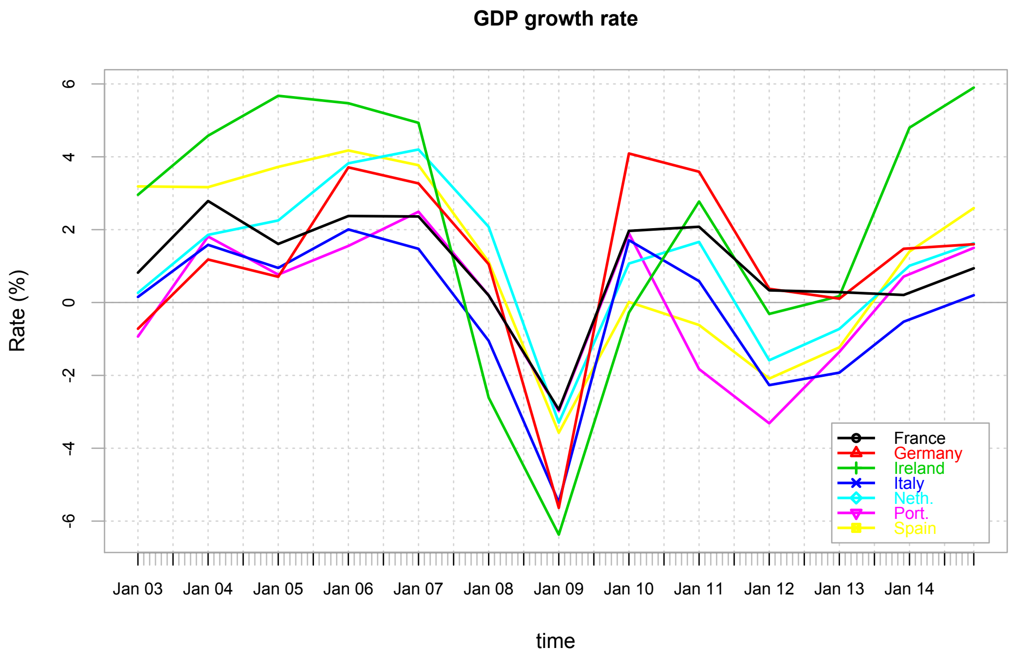
Notes: Interest rates on 10-years government bonds, from January 2003 until December 2014. In the first period interest rates on government bonds in different countries are extremely similar to each other, while after 2008 a clustering effect started emerging, differentiating between core and peripheral countries.

Figure 3.6.9: Interest rates on deposits



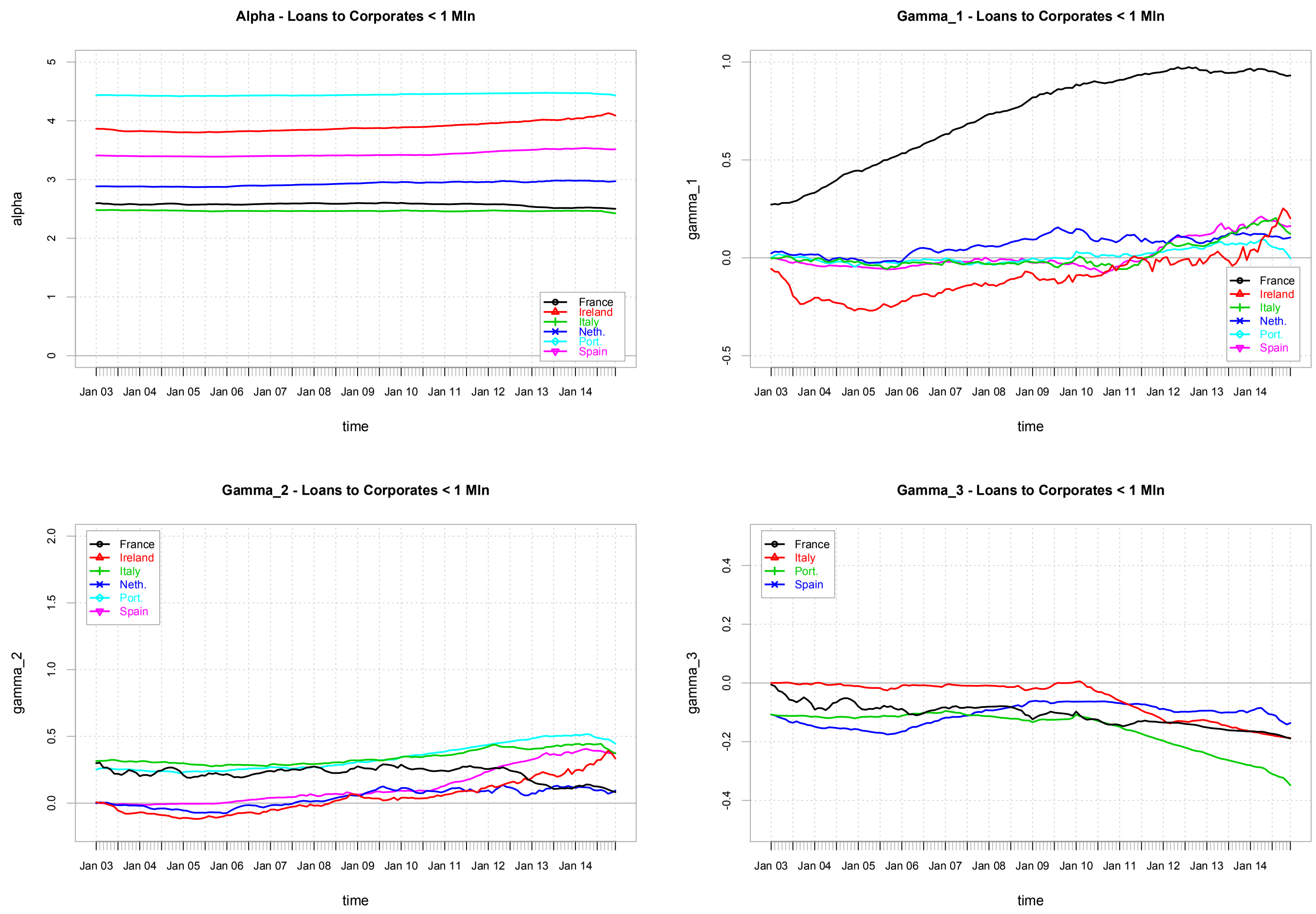
Notes: Aggregate interest rates on deposits, from January 2003 until December 2014. These rates show a heterogeneous behaviour: they all increased during the financial and the sovereign crisis, with the exception of France where they remained, on average, almost constant.

Figure 3.6.10: GDP growth rates



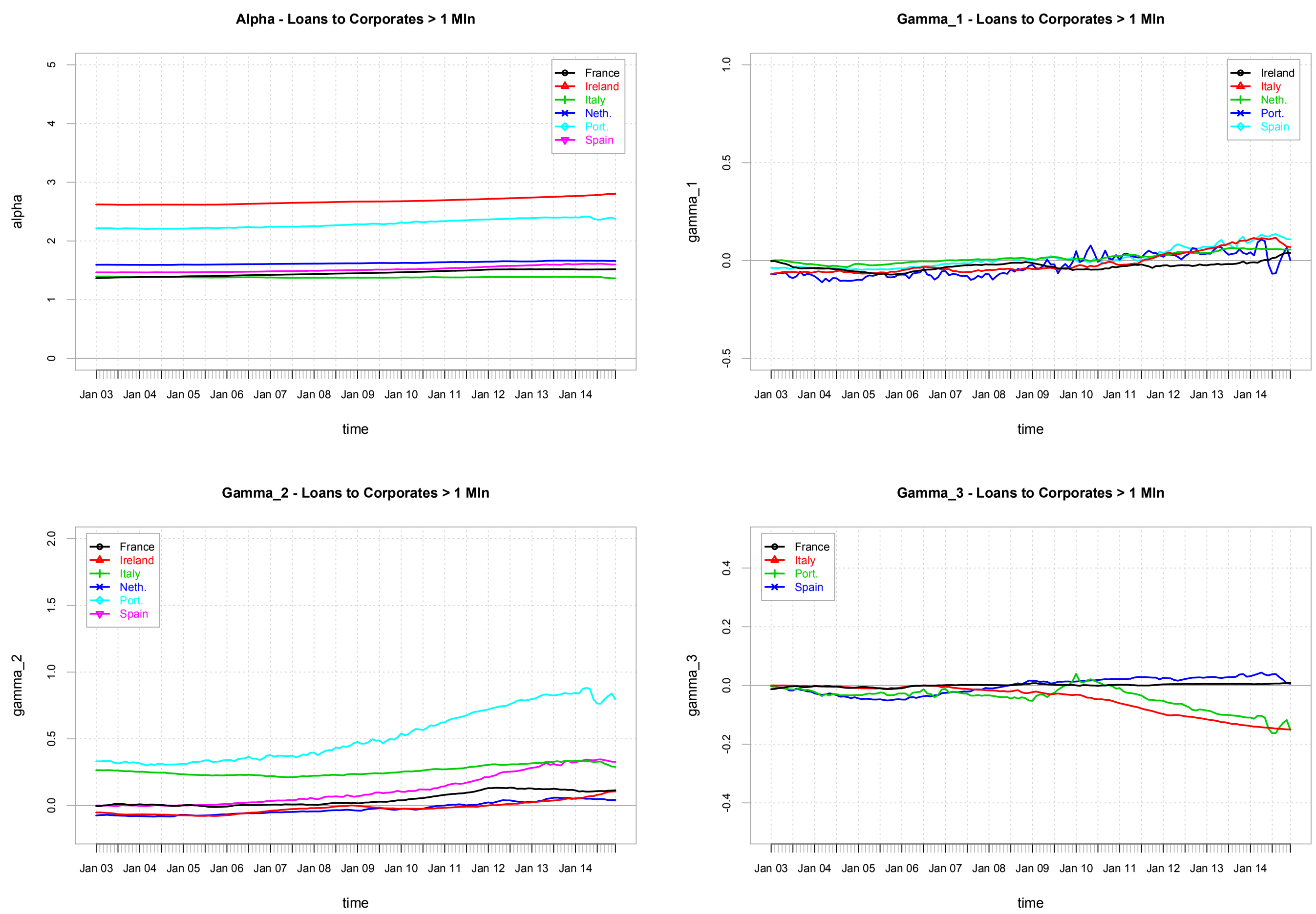
Notes: Annual GDP growth rates, from January 2003 until December 2014. They all decreased during the financial and the sovereign crisis, with core (peripheral) countries suffering less (more) the sovereign crisis. Ireland shows a peculiar behaviour, being characterised by a strong increase in its GDP started in 2013.

Figure 3.6.11: Corporates 1: α and γs coefficients



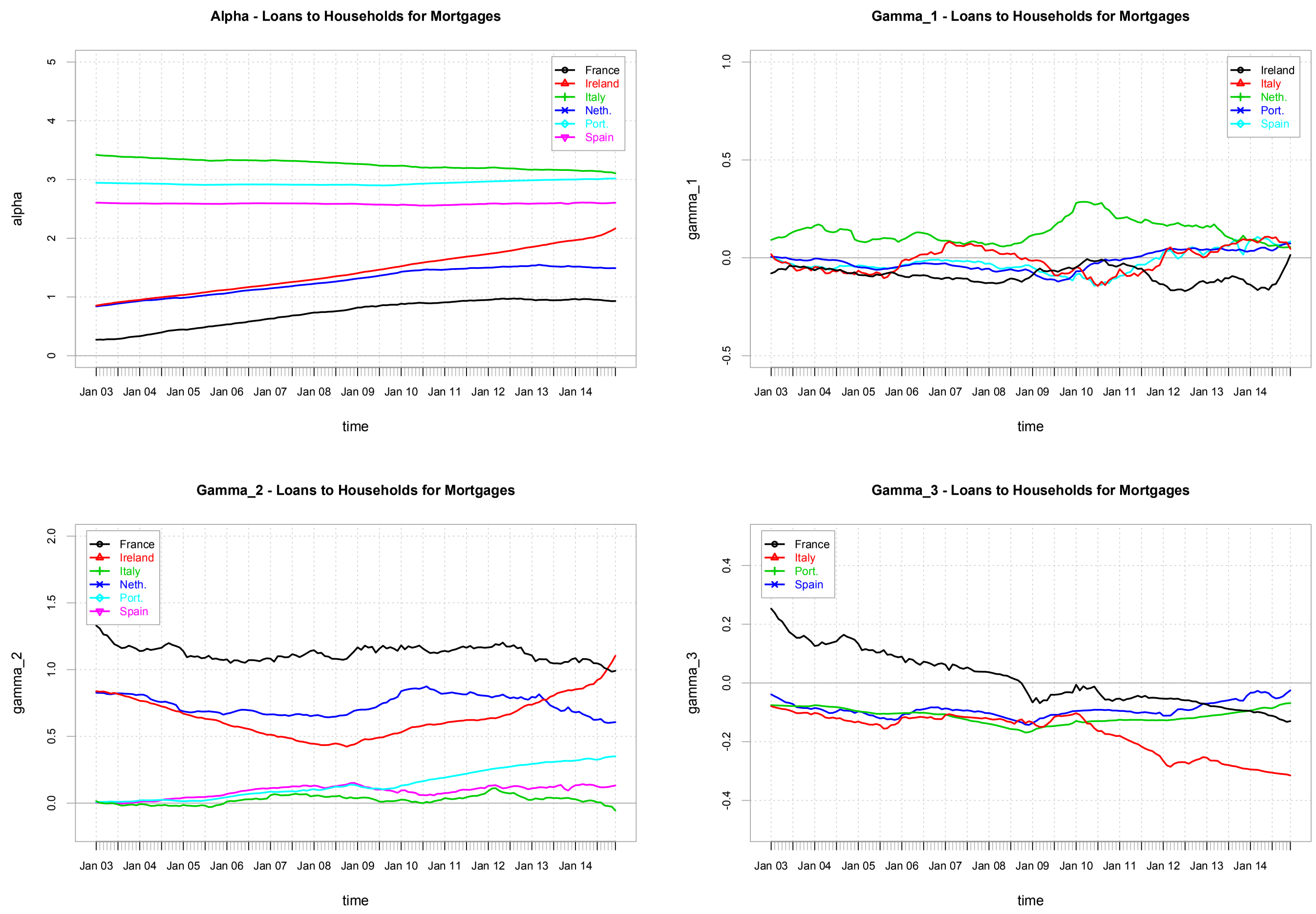
Notes: Regression coefficients obtained through the multiple dynamic model for $\Theta_t^{j,h}$ *to non-financial corporates up to 1 Mln euros, from January 2003 until December 2014. Interest rates on government bonds seem to have a significant effect only in France, Ireland and the Netherlands during the entire period, and in all countries after 2012; the contribution of interest rates on deposits is particularly significant in Italy and Spain during the whole years, and in Ireland since 2012; the contribution of the GDP growth rate became more significant after the financial crisis of 2008, not only for peripheral countries but also for France.*

Figure 3.6.12: Corporates 2: α and γs coefficients



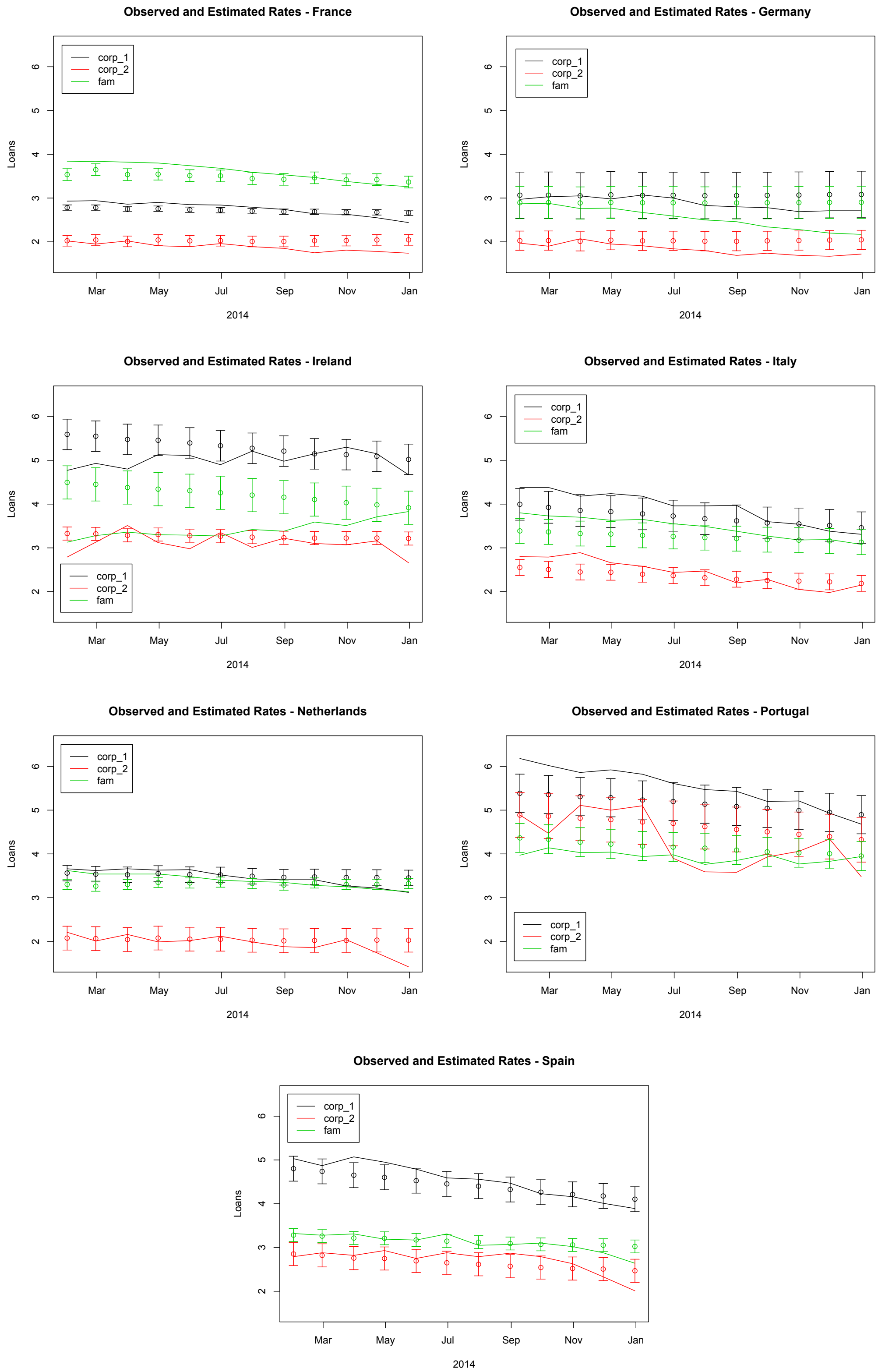
Notes: Regression coefficients obtained through the multiple dynamic model for $\Theta_t^{j,h}$ *to non-financial corporates over 1 Mln euros, from January 2003 until December 2014. Interest rates on loans to large corporates are much more explained by monetary rates with respect to loans to small-medium enterprises; the contribution due to interest rates on deposits is very high in peripheral countries.*

Figure 3.6.13: Households: α and γs coefficients



Notes: Regression coefficients obtained through the multiple dynamic model for $\Theta_t^{j,h}$ *to households for mortgages, from January 2003 until December 2014. The contribution of interest rates on government bond is significant in all countries (with the exception of France); the contribution of interest rates on deposits is high in France and the Netherlands and low in Italy, Portugal and Spain, while in Ireland it started increasing after the financial crisis; the contribution of GDP growth rates is decreasing in France and Italy, while it is stable in Portugal and Spain.*

Figure 3.6.14: Estimated interest rates



Notes: Observed and estimated interest rates on loans, from January 2003 until December 2014. The results show that the hierarchical dynamic procedure proposed in this work predicts quite well future values of interest rates on loans, in almost all countries (some problems can be detected only in Ireland for loans to households, and in Portugal for loans to small-medium corporates.

Chapter 4

Sovereign Risk in the Euro Area: a multivariate stochastic process approach[1]

Quantitative Finance[2]

Abstract

In this paper we aim at jointly modelling financial and real systemic effects of sovereign risk by means of correlated stochastic processes, with an application to Eurozone countries. To achieve this aim, for each country we consider a leverage measure, the Debt/GDP ratio. We model the time dynamic of both the Debt and the GDP by means of a linear combination of two stochastic equations: an Euro Area systematic process and a country specific idiosyncratic process. Doing so, we provide the evolution of the sovereign debt sustainability in an endogenous way, considering both the financial and the real economy sides, and in terms of both common and country-specific factors. We finally provide an estimation procedure for the parameters of the processes, and we derive the implied default probabilities for each country. The empirical findings show a clear clustering effect between northern and southern countries, especially on the financial side. The inclusion of the GDP growth rate in the derivation of default probabilities partially changes that clustering structure, showing that the sovereign risk of some countries (especially France and Italy) is strongly affected not only by financial data, but also by the leverage ratio.

Keywords: Systemic risk, Stochastic processes, Partial correlation network, Macroeconomic fundamentals.

JEL: C32, C58, E43, F36.

4.1 Introduction

4.1.1 Motivation

The last few years have witnessed an increasing research literature on systemic risk (for a definition see, for example, Allen and Gale, 2000; Acharya, 2009), with the aim of identifying the most contagious institutions and their transmission channels, and of studying

[1]Joint work with P. Giudici (University of Pavia).

[2]Second, major revision.

the impact of monetary policies on default probabilities, especially during crisis periods (see, for example, Chong et al., 2006; Longstaff, 2010; Shleifer and Vishny, 2010).

Following a historical perspective, the first specific measures of systemic risk have been proposed for the banking sector; in particular, by Acharya et al. (2010), Adrian and Brunnermeier (2011), Brownlees and Engle (2012), Acharya et al. (2012), Dumitrescu and Banulescu (2014) and Hautsch et al. (2015) who, on the basis of market share prices, calculate the quantiles of the estimated loss probability distribution of a bank, conditional on the occurrence of an extreme event in the financial market. A similar approach has been recently applied for modelling sovereign systemic risk: in particular, Popescu and Turcu (2014) use the marginal and the component expected shortfall (introduced by Acharya et al. (2010) and firstly applied in the banking sector) in order to derive the marginal or the absolute contribution of a country to the overall risk of a system, by using bond interest rates and macroeconomic data.

Similarly to Popescu and Turcu (2014), we will address the issue of modelling sovereign risk, but we will follow a different strategy. Conditional probabilities, in fact, are a useful tool in order to measure the dependence of a single institution on the entire market or economic system. However, the bivariate structure of such methodologies does not allow the joint modelling of a set of agents operating in the market, as well as the identification of contagion channels; in addition, they are all based on market data, thus not considering the real economy side.

The dependence of systemic risk on the real side of the economy has typically been studied using causal models; according to this stream of research, the financial and the real components of an economy have been modelled separately and independently. Noticeable reference papers are Gray et al. (2013), Billio et al. (2015) and Schwaab et al. (2015), who model systemic risk in terms of econometric models based on the correlations between systemic and idiosyncratic exogenous risk factors.

A different stream of research has been followed by Ang and Longstaff (2012) and Brownlees et al. (2014): they both model systemic risk in terms of endogenous stochastic processes, that may depend on a common systematic factor.

In this work we will combine the approaches introduced so far: we will concentrate on sovereign risk, and in order to avoid endogeneity and non-linearity issues we will use multivariate stochastic processes. As in Ang and Longstaff (2012) and Brownlees et al. (2014), we will model each country as a combination of two components: an idiosyncratic and a systematic one, both based on market data. In addition, we will combine such financial information with real economy ones: more precisely, for each country we will consider a further combination of stochastic processes, able to describe their real economy side by using, again, a multivariate stochastic approach. The combination of financial and economic information will be achieved in the derivation of default probabilities. For each country, in fact, we will derive two default probabilities: one only based on market data; the second one incorporating both market and economic variables.

Such inclusion of both financial and economic data is justified by recent papers. The financial crisis, in fact, has shown that shocks in the financial and in the real side of an economy are indeed interrelated, and should be jointly measured and modelled. Consistently, Ramsay and Sarlin (2015) have introduced to the field of systemic risk a number of financial leverage measures used in corporate finance, such as the Debt/GDP ratio and the Debt/Cash Flow ratio, as descriptive indicators of a crisis. They have shown that not only leverage, but also such ratios provide additional measures of vulnerability for sovereign risk with respect to conventional indicators, especially in an early-warning perspective.

Our aim here is to extend, in a stochastic process framework, the approach of Ramsay and Sarlin (2015). In particular, we will model the dynamic of debt by using market data, such as interest rates on government bonds; for the real economy side, we will introduce the dynamic of the GDP growth rate, so that the combination of the two will give rise to the Debt/GDP ratio. This indicator is the most commonly used in order to understand the ability of each country to repay its debt, since it describes the financial leverage of an economy. In addition, it incorporates both financial and real economy information, thus providing a full picture of the sovereign risk of each country. Other recent papers on systemic and sovereign risk has concentrated on that leverage ratio, showing its importance in determining the vulnerability of countries or institutions: among others, Giordano et al. (2012), Chuang and Ho (2013), Hurlin et al. (2013).

More specifically, we will model each term of the Debt/GDP ratio by means of stochastic processes as in Ang and Longstaff (2012) and Brownless et al. (2014). We will extend their approach by considering multivariate simultaneous equation processes, following the methodology suggested in Kalogeroupolos et al. (2011) but employing, rather than a single process, two systems of equations: one system for describing the financial side (leverage); one system for describing the dynamics of the real economy side (GDP). Moreover, each of the two systems will be multivariate (N simultaneous equations, one for each country), and each equation will consist of a linear combination of two stochastic processes:

a systematic and an idiosyncratic one. Finally, for each country we will derive two kinds of time-dependent default probabilities: one reflecting only the financial side (debt); the second one including both the financial and real economy side (Debt/GDP ratio). The comparison between the two across countries and through time will allow us to study the impact of the GDP growth rate on sovereign risk: we believe this is an important result, since leverage ratios are usually not considered in traditional systemic risk measures, even if other papers have demonstrated their importance in determining the vulnerability of countries and financial institutions.

From an economic viewpoint, we will apply our methodology to the main Euro area countries during the post-crisis period, in order to understand whether, how and when their sovereign risks have been transmitted within a systemic risk and integrated perspective.

In terms of relevance, our model should be useful to study the impact of macro prudential policies on the sovereign risk of a country, within a multi-correlated framework in which such policies affect countries at both the real and the financial level; moreover, such transmission mechanisms are allowed to be not only direct, but also indirect by means of systemic transmission effects. In the European Union, characterised by one monetary authority (the European Central Bank), that regulates still fragmented national markets, the importance of this study is particularly evident: for example, southern european countries, differently from northern ones, have benefited very little from the drop of monetary rates that has followed the financial crisis. By explicitly modelling the correlations that describe transmission effects between countries, we aim at understanding the main factors that may constrain the transmission of the monetary impulse at the systemic level.

The main findings can be summarised as follows. Firstly, by looking at the financial side during the recent time period a clear clustering effect, differentiating core and peripheral countries, has emerged: these two clusters are characterised by strong inner correlations and weak, or negative cross correlations. Secondly, the inclusion of the GDP growth rate in the calculation of the default probability partially changes such clustering effect, as well as the ranking of some countries: in particular, France and Italy exchange their roles, with the former getting closer to peripheral countries and the latter approaching an intermediate position between peripheral countries and Germany.

4.1.2 Background

Kalogeropoulos et al. (2011) introduced a multivariate Cox-Ingersoll-Ross (CIR) process to describe the dynamic of exchange rates: from a methodological viewpoint, we will extend their work in order to estimate our proposed model. More precisely, their model can be specified starting from a general family of non-parametric, time-homogeneous and continuous equations for the dynamic of the interest rate Y_t:

$$\mathrm{d}\,Y_t = (\theta_1 - \theta_2 Y_t)\,\mathrm{d}\,t + \theta_3 (Y_t)^{\beta}\,\mathrm{d}\,W_t, \tag{4.1.1}$$

where $\beta = 0.5$ corresponds to the CIR process, while $\beta = 0$ represents the Vasicek model.

The previous process can be applied to model the joint dynamic of the interest rates of a group of countries. For example, we can represent the variations of the bond interest rates in a group of countries as functions of the variation of monetary rates, described as the increments of a Wiener process and denoted by $\mathrm{d}\,W_t$, as in the CIR formulation.

Mathematically, for a group $Y_t = (y_t^1, ..., y_t^N)$ of countries, each of the N stochastic processes can be expressed as follows:

$$\mathrm{d}\,y_t^i = (\theta_1^i - \theta_2^i y_t^i)\,\mathrm{d}\,t + \theta_3^i \sqrt{y_t^i}\,\mathrm{d}\,W_t, \qquad \text{i = 1, ..., N} \tag{4.1.2}$$

where each parameter $\theta^i_{\{1,2,3\}}$ is process-specific.

The structure of (4.1.2) can be enriched by introducing correlation coefficients between the N stochastic processes, leading to a multivariate CIR:

$$\begin{cases} \mathrm{Corr}(\mathrm{d}\,y^i, \mathrm{d}\,y^j) = \rho^{ij} & \\ \rho^{ij} \neq 1 & \text{for } i \neq j, \\ \rho^{ij} = 1 & \text{for } i = j. \end{cases} \tag{4.1.3}$$

The CIR process is characterised by its variance-covariance structure. The variance of each CIR process can be shown to be equal to:

$$Var\left[y_t^i | y_0^i\right] = y_0^i \left(\frac{\theta_3^i}{\theta_2^i}\right)^2 \left(e^{-\theta_2^i t} - e^{-2\theta_2^i t}\right) + \frac{\theta_1^i}{2}\left(\frac{\theta_3^i}{\theta_2^i}\right)^2. \tag{4.1.4}$$

The limit of the above variance can be calculated for an adjustment speed that tends to zero:

$$\lim_{\theta_2^i \to 0} Var\left[y_t^i | y_0^i\right] = y_0^i (\theta_3^i)^2 t. \tag{4.1.5}$$

Then, using the correlation coefficients in (4.1.3), the instantaneous covariance matrix can be shown to be equal to:

$$A = \begin{bmatrix} y_0^1(\theta_3^1)^2 & \rho^{12}\sqrt{y_0^1 y_0^2}\theta_3^1\theta_3^2 & \dots & \rho^{1N}\sqrt{y_0^1 y_0^N}\theta_3^1\theta_3^N \\ \rho^{21}\sqrt{y_0^2 y_0^1}\theta_3^2\theta_3^1 & y_0^2(\theta_3^2)^2 & \dots & \rho^{2N}\sqrt{y_0^2 y_0^N}\theta_3^2\theta_3^N \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{N1}\sqrt{y_0^N y_0^1}\theta_3^N\theta_3^1 & \rho^{N2}\sqrt{y_0^N y_0^2}\theta_3^N\theta_3^2 & \dots & y_0^N(\theta_3^N)^2 \end{bmatrix} \tag{4.1.6}$$

Note that the stochastic process described above can be written in a compact multi-dimensional form:

$$\mathrm{d}\,Y_t = M(Y_t, \Theta_{1,2})\,\mathrm{d}\,t + \Sigma(Y_t, \Theta_3)\,\mathrm{d}\,W_t, \tag{4.1.7}$$

where

$$[M]^i = \theta_1^i - \theta_2^i y_t^i, \quad [\Sigma]^i = \theta_3^i\sqrt{y_t^i \rho^i}, \quad \Theta_{1,2} = \begin{bmatrix} \theta_1^1 & \theta_2^1 \\ \vdots & \vdots \\ \theta_1^N & \theta_2^N \end{bmatrix}, \quad \Theta_3 = \begin{bmatrix} \theta_3^1 \\ \vdots \\ \theta_3^N \end{bmatrix}.$$

Our aim is to broaden the process proposed by Kalogeropoulos et al. (2011), extending their multivariate CIR stochastic process in a more general process able to capture both the systematic and the idiosyncratic components that may affect interest rate dynamics. Section 4.2 describes our proposal, with Section 4.2.2 deriving the implied default probabilities. Our models will be applied and compared to data that concern the recent post-crisis period (2010-2014) and the countries belonging to the Eurozone. As the validity of a model ought to be tested in terms of its predictive performance, we will also develop an appropriate model assessment methodology based on out-of-sample predictions of interests and growth rates, for a given Monte Carlo path of monetary and real reference rates. Section 4.3 presents the empirical evidence obtained from the application of our models, with Section 4.3.1 describing data, Section 4.3.2 providing details about the model estimation and validation, Section 4.3.3 presenting partial correlation networks and Section 4.3.4 showing the estimated PD values. Section 4.3.5 will describe the predictive performance of our models and Section 4.4 will conclude with some final remarks.

4.2 Proposal

We assume that the dynamic of the debt of each country expressed, for simplicity, by the evolution of the associated interest rate, can be described by a linear combination of two stochastic processes. We assume, in fact, that they follow the same diffusion mechanism, that can be considered as the systematic process; in addition, we assume that they are also characterised by another stochastic equation, that can be considered as an idiosyncratic evolution. For each country i, the complete process is the following:

$$Z_t^i = -\alpha^i S_t + \beta^i y_t^i, \tag{4.2.1}$$

where S_t stands for the systematic process, while y_t^i represents the idiosyncratic process referred to country i. Furthermore, α^i measures the weight of the systematic process on country i, while β^i is a weight variable which measures the influence of the idiosyncratic equation on the general, complete process Z_t^i, that describes the evolution of interest rates. From an economic viewpoint, the previous equation can be interpreted in two ways, giving two different measures (that we will call $Z_{t,1}$ and $Z_{t,2}$). More precisely, $Z_{t,1}$ can express the financial side of each country, being the difference between interest rates on government bonds and Euribor rates; $Z_{t,2}$ can be interpreted as a measure of the real-economy side, being the difference between the GDP growth rate of each country and the average GDP growth rate of the Eurozone. This means that $Z_{t,1}$, from the perspective of an investor, can be interpreted as a financial premium, or the extra-return of investing in that country with respect to the cost of liquidity (measured by the systematic process S_t, which consists in the Euribor rate). By using the perspective of the country, $Z_{t,1}$ expresses the extra-payment that country provides (the cost of its debt) with respect to the cost of liquidity. On the other side, $Z_{t,2}$ expresses the real-economy premium, that is to say how much the real side of the economy of each country grows with respect to the others (or their average). From the perspective of an investor, $Z_{t,2}$ measures how safe is investing in that country with respect to the others. Both the extra-payments and the extra-premium take into account their correlations, expressed by the weights α and β. Finally, as we will better explain in Section 4.2.2 , the combination of $Z_{t,1}$ and $Z_{t,2}$ will provide a measure of the debt sustainability of each country.

Both the systematic and the idiosyncratic processes can be formulated as stochastic differential equations, through a CIR specification:

$$\begin{cases} \mathrm{d}\, S_t = (a - vS_t)\,\mathrm{d}\, t + b\sqrt{S_t}\,\mathrm{d}\, B_t, \\ \mathrm{d}\, y_t^i = (\theta_1^i - \theta_2^i y_t^i)\,\mathrm{d}\, t + \theta_3^i \sqrt{y_t^i}\,\mathrm{d}\, W_t, \end{cases} \tag{4.2.2}$$

where $\mathrm{d}\, B_t$ and $\mathrm{d}\, W_t$ are two independent Brownian motions.

The previous equation derives from an important assumption: the systematic process is the same for all the countries considered in the sample, but it differently influences each generic country-specific process Z_t^i, through the weight α^i.

The next step consists in deriving the covariance matrix of the process. To achieve this objective we introduce the following assumptions on the correlation structure:

$$\begin{cases} \mathrm{Corr}[\mathrm{d}\, y_t^i, \mathrm{d}\, y_t^j] = \rho^{ij}, \\ \mathrm{Corr}[\mathrm{d}\, S_t, \mathrm{d}\, y_t^j] = \gamma^j. \end{cases} \tag{4.2.3}$$

The first equation is consistent with the assumptions used in the formulation of multidimensional CIR processes; the second one describes the correlation between each idiosyncratic process and the systematic process S_t.

We can thus obtain the covariance $\mathrm{Cov}(Z_t^i, Z_t^j)$, where

$$\begin{cases} \mathrm{d}\, Z_t^i = -\alpha^i \,\mathrm{d}\, S_t + \beta^i \,\mathrm{d}\, y_t^i, \\ \mathrm{d}\, Z_t^j = -\alpha^j \,\mathrm{d}\, S_t + \beta^j \,\mathrm{d}\, y_t^j, \\ i, j = \text{countries}. \end{cases} \tag{4.2.4}$$

After some calculations the following expression for the instantaneous covariance can be shown to be equal to:

$$\mathrm{Cov}(Z^i, Z^j) = \alpha^i \alpha^j b^2 S_0 + \sqrt{S_0} b \cdot \left[\alpha^i \beta^j \gamma^j \sqrt{y_0^j}\theta_3^j + \alpha^j \beta^i \gamma^i \sqrt{y_0^i}\theta_3^i\right] + \beta^i \beta^j \sqrt{y_0^i y_0^j}\theta_3^i \theta_3^j \rho^{ij}. \tag{4.2.5}$$

Note that the previous equation can be simplified if the two countries coincide ($i = j$):

$$\mathrm{Cov}(Z^i, Z^i) = (\alpha^i)^2 b^2 S_0 b + 2\sqrt{S_0}\sqrt{y_0^i}\alpha^i \beta^i b \theta_3^i \gamma^i + (\beta^i)^2 y_0^i (\theta_3^i)^2. \tag{4.2.6}$$

A further development can be achieved by deriving a compact formulation for the instantaneous covariance matrix. Consider the correlation matrix of the idiosyncratic processes:

$$P = \begin{bmatrix} 1 & \rho^{12} & \dots & \rho^{1N} \\ \rho^{21} & 1 & \dots & \rho^{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{N1} & \rho^{N2} & \dots & 1 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} \gamma^1 \\ \vdots \\ \gamma^i \\ \vdots \\ \gamma^N \end{bmatrix}, \tag{4.2.7}$$

where each element in P consists in the correlation coefficient between the idiosyncratic processes of two countries, while Γ is a column vector which includes the correlation coefficients between each institution i and the systematic process S_t.

Through the previous specification we can rewrite the instantaneous covariance matrix A with the following, simple decomposition:

$$A = \Phi \cdot \Theta^T, \tag{4.2.8}$$

where

$$[\Phi]^i = \left[\alpha^i b\sqrt{S_0}, \quad \alpha^i, \quad \beta^i\sqrt{S_0 y_0^i} b\theta_3^i [\Gamma]^i, \quad \beta^i\sqrt{y_0^i}\theta_3^i\sqrt{[P]^i}\right],$$

$$[\Theta^T]^j = \begin{bmatrix} \alpha^j b\sqrt{S_0} \\ \\ \beta^j\sqrt{S_0 y_0^j} b\theta_3^j [\Gamma]^j \\ \\ \alpha^j \\ \\ \beta^j\sqrt{y_0^j}\theta_3^j\sqrt{[P]^j} \end{bmatrix}.$$

We remark that, in the general case of correlations between the idiosyncratic and the systematic components significantly different from zero ($\gamma^i, \gamma^j, \rho^{ij} \neq 0$), equation (4.2.8) expresses the variance-covariance matrix as a mix between four components, deriving from the cross correlations between the idiosyncratic components (driven by ρ^{ij}) and between each of them and the systematic factor (driven by γ^i and γ^j). In case both the systematic factors are not correlated with the systematic one ($\gamma^i = \gamma^j = 0$), equation (4.2.8) simplifies, becoming the following:

$$\mathrm{Cov}(Z^i, Z^j) = \alpha^i\alpha^j b^2 S_0 + \beta^i\beta^j\sqrt{y_0^i y_0^j}\theta_3^i\theta_3^j\rho^{ij}. \tag{4.2.9}$$

Note that (4.2.9) reports the covariance as the sum of two separate and disentangled addends: one referred to the systematic (and common) factor, the other deriving from the interaction between the two idiosyncratic components.

We now aim at extending the model described in (4.2.1) in a multivariate framework that can allow the jointly estimation of the time evolution of two systems of stochastic processes: these two systems of stochastic equations (each of them is composed by an idiosyncratic and a systematic factor) will describe, respectively, the evolution of the financial and the real side of the economy and, specifically, their impact on the sustainability of a country's debt.

From a mathematical viewpoint, consider that $f(t) = Debt(t)/GDP(t)$ is a two-variables function that depends on time through the time dependence of its two components Debt and GDP. It can be easily shown that its total derivative is the following:

$$\mathrm{d}\, f = \frac{\partial f}{\partial Debt} \cdot \mathrm{d}\, Debt + \frac{\partial f}{\partial GDP} \cdot \mathrm{d}\, GDP = \frac{\frac{\partial Debt}{\partial t} \cdot GDP - Debt \cdot \frac{\partial GDP}{\partial t}}{GDP^2}. \tag{4.2.10}$$

The previous equation suggests to model the time evolution of the Debt/GDP ratio and, therefore, the sustainability of a country's debt, by looking at the evolution of both the financial liability side and the real asset side of its underlying economy.

More formally, assume that the evolution of the Debt/GDP ratio is described by the process $Z^i_{t,1} - Z^i_{t,2}$, whose first component $Z^i_{t,1}$ (the Debt process) is independent from the second component $Z^i_{t,2}$ (the GDP process). The evolution of Debt, simplified by the cost of the debt service, is described by the following process:

$$Z^i_{t,1} = -\alpha^i_1 S_{t,1} + \beta^i_1 y^i_{t,1}, \tag{4.2.11}$$

where $S_{t,1}$ represents the Euribor interest rate evolution, while $y^i_{t,1}$ describes the interest rate of 10-years maturity government bonds: thus, $Z^i_{t,1}$ measures the weighted spread between bond interest rates and monetary rates.

On the other hand, the evolution of the GDP of a country is described by the following process:

$$Z^i_{t,2} = -\alpha^i_2 S_{t,2} + \beta^i_2 y^i_{t,2}, \tag{4.2.12}$$

which represents the spread between the country-specific GDP growth rate ($y^i_{t,2}$) and the GDP growth rate of the overall Euro area ($S_{t,2}$).

4.2.1 Model estimation and validation

All the proposed CIR time-homogeneous processes introduced in the previous subsection need a specific parameter estimation procedure. For this aim we can define the following variables:

$$c_t^i = \frac{2\theta_2^i}{(\theta_3^i)^2(1 - e^{-\theta_2^i t})}, \quad u_t^i = c_t^i y_t^i e^{-\theta_2^i t}, \quad q_t^i = \frac{2\theta_1^i}{(\theta_3^i)^2} - 1, \quad v_t^i = c_t^i y_{t+1}^i.$$

The log-likelihood function of the process, consistently with the approach proposed by Iacus (2008), can be derived as:

$$\ln L(\Theta^i) = \sum_{j=1}^{N-1} \left[\ln c_{t_j}^i - u_{t_j}^i - v_{t_j}^i + \frac{q_{t_j}^i}{2} ln \left(\frac{v_{t_j}^i}{u_{t_j}^i} \right) + \ln[I_q(2\sqrt{u_{t_j}^i v_{t_j}^i})] \right], \tag{4.2.13}$$

where $I_q(2\sqrt{u_{t_j}^i v_{t_j}^i})$ is the modified Bessel function of order q. The parameter vector $\hat{\Theta}$ is thus found by maximising the log-likelihood function:

$$\hat{\Theta}^i = (\hat{\theta}_1^i, \hat{\theta}_2^i, \hat{\theta}_3^i) = \arg\max_{\Theta^i} \ln L(\Theta^i). \tag{4.2.14}$$

Our aim is to develop models that can be used to predict the time evolution of the Debt/GDP ratio and, accordingly, the default probability of each country, in an early warning perspective. To this aim, we ought to develop a predictive assessment procedure. This is particularly meaningful especially in the light of the necessity to forecast ahead of time the levels of the systemic rates, that are the main explanatory components of the model.

In order to predict financial or economic spreads ($Z_{t,1}^i$, $Z_{t,2}^i$) for the countries considered in the sample, we need to estimate not only the parameters of the stochastic equations (as in (4.2.14)), but also the weights of the systematic (α^i) and the idiosyncratic (β^i) processes. Let us call $d_{\{1,2\}}^i(real)$ the observed spreads for the period under analysis. Then

$$\begin{cases} \beta_{\{1,2\}}^i = \mathrm{Corr}\big(d_{\{1,2\}}^i(real), y_{t,\{1,2\}}^i\big), \\ \\ \alpha_{\{1,2\}}^i = \mathbb{E}\Big(\frac{\beta_{\{1,2\}}^i y_{t,\{1,2\}}^i - d_{\{1,2\}}^i(real)}{S_{t,\{1,2\}}}\Big) \end{cases} \tag{4.2.15}$$

From an economic viewpoint, it is important to understand the meaning of the weight coefficients α^i and β^i: if they are both positive, it means that both the correlations between d^i and y^i and that between d^i and S are positive, which means that the idiosyncratic process part of d^i increases faster than the systematic component part: $|\frac{\partial y^i}{\partial t}| > \frac{\partial S}{\partial t}$. If the two coefficients are both negative, the systematic component, instead, changes faster than the idiosyncratic one: $|\frac{\partial S}{\partial t}| > \frac{\partial y^i}{\partial t}$.

To obtain a robust measurement we can indeed generate N scenarios of the general processes, using the estimated parameters and weights, and obtain the corresponding values using either (4.2.11) and (4.2.12). On the basis of them we can calculate and approximate the Monte Carlo expected values and variances for the predicted values, as follows.

Let $Z^i_{t,\{1,2\}}$ be a spread to be predicted at time t, with unknown density function $f_Y(y)$. The expected value of Y can then be approximated with

$$\widehat{\mathbb{E}(Y)} = \frac{1}{N}\sum_{k=1}^{N} y^{(k)}, \tag{4.2.16}$$

and its variance with

$$\widehat{var(Y)} = \frac{1}{N^2}\sum_{k=1}^{N}[y^{(k)} - \widehat{\mathbb{E}(Y)}]^2. \tag{4.2.17}$$

Similarly, for each generated scenario we can calculate the corresponding default probability, as well as we can generate and predict future values according to equations (4.2.20) and (4.2.22).

4.2.2 Default probability estimation

The process introduced in the previous Subsection can be employed to derive the probability of default (PD) of each country, that will, therefore, depend on the joint dynamic of its debt and GDP. To achieve this aim we can proceed as follows.

Assume that we are in an arbitrage-free context. According to the two specifications of the general process $Z^i_{t,\{1,2\}}$, two PDs can then be obtained.

The first (PD^i_1) exclusively depends on the debt side, through the interest rate spread, and it can be derived considering:

$$D^i_{t+1} = (1 - PD^i_1)e^{S_{t,1}+d^i_1}D^i_t, \tag{4.2.18}$$

where D^i_{t+1} (D^i_t) is the total debt of country i at time $t+1$ (t), and d^i_1 is the spread between the idiosyncratic and the systematic interest rate. The analogous risk-free expression is the following:

$$D^i_{t+1} = D^i_t e^{S_{t,1}}. \tag{4.2.19}$$

Equating (4.2.18) with (4.2.19) we can obtain PD_1:

$$PD^i_{t,1} = 1 - e^{-d^i_{t,1}} = 1 - e^{-Z^i_{t,1}}. \tag{4.2.20}$$

The second expression of the PD (PD^i_2) depends on both the financial and the real side, and can be obtained by considering the processes $Z^i_{t,1}$ and $Z^i_{t,2}$ together and deriving the probability of default from the ratio between the liability (debt) and the asset (GDP) components. Doing so equations (4.2.18) and (4.2.19) become:

$$\begin{cases} \frac{D^i_{t+1}}{A^i_{t+1}} = (1 - PD^i_2)\frac{D^i_t}{A^i_t}\frac{e^{S_{t,1}+d^i_1}}{e^{S_{t,2}+d^i_2}}, \\ \\ \frac{D^i_{t+1}}{A^i_{t+1}} = \frac{D^i_t}{A^i_t}e^{S_{t,1}-S_{t,2}}. \end{cases} \tag{4.2.21}$$

Therefore, after having equated the two expressions, we obtain:

$$PD^i_{t,2} = 1 - e^{-(d^i_{t,1}-d^i_{t,2})} = 1 - e^{-(Z^i_{t,1}-Z^i_{t,2})}. \tag{4.2.22}$$

From the above equation some comments can be made: (a) if d^i_1 decreases, the probability of default decreases, which is consistent with the definition of d^i_1 as the spread between the country government bond interest rates and the monetary rates (the higher $y^i_{t,1}$ and $Z^i_{t,1}$, the riskier the country); (b) similarly, if d^i_2 decreases the probability of default increases, which is consistent with the definition of d^i_2 as the spread between the idiosyncratic GDP growth rate and the European GDP growth rate (the lower $y^i_{t,2}$ and $Z^i_{t,2}$, the riskier the country).

4.3 Application

4.3.1 Data and descriptive statistics

The recent financial crisis, together with the sovereign crisis, has had a great impact in the Euro area. The volatility of the default probability of each country has significantly increased, and the relationships between countries have substantially changed: southern european countries are very close to each other; similarly, northern economies are strongly interconnected and characterised by limited relations with southern countries.

To investigate the above issues we focus on seven european countries: France, Germany, Greece, Ireland, Italy, Portugal and Spain, for the post-crisis period, ranging from January 2010 to December 2014.

For the purposes of our analysis, the systematic process is the 1-month Euribor, while the idiosyncratic process is defined by the interest rates of 10-years government bonds. All the data collected and used in this analysis have monthly frequencies. As the GDP growth rates are quarterly released, to obtain monthly data we have performed a linear interpolation of the available values.

The time evolution of the systematic and idiosyncratic processes can be observed in Figure 4.6.1.

[Figure 4.6.1]

From Figure 4.6.1 it is clear that the Euribor is the lowest interest rate (at the moment very close to zero); Greek bond rates, on the contrary, are characterised by the highest values for the whole period and by a very high volatility, with a strong peak during 2012. This feature is obviously consistent with the Greece sovereign crisis. Similarly, Portugal presents the highest values concentrated around 2012. Ireland shows a peculiar behaviour, with a strong increase during 2011, followed by a remarkable drop during the next years. Spain and Italy seem to have very similar curves and, finally, Germany and France bond rates are quite homogeneous.

The evolution of the GDP growth rates is represented in Figure 4.6.2.

[Figure 4.6.2]

Figure 4.6.2 shows that during the first post-crisis years almost all the GDP growth rates were negative, with a strong decrease for Greece; since 2013 the trend has changed and the GDPs have started increasing again in all countries, with the exception of France. Again, Ireland seems to have a peculiar trend, being now characterised by the most significant increase in its GDP with respect to the other economies.

The correlation matrices between the processes can be calculated, and are reported in Table 4.6.1 ($S_{t,1}$ and $y^i_{t,1}$) and in Table 4.6.2 ($S_{t,2}$ and $y^i_{t,2}$), together with the corresponding $p-values$.

[Table 4.6.1]

From Table 4.6.1 one can notice that almost all the correlation coefficients are positive and significant, meaning a strong relationship between the bond interest rates of the seven european countries considered in the sample. The most positive links are between France and Germany on one side, and between Greece, Italy, Portugal and Spain on the other side. In addition, Germany seems to be not significantly connected to southern countries, such as Italy, Portugal and Greece, while its correlation with Spain is, even if significant, very low in value. This result would suggest us to divide the sample into two, independent clusters, one composed by northern economies (France and Germany), the other one including southern economies (Spain, Portugal, Italy and Greece) and Ireland staying in between.

[Table 4.6.2]

Table 4.6.2, on the contrary, shows a different scenario. By analysing the correlations between the GDP growth rates one can notice that Germany and France are still positively related, but now Italy has radically changed its position: in fact, it is positively, significantly and strongly linked with both France and Germany, while its correlation with Portugal and Spain still remains positive and significant, but lower in value, meaning that its GDP growth rate presents a behaviour much more similar to that of northern economies. This is consistent with the presence of strong real trading of Italy with Germany and France. On the other hand, other southern economies, such as Spain, Portugal, Greece, as well as Ireland, remain strongly related to each other also in terms of GDP growth. In terms of significance levels, Ireland does not seem to be connected to core countries like Germany and France; similarly, both Portugal and Spain present not significant correlations with France.

4.3.2 Model estimation and validation

The first step in the model estimation consists in deriving the CIR coefficients for all the countries and for the two general processes $Z^i_{t,\{1,2\}}$ through the maximisation of the log-likelihood function. In order to perform out-of-sample tests, we have used data from 2010 until 2013: in this way we can generate all the processes for 2014, and we can predict the values of the spreads for all the countries (see Section 3.4). The estimated parameter values obtained for the two systematic processes $S_{t,1}$ (Euribor interest rate) and $S_{t,2}$ (Euro area GDP growth rate) are reported in Table 4.6.3.

[Table 4.6.3]

In Table 4.6.4 we have reported the estimated parameters of the idiosyncratic processes $y^i_{t,1}$ (10-years bond interest rate) and $y^i_{t,2}$ (GDP growth rate), where i refers to each of the seven countries considered in this analysis.

[Table 4.6.4]

Table 4.6.4 shows that Greece has the highest volatility parameter for the process that describes bond interest rates ($\theta_{3,1}$): this is consistent with the descriptive statistics and with the density plot presented in Figure 4.6.3. Similarly, Ireland presents the highest volatility parameter for the process that describes the GDP growth rate: this is again consistent with the density plot in Figure 4.6.3.

[Figure 4.6.3]

Secondly, we have to derive the weight coefficients of the systematic and the idiosyncratic processes, consistently with equation (4.2.15): they are reported in Table 4.6.5.

[Table 4.6.5]

From Table 4.6.5 note that, for the first process, both weights α_1 and β_1 are positive in Germany, Greece, Ireland and Portugal: the idiosyncratic component, therefore, changes faster than the systematic one, consistently with the actual situation of almost zero monetary rates. For the second process, instead, such behaviour can be detected in Germany, Greece, Italy and Spain.

Through the specification of the parameters obtained so far we are now able to generate the total processes $Z^i_{t,1}$ and $Z^i_{t,2}$ for the period 2010-2013 or for 2014, and for all the countries. An interesting point consists in the analysis of the correlation coefficients between them. Table 4.6.6 represents the correlation matrix between $Z^i_{t,1}$ for $t = 2010, ..., 2014$.

[Table 4.6.6]

Table 4.6.6 (which shows the correlations between the processes that describe the spread between bond interest rates and monetary rates) is absolutely consistent with Table 4.6.1, showing again two distinct clusters characterised by strong and significant inner correlations: France and Germany on one side, and Spain, Italy, Portugal and Greece on the other one, with the peculiar case of Ireland, positively correlated with both southern and northern economies. Cross correlations between these two clusters are instead mixed in their behaviour: Germany, in fact, is significantly and negatively related to Greece, while it is not significantly related to the other southern economies (Italy, Portugal, Spain); similarly, France does not present significant correlation coefficients with both Greece and Portugal.

Differently from Table 4.6.1, Table 4.6.6 reports the correlation coefficients between the whole processes $Z^i_{t,1}$, which include both the idiosyncratic and the systematic components of each country, being the latter described by monetary rates: the persistence of the two clusters (core and peripheral countries) even after the inclusion of monetary rates means that the recent monetary policy and the choice of close-to-zero monetary rates has not really affected the relationships between Eurozone economies.

Table 4.6.7 reports the correlation coefficients between $Z^i_{t,2}$, which is the process that describes the difference between the idiosyncratic GDP growth rate and the global Eurozone GDP growth rate.

[Table 4.6.7]

Comparing Table 4.6.6 with Table 4.6.7, it is interesting to note that some coefficients have changed their sign, meaning the relationships between some pairs of countries may change depending on the variables under analyses. For example, if we look at the matrix $Z^i_{t,1}$, Italy is positively correlated to Spain and negatively correlated to Germany; but if we change perspective and look at the GDP growth rate, we can notice that those two relationships radically change. Similarly, the correlation coefficients of Italian bonds with the Greek and Irish ones (both positive) become negative when looking at the correlations between the GDP growth rates. Another interesting case regards France: its bond interest rates, in fact, are positively correlated to the German ones, but its GDP decreases when the German GDP increases.

In addition, the correlation matrix presented in Table 4.6.7 (which is based on the whole spread measures $Z^i_{t,2}$) shows some differences with respect to Table 4.6.2 (based only on the idiosyncratic components of the total processes $Z^i_{t,2}$): this means that while the latter looks at the relationships between countries based on the GDP growth rate by itself, the former analyses the correlation coefficients between the GDP growth rates with respect to the Eurozone average. The comparison between the two reveals that the French GDP is growing less than the Eurozone average, and in this sense France is getting closer to southern economies. Spain seems to be very close to France, thus finding itself in an intermediate situation between Germany and peripheral countries such as Portugal and Greece. On the other side, the Italian GDP growth rate seems to be, with respect to the Eurozone average, much closer to the German one.

Finally, Table 4.6.8 shows the correlation coefficients of the total processes $Z^i_{t,1} - Z^i_{t,2}$.

[Table 4.6.8]

Table 4.6.8 emphasises again the peculiar case of France, now negatively and significantly related to both northern economies (Germany) and southern ones (Italy and Portugal): the other five countries (Spain, Portugal, Italy, Ireland and Greece) remain positively related to each other, while Germany does not seem significantly related to any of them.

4.3.3 Network analysis

From the correlation matrices reported in Tables 4.6.6, 4.6.7 and 4.6.8 we can calculate their inverse and, therefore, obtain the partial correlations between countries. This, following Giudici and Spelta (2015) allows to build a graphical Gaussian network between the default probabilities of different countries, which gives an important representation of systemic risk channels.

For simplicity we report such partial correlations referred only to $Z^i_{t,1}$ (Table 4.6.9) and to $Z^i_{t,1} - Z^i_{t,2}$ (Table 4.6.10), for $t = 2010, ..., 2013$, along with the p-values that correspond to the hypotheses of them being equal to zero (no connection).

[Table 4.6.9]

Table 4.6.10 reports the partial correlations of the spread between the two processes, consistently with the formulation of PD^i_2.

[Table 4.6.10]

By considering a significance level $\alpha = 0.01$, we can select the most significant correlations, and thus derive the graphical Gaussian networks for $Z_{t,1}$ and for the spread $Z_{t,1} - Z_{t,2}$, with $t = 2010, ...2013$, as in Figure 4.6.4.

[Figure 4.6.4]

In Figure 4.6.4 the green lines stand for positive correlations, while red connectors indicate negative correlations.

The comparison between the two networks, calculated on past data, reflects what has been underlined in the previous Section: the inclusion of the GDP growth rate, and so of a macroeconomic variable, in the study of the default probability is important and necessary in order to capture all the correlations and the direct links between the countries.

Firstly, the network on the left seems more complex and composed by a large number of connections with respect than the other one. Secondly, in the first graph almost the correlations are positive, with a high level of connections between Spain, Portugal, Italy and Ireland.

The final network on the right, referred to the process $Z_{t,1} - Z_{t,2}$, shows very interesting results, being very different from the previous one. As underlined before, France has completely changed its position, being now negatively related both to Germany and Portugal. The negative relations between Germany and both Italy and Portugal have disappeared, as well as two negative correlations have emerged between Germany and Greece and between Italy and Greece. Finally, Germany seems the less interrelated country, while Spain, Portugal, Italy, Ireland and Greece present a higher level correlations.

4.3.4 Default probability estimation

After having analysed the correlation coefficients, we can now calculate the two default probabilities of the seven countries: the first probability ($PD^i_{t,1}$) considers only the spread between bond interest rates and monetary rates and it is calculated with equation (4.2.20); the second default probability ($PD^i_{t,2}$) incorporates both the spread between interest rates and the spread between the country-specific GDP growth rate and the Eurozone GDP growth rate, and it is based on equation (4.2.22). The first graph in Figure 4.6.5 refers to $PD_{t,1}$, while the second one describes $PD_{t,2}$.

[Figure 4.6.5]

From Figure 4.6.5 it is clear that the inclusion of the GDP growth rate, together with the spread between interest rates, changes the default probabilities during the period 2010-2014. This is especially evident for Greece, which experienced an increase in the PD after the addition of the GDP. This is consistent with the fact that its GDP growth rate has been strongly negative for the first years after the crisis. The same reasoning can be applied also to France, Italy, Portugal and Spain: they all present an increase in their default probability because of their decrease in the GDP. On the contrary, Ireland shows a lower curve in the second graph, meaning that its GDP increasing during the last years has weakened its PD. Finally, Germany has always been characterised by a positive GDP growth rate, and for this reason its default probability decreases after having included $Z_{t,2}$ in the derivation of the PD for the whole period under analysis. Figure 4.6.5 also shows a decrease in the probabilities of default for almost all the countries during 2014. This is explained by the radical drop in the interest rates of the 10-years bonds during the last period. Moreover, by comparing the several curves, it is clear that Spain, Ireland and

Germany have $PD_{2014,2} < PD_{2014,1}$: this is consistent with their actual increase (very strong for Ireland) in the GDP during the last year. Italy and France, on the contrary, present $PD_{2014,2} > PD_{2014,1}$, and again this is due to the actual negative values of their GDP growth rates.

It is important to remark that Figure 4.6.5 shows a change in the default probabilities after having included the GDP growth rates in the analyses. This is a clear evidence of the importance of including an economic perspective, together with a financial one, in the analysis of the debt sustainability of a country.

Finally, we can calculate the correlation matrices between the two default probabilities, and see how they change considering only the financial viewpoint ($PD_{t,1}$) or both a financial and an economic perspective ($PD_{t,2}$).

[Tables 4.6.11, 4.6.12]

By comparing Table 4.6.11 and 4.6.12 it is clear that France and Italy are the most interesting situations. On the liability side, France si significantly and positively related to Germany, while it is negatively connected to Portugal; by implementing the GDP growth rate, both these relationships become opposite in sign. Similarly, while France and Italy are positively correlated on the liability side, they become negatively related after the inclusion of the GDP. Peripheral countries such as Greece, Portugal and Spain still remain positively related to each other, while Germany shows a peculiar behaviour, since in Table 4.6.12 it does not show any significant relationship with other economies. Ireland lies in between core and peripheral countries, being positively related to all countries (with the exception of Germany) even if to a lesser extent.

From this Section an important conclusion emerges: correlations between idiosyncratic processes are significant, but it is even more important to consider correlations between each country-specific parameter and the overall european level of the same quantity. This final remark justifies our choice of including an european level within our model, looking at the spread between an idiosyncratic variable in each country and its mean value in the Eurozone.

4.3.5 Predictive Performance Assessment

In order to test the precision of the model, we have computed out-of-sample tests on the spread measures $Z^i_{t,1}$ and $Z^i_{t,2}$. More precisely, we have estimated the parameters of the idiosyncratic ($y^i_{t,1}$ and $y^i_{t,2}$) and systematic ($S_{t,1}$ and $S_{t,2}$) processes on real data for the period January 2010-December 2013; we have then used such parameters in order to generate, through Monte Carlo simulations, the stochastic processes for the following twelve months. In such a way we have been able to predict the values of the spread measures $Z^i_{t,1}$ and $Z^i_{t,2}$ for the year 2014. The comparison between observed and predicted spread measures can be observed in Figure 4.6.6: solid lines refer to the observed values, while points and confidence intervals are used for the estimated values; the black color indicates $Z^i_{t,1}$, while the red color is used for $Z^i_{t,2}$.

[Figure 4.6.6]

Figure 4.6.6 clearly shows that the CIR stochastic processes used for modelling the two components (idiosyncratic and systematic) of the two spread measures ($S_{t,1}$ and $S_{t,2}$) perform quite well from a predictive viewpoint. Almost all the predicted values, in fact, are included within the confidence bands.

Table 4.6.13 reports the root mean square errors (RMSE) and the mean percentage errors (MAPE) referred to the processes $Z_{t,1}$ (first two columns) and $Z_{t,2}$ (last two columns) estimated for 2014.

[Table 4.6.13]

Table 4.6.13 shows that, on average, the errors referred to the second spread measures $Z_{t,2}$ are bigger than the first ones, both in absolute and relative terms. This may be due to the fact that the GDP growth rates have changed very little during the last year with respect to interest rates on government bonds, as well as they have stopped increasing in 2014: stochastic processes estimated during the previous years may have misunderstood the time evolution of that variable.

4.4 Conclusions

We have demonstrated that correlated stochastic processes can be very useful in the joint modelling of the dynamic of debt sustainability of an economy, as measured by the Debt/GDP ratio. They are relatively easy to be interpreted and, in addition, they show a good predictive performance.

The application of our approach to the Euro area indicates the existence of a "clustering" effect between countries: by considering interest rate spreads, in fact, the Eurozone appears to be divided into northern (core) and southern (peripheral) economies. Countries appear to be strongly interrelated within each cluster, while cross-cluster correlations are not significant or negative.

Interesting results also emerge from the analysis of default probabilities. When the PD of a country is calculated not only on the basis of the interest rate spread, but also in terms of the GDP growth rate, the clustering effect above described changes: in particular, France and Italy exchange their roles, with the former getting closer to peripheral countries, and the latter approaching an intermediate situation between peripheral economies and Germany. Furthermore, Ireland increases its regime switching performance along time: together with the drop of interest rates on government bonds, in fact, Ireland had a strong increase in its GDP growth rate (the highest one in the Euro area) in the last two years: the combination of these two effects thus justifies its radical positioning change between clusters.

4.5 References

Acharya, V.V., 2009. A theory of systemic risk and design of prudential bank regulation. Journal of Financial Stability, 5(3), 224-255.

Acharya, V.V., Pedersen, L.H., Philippon, T., Richardson, M., 2010. Measuring Systemic Risk. Technical Report. New York University.

Acharya, V.V., Engle, R., Richardson, M., 2012. Capital Shortfall: A New Approach to Ranking and Regulating Systemic Risks. American Economic Review: Papers and Proceedings, 102(3), 59-64.

Adrian, T., Brunnermeier, M.K., 2011. CoVar. NBER Working Paper No. 17454. National Bureau of Economic Research.

Ahelegbey, D.F., Billio, M., Casarin, R., 2016. Bayesian Graphical Models for Structural Vector Autoregressive Processes. Journal of Applied Econometrics, 31(2), 357-386.

Allen, F., Gale, D., 2000. Financial Contagion. Journal of Political Economy, 108(1), 1-33.

Ang, A., Longstaff, F.A., 2012. Systemic sovereign credit risk: lessons from the U.S. and Europe. NBER Working Paper No. 16982. National Bureau of Economic Research.

Billio, M., Getmansky, M., Lo, A.W., Pelizzon, L., 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of Financial Economics, 104, 535-559.

Billio, M., Caporin, M., Panzica, R., Pelizzon, L., 2015. Network Connectivity and Systematic Risk. Technical Report. University Ca' Foscari.

Brownlees, C., Engle, R., 2012. Volatility, Correlation and Tails for Systemic Risk Measurement. Technical Report. NYU Stern School of Business.

Brownlees, C., Hans, C., Nualart, E., 2014. Bank Credit Risk Networks: Evidence from the Eurozone Crisis. Technical Report.

Chong, B.S., Liu, M., Shrestha, K., 2006. Monetary transmission via the administered interest rates channel. Journal of Banking and Finance, 30(5), 1467-1484.

Chuang, H., Ho, H.C., 2013. Measuring the default risk of sovereign debt from the perspective of network. Physica A: Statistical Mechanics and its Applications, 392(9), 2235-2239.

Cox, J.C., Ingersoll, J.E.Jr, Ross, S.A., 1985. A Theory of Term Structure of Interest Rates. Econometrica, 53(2), 385-408.

Dumitrescu, E., Banulescu, D.G., 2014. Which are the SIFIs? A Component Expected Shortfall (CES) Approach to Systemic Risk. Journal of Banking and Finance, 50, 575-588.

Giordani, L., Linciano, N., Soccorso, P., 2012. The determinants of government yield spreads in the euro area. Technical Report No. 71. Consob.

Giudici, P., Spelta, A., 2015. Graphical Network Models for International Financial Flows. Journal of Business and Economic Statistics, to appear.

Gray, D., Gross, M., Paredes, J., Sydow, M., 2013. Modeling Banking, Sovereign and Macro Risk in a CCA Global VAR. Technical Report. International Monetary Fund.

Hautsch, N., Schaumburg, J., Schienle, M., 2015. Financial Network Systemic Risk Contributions. Review of Finance, 19(2), 685-738.

Hurlin, C., Popescu, A., Turcu, C., 2013. Sovereign Debt and Systemic Risk. Technical Report. University of Orleans.

Iacus, S.M., 2008. Simulation and Inference for Stochastic Differential Equations. Springer.

Kalogeropoulos, K., Dellaportas, P., Roberts, G.O., 2011. Likelihood-based inference for correlated diffusions. The Canadian Journal of Statistics, 39(1), 52-72.

Longstaff, F.A., 2010. The subprime credit crisis and contagion in financial markets. Journal of Financial Economics, 97(3), 436-450.

Popescu, A., Turcu, C., 2014. Systemic Sovereign Risk in Europe: an MES and CES Approach. Revue d'economie politique, 124(6), 899-925.

Ramsay, B.A., Sarlin, P., 2016. Ending over-lending: assessing systemic risk with debt to cash flow. International Journal of Finance and Economics, 21(1), 36-57.

Schwaab, B., Koopman, S.J., Lucas, A., 2015. Global Credit Risk: World, Country and Industry Factors. Technical Report. Duisenberg School of Finance - Tinbergen Institute.

Shleifer, A., Vishny, R.W., 2010. Unstable banking. Journal of Financial Economics, 97(3), 306-318.

4.6 Appendix C: Tables and Figures

Table 4.6.1: Correlation matrix, bond interest rates

	France	Germany	Greece	Ireland	Italy	Portugal	Spain	Euribor
France	1.000							
Germany	0.908	1.000						
	(0.000)							
Greece	0.283	-0.066	1.000					
	(0.028)	(0.615)						
Ireland	0.805	0.647	0.563	1.000				
	(0.000)	(0.000)	(0.000)					
Italy	0.589	0.243	0.828	0.713	1.000			
	(0.000)	(0.061)	(0.000)	(0.000)				
Portugal	0.504	0.150	0.891	0.758	0.915	1.000		
	(0.000)	(0.253)	(0.000)	(0.000)	(0.000)			
Spain	0.562	0.284	0.793	0.714	0.918	0.818	1.000	
	(0.000)	(0.028)	(0.000)	(0.000)	(0.000)	(0.000)		
Euribor	0.747	0.658	0.308	0.895	0.520	0.578	0.426	1.000
	(0.000)	(0.000)	(0.022)	(0.000)	(0.000)	(0.000)	(0.000)	

Notes: Correlation matrix between the interest rates on 10-years government bonds and the Euribor. Almost all the correlation coefficients are positive and significant. The most positive links are between France and Germany on one side, and between Greece, Italy, Portugal and Spain on the other side. Germany seems to be not significantly connected to southern countries, such as Italy, Portugal and Greece.

Table 4.6.2: Correlation matrix, GDP growth rates

	France	Germany	Greece	Ireland	Italy	Portugal	Spain	Eurozone
France	1.000							
Germany	0.903	1.000						
	(0.000)							
Greece	-0.749	-0.414	1.000					
	(0.000)	(0.001)						
Ireland	-0.253	0.130	0.697	1.000				
	(0.051)	(0.323)	(0.000)					
Italy	0.782	0.966	-0.180	0.283	1.000			
	(0.000)	(0.000)	(0.169)	(0.028)				
Portugal	0.009	0.389	0.641	0.633	0.611	1.000		
	(0.948)	(0.002)	(0.000)	(0.000)	(0.000)			
Spain	-0.171	0.256	0.763	0.904	0.467	0.902	1.000	
	(0.193)	(0.049)	(0.000)	(0.000)	(0.001)	(0.000)		
Eurozone	0.746	0.955	-0.130	0.383	0.993	0.621	0.528	1.000
	(0.000)	(0.000)	(0.320)	(0.003)	(0.000)	(0.000)	(0.000)	

Notes: Correlation matrix between the GDP growth rates and the Eurozone GDP growth rate. Italy is positively, significantly and strongly linked with both France and Germany, while its correlation with Portugal and Spain still remains positive and significant, but lower in value. The other southern economies, such as Spain, Portugal, Greece, as well as Ireland, remain strongly related to each other also in terms of GDP growth.

Table 4.6.3: Parameters, systematic processes

	a_1	v_1	b_1	a_2	v_2	b_2
All countries	0.011	0.028	0.124	0.053	0.066	0.140

Notes: Estimated parameters of the systematic processes $S_{t,1}$ (Euribor interest rates) and $S_{t,2}$ (Euro area GDP growth rate).

Table 4.6.4: Parameters, idiosyncratic processes

	France	Germany	Greece	Ireland	Italy	Portugal	Spain
$\theta_{1,1}$	0.194	0.123	1.309	1.114	0.441	0.485	0.549
$\theta_{2,1}$	0.078	0.073	0.086	0.022	0.091	0.054	0.108
$\theta_{3,1}$	0.116	0.124	0.548	0.235	0.150	0.234	0.152
$\theta_{1,2}$	0.010	0.021	0.004	0.587	0.020	0.039	0.016
$\theta_{2,2}$	0.038	0.044	0.001	0.001	0.113	0.197	0.001
$\theta_{3,2}$	0.127	0.137	0.126	0.918	0.081	0.190	0.125

Notes: Estimated parameters of the idiosyncratic processes $y^i_{t,1}$ (10-years bond interest rate) and $y^i_{t,2}$ (GDP growth rate). Greece has the highest volatility parameter for the process that describes bond interest rates ($\theta_{3,1}$), while Ireland presents the highest volatility parameter for the process that describes the GDP growth rate ($\theta_{3,2}$).

Table 4.6.5: Weight coefficients

	France	Germany	Greece	Ireland	Italy	Portugal	Spain
α_1	-0.978	0.126	0.935	0.944	-0.276	0.834	-0.366
β_1	0.612	0.772	0.998	0.995	0.855	0.989	0.851
α_2	-1.416	0.779	1.448	-0.385	1.058	-0.069	0.581
β_2	-0.267	0.936	0.958	0.689	0.959	0.720	0.278

Notes: Weight coefficients of the two general processes $Z^i_{t,\{1,2\}}$. Both weights α_1 and β_1 are positive in Germany, Greece, Ireland and Portugal: the idiosyncratic component, therefore, changes faster than the systematic one. For the second process, instead, such behaviour can be detected in Germany, Greece, Italy and Spain.

Table 4.6.6: Correlation matrix, processes $Z_{t,1}$

	France	Germany	Greece	Ireland	Italy	Portugal	Spain	Euribor
France	1.000							
Germany	0.702	1.000						
	(0.000)							
Greece	0.008	-0.504	1.000					
	(0.956)	(0.001)						
Ireland	0.826	0.378	0.389	1.000				
	(0.000)	(0.008)	(0.006)					
Italy	0.410	-0.257	0.776	0.614	1.000			
	(0.004)	(0.078)	(0.000)	(0.000)				
Portugal	0.248	-0.403	0.878	0.560	0.887	1.000		
	(0.089)	(0.005)	(0.000)	(0.000)	(0.000)			
Spain	0.300	-0.250	0.787	0.688	0.841	0.778	1.000	
	(0.038)	(0.087)	(0.000)	(0.000)	(0.000)	(0.000)		
Euribor	0.944	0.496	0.083	0.849	0.491	0.345	0.392	1.000
	(0.000)	(0.000)	(0.631)	(0.000)	(0.002)	(0.030)	(0.007)	

Notes: Correlation coefficients between the processes $Z^i_{t,1}$ (spread between bond interest rates and monetary rates). The results show two distinct clusters characterised by strong and significant inner correlations: France and Germany on one side, and Spain, Italy, Portugal and Greece on the other one, with the peculiar case of Ireland, positively correlated with both southern and northern economies.

Table 4.6.7: Correlation matrix, processes $Z_{t,2}$

	France	Germany	Greece	Ireland	Italy	Portugal	Spain	Eurozone
France	1.000							
Germany	-0.927	1.000						
	(0.000)							
Greece	0.749	-0.931	1.000					
	(0.000)	(0.000)						
Ireland	0.072	-0.315	0.400	1.000				
	(0.627)	(0.029)	(0.005)					
Italy	-0.859	0.814	-0.599	-0.456	1.000			
	(0.000)	(0.000)	(0.000)	(0.001)				
Portugal	-0.563	0.269	0.088	0.101	0.689	1.000		
	(0.000)	(0.064)	(0.553)	(0.495)	(0.000)			
Spain	0.940	-0.995	0.924	0.310	-0.825	-0.299	1.000	
	(0.000)	(0.000)	(0.000)	(0.032)	(0.000)	(0.039)		
Eurozone	-0.998	0.906	-0.712	-0.037	0.857	0.601	-0.920	1.000
	(0.000)	(0.000)	(0.000)	(0.805)	(0.000)	(0.000)	(0.000)	

Notes: Correlation coefficients between the processes $Z^i_{t,2}$ (spread between the GDP growth rate of each country and the global Eurozone GDP growth rate). The results show significant changes in the correlation pattern between countries with respect to the co-movements of interest rates on government bonds.

Table 4.6.8: Correlation matrix, processes $Z_{t,1} - Z_{t,2}$

	France	Germany	Greece	Ireland	Italy	Portugal	Spain
France	1.000						
Germany	-0.594	1.000					
	(0.000)						
Greece	-0.076	-0.197	1.000				
	(0.606)	(0.179)					
Ireland	0.342	-0.214	0.770	1.000			
	(0.017)	(0.144)	(0.000)				
Italy	-0.448	0.275	0.692	0.519	1.000		
	(0.001)	(0.059)	(0.000)	(0.000)			
Portugal	-0.441	0.239	0.789	0.600	0.931	1.000	
	(0.002)	(0.102)	(0.000)	(0.000)	(0.000)		
Spain	0.317	-0.280	0.765	0.895	0.518	0.512	1.000
	(0.028)	(0.043)	(0.000)	(0.000)	(0.000)	(0.000)	

Notes: Correlation coefficients between the processes $Z^i_{t,1} - Z^i_{t,2}$. France is now negatively and significantly related to both northern economies (Germany) and southern ones (Italy and Portugal): the other five countries (Spain, Portugal, Italy, Ireland and Greece) remain positively related to each other, while Germany does not seem significantly related to any of them.

Table 4.6.9: Inverse correlation matrix, processes $Z_{t,1}$

	France	Germany	Greece	Ireland	Italy	Portugal	Spain
France	1.000						
Germany	0.924	1.000					
	(228.33)						
Greece	0.032	-0.001	1.000				
	(0.041)	$(2.57 \cdot 10^{-5})$					
Ireland	-0.061	0.323	-0.028	1.000			
	(0.145)	(4.533)	$(2.97 \cdot 10^{-2})$				
Italy	0.506	-0.341	-0.033	-0.517	1.000		
	(13.424)	(5,120)	$(4.19 \cdot 10^{-2})$	(14.263)			
Portugal	0.335	-0.522	0.503	0.575	0.373	1.000	
	(4.921)	(14.598)	(13.192)	(19.241)	(6.284)		
Spain	-0.174	-0.057	0.309	0.779	0.689	-0.487	1.000
	(1.222)	(0.127)	(4.104)	(60.348)	(35.187)	(12.135)	

Notes: Inverse correlation matrix for the processes $Z^i_{t,1}$ (spread between bond interest rates and monetary rates).

Table 4.6.10: Inverse correlation matrix, processes $Z_{t,2}$-$Z_{t,2}$

	France	Germany	Greece	Ireland	Italy	Portugal	Spain
France	1.000						
Germany	-0.449	1.000					
	(9.856)						
Greece	-0.213	-0.493	1.000				
	(1.846)	(12.510)					
Ireland	0.643	0.237	-0.008	1.000			
	(27.426)	(2.330)	$(2.761 \cdot 10^{-2})$				
Italy	0.056	0.041	-0.345	-0.424	1.000		
	(0.125)	(0.065)	(5.262)	(8.562)			
Portugal	-0.381	0.078	0.529	0.631	0.823	1.000	
	(6.641)	(0.238)	(15.142)	(25.859)	(82.131)		
Spain	-0.081	-0.025	0.464	0.686	0.600	-0.610	1.000
	(0.259)	(0.024)	(10.720)	(34.589)	(21.972)	(23.060)	

Notes: Inverse correlation matrix for the processes $Z^i_{t,2}$-$Z^i_{t,2}$ (consistently with the formulation of PD^i_2).

Table 4.6.11: Correlation matrix, PD_1

	France	Germany	Greece	Ireland	Italy	Portugal	Spain
France	1.000						
Germany	0.703	1.000					
	(0.000)						
Greece	0.014	-0.508	1.000				
	(0.925)	(0.000)					
Ireland	0.826	0.377	0.405	1.000			
	(0.000)	(0.008)	(0.004)				
Italy	0.410	-0.257	0.785	0.618	1.000		
	(0.004)	(0.078)	(0.000)	(0.000)			
Portugal	0.246	-0.405	0.886	0.564	0.888	1.000	
	(0.092)	(0.004)	(0.000)	(0.000)	(0.000)		
Spain	0.299	-0.250	0.798	0.691	0.843	0.783	1.000
	(0.005)	(0.004)	(0.000)	(0.000)	(0.000)	(0.000)	

Notes: Correlation coefficients between the default probabilities PD_1^i.

Table 4.6.12: Correlation matrix, PD_2

	France	Germany	Greece	Ireland	Italy	Portugal	Spain
France	1.000						
Germany	-0.592	1.000					
	(0.000)						
Greece	-0.059	-0.211	1.000				
	(0.692)	(0.150)					
Ireland	0.345	-0.220	0.787	1.000			
	(0.017)	(0.134)	(0.000)				
Italy	-0.448	0.278	0.686	0.517	1.000		
	(0.001)	(0.056)	(0.000)	(0.000)			
Portugal	-0.443	0.243	0.782	0.597	0.932	1.000	
	(0.002)	(0.097)	(0.000)	(0.000)	(0.000)		
Spain	0.316	-0.280	0.781	0.897	0.517	0.514	1.000
	(0.012)	(0.055)	(0.000)	(0.000)	(0.000)	(0.000)	

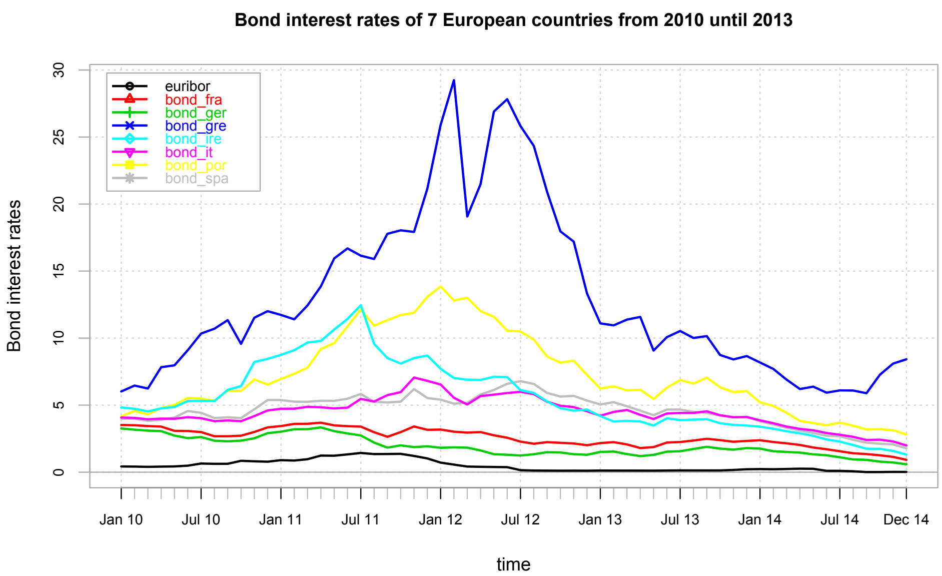
Notes: Correlation coefficients between the default probabilities PD_2^i. France and Italy change their relationships to other countries with respect to the results reported inTable 4.6.11; other peripheral countries such as Greece, Portugal and Spain still remain positively related to each other; Germany does not show any significant relationship with other economies; finally, Ireland lies in between core and peripheral countries.

Table 4.6.13: RMSE and MAPE, processes $Z_{t,1}$ and $Z_{t,2}$

Country	$Z_{t,1}$		$Z_{t,2}$	
	RMSE	MAPE (%)	RMSE	MAPE (%)
France	0.100	16.7%	0.053	28.5%
Germany	0.035	18.4%	0.002	5.0%
Greece	2.464	22.2%	0.087	28.4%
Ireland	0.100	12.3%	0.412	16.4%
Italy	0.151	13.9%	0.043	15.0%
Portugal	4.086	37.5%	0.111	45.6%
Spain	0.230	18.9%	0.025	28.0%

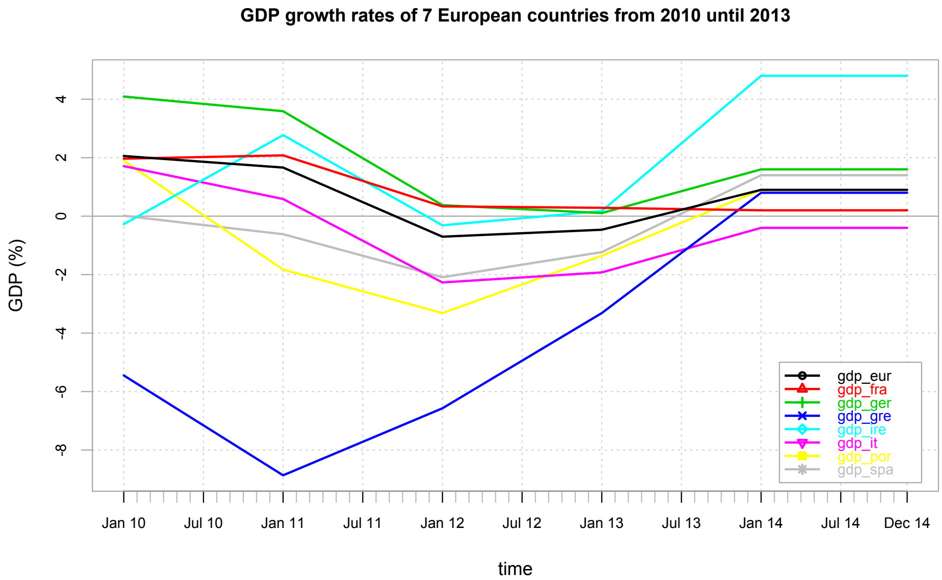
Notes: Root mean square errors (RMSE) and Mean percentage errors (MAPE) referred to the processes $Z^i_{t,1}$ and $Z^i_{t,2}$ estimated for 2014.

Figure 4.6.1: Bond interest rates



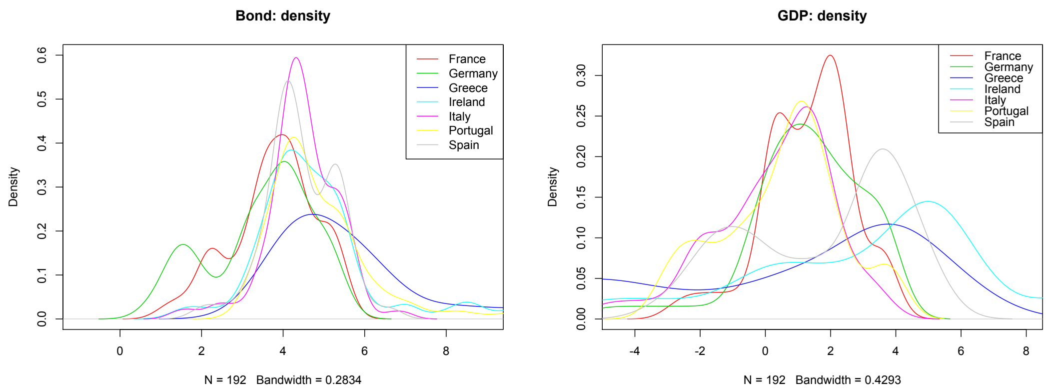
Notes: Monthly time evolution of 10-years maturity bond interest rates, from January 2010 until December 2014. Greece and Portugal present the highest values and volatility, while Germany and France have the lowest ones; Italy and Spain present similar curves, while Ireland has a peculiar behaviour, being characterised by very high values in 2011, followed by a remarkable drop during the following years.

Figure 4.6.2: GDP growth rates



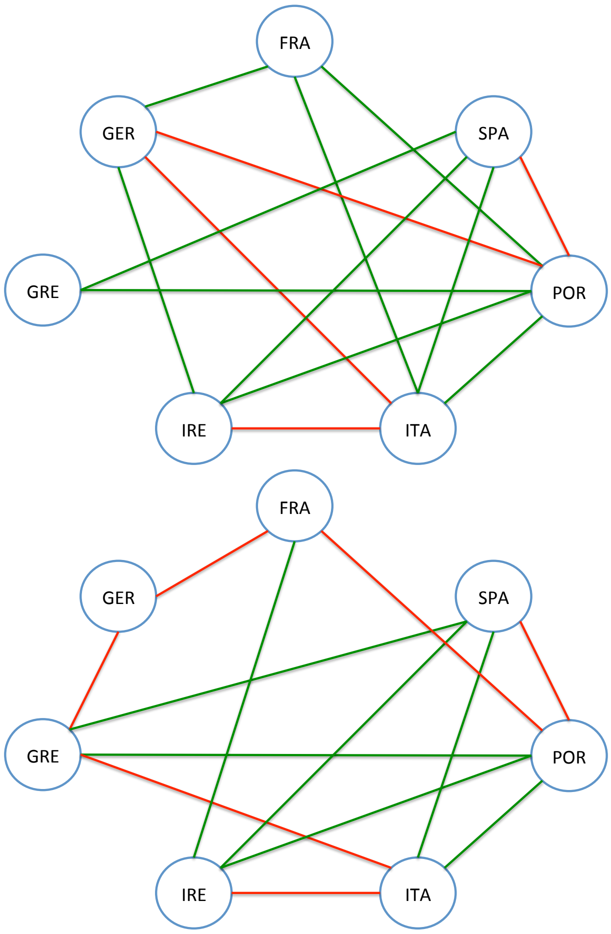
Notes: Monthly time evolution of GDP growth rates, from January 2010 until December 2014. During the first post-crisis years almost all the GDP growth rates were negative, with a strong decrease for Greece; since 2013 the trend has changed and the GDPs have started increasing again in all countries, with the exception of France. Ireland has a peculiar trend, being now characterised by the most significant increase in its GDP with respect to the other economies.

Figure 4.6.3: Density, bond interest rates and GDP growth rates



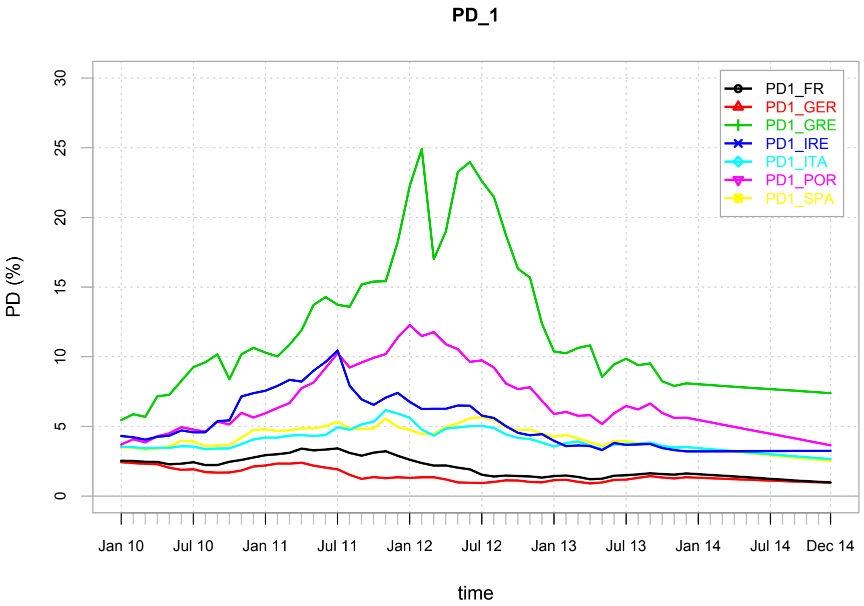
Notes: Density plots referred to the observed interest rates on government bonds (left) and the GDP growth rates (right), for the period from January 2010 until December 2014.

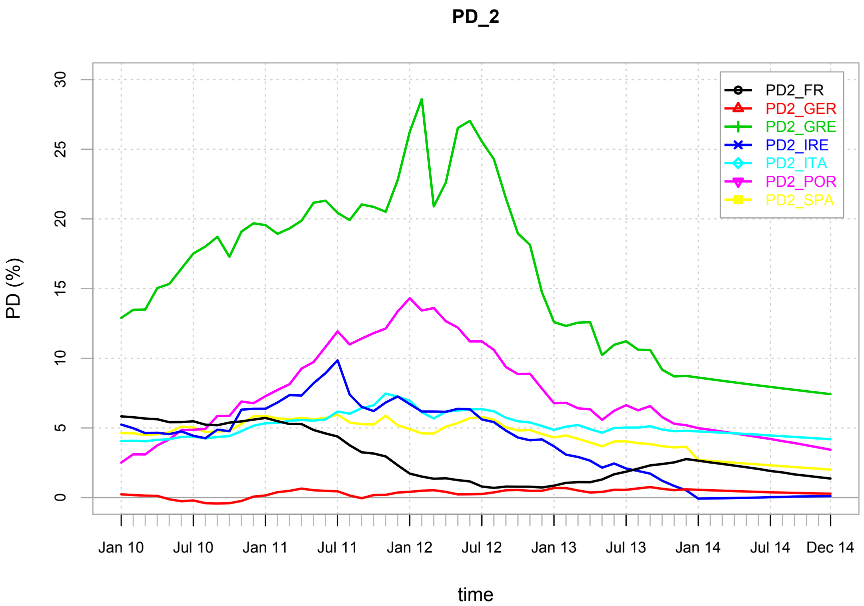
Figure 4.6.4: Networks, processes $Z_{t,1}$ and $Z_{t,1} - Z_{t,2}$



Notes: Partial correlation networks referred to the processes $Z_{t,1}$ *(top) and* $Z_{t,1} - Z_{t,2}$ *(bottom). Green lines stand for positive correlations, while red connectors indicate negative partial correlations. The comparison between the two networks show strong differences: as an example, France has completely changed its relationship between peripheral and core economies, thus suggesting that the the inclusion of a macroeconomic variable as the GDP growth rate is important and necessary in order to capture all the correlations and the direct links between countries.*

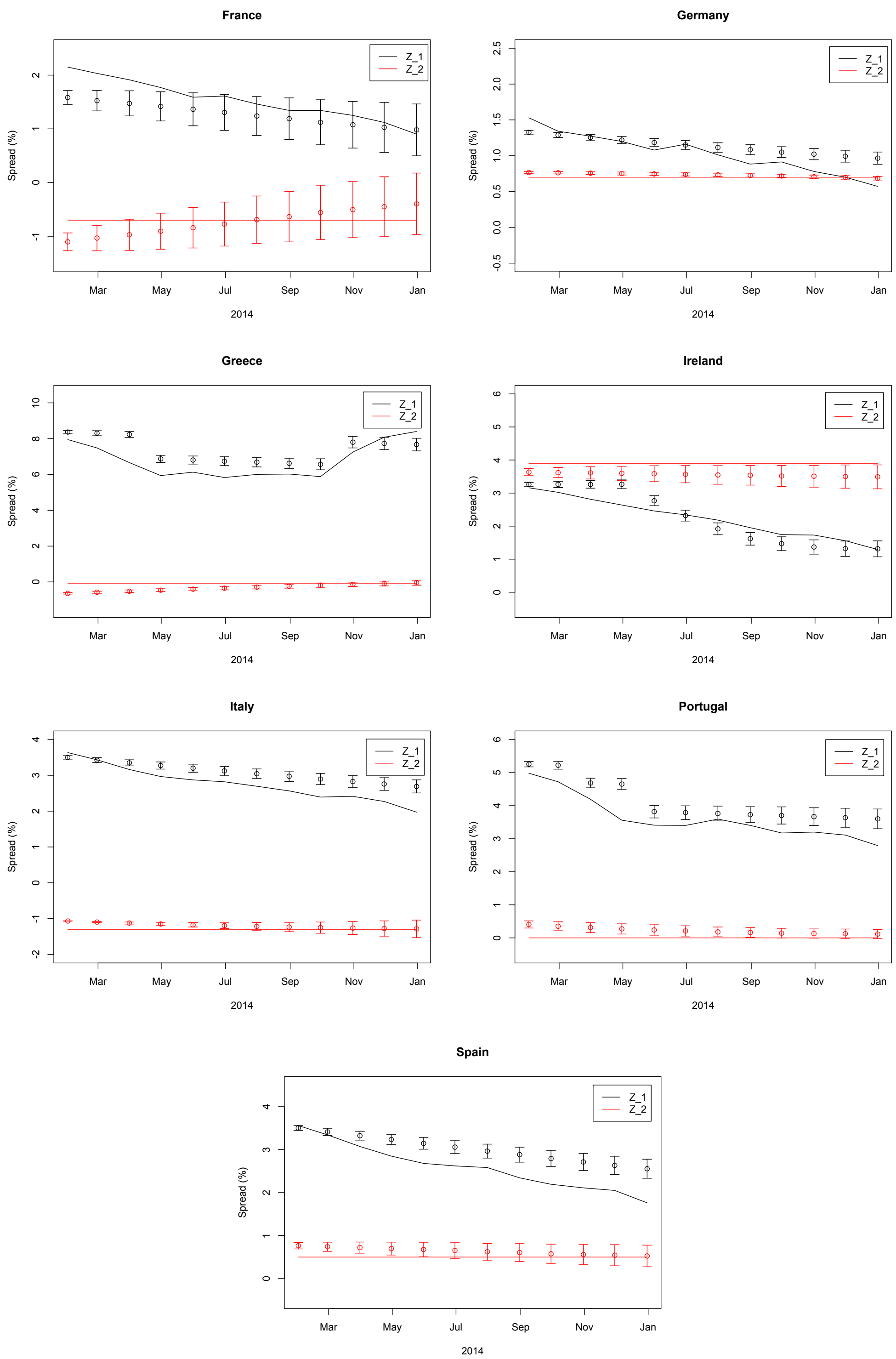
Figure 4.6.5: Default probabilities





Notes: Default probabilities from 2010 until 2014: $PD_{t,1}$ (top) and $PD_{t,2}$ (bottom). The inclusion of the GDP growth rate strongly changes the default probabilities, increasing the PDs of Greece, France, Italy, Portugal and Spain, while decreasing the PDs of Ireland and Germany.

Figure 4.6.6: Predictive performance, processes $Z_{t,1}$ and $Z_{t,2}$



Notes: Observed (solid line) and estimated (points and confidence intervals) spread measures $Z^i_{t,1}$ (spread between bond interest rates and monetary rates) and $Z^i_{t,2}$ (spread between the GDP growth rate of each country and the global Eurozone GDP growth rate) for the year 2014; the black color indicates $Z^i_{t,1}$, while the red color is used for $Z^i_{t,2}$.

Chapter 5

CoRisk: measuring systemic risk through default probability contagion[1]

Journal of Applied Econometrics[2]

Abstract

We propose a novel systemic risk measurement model based on stochastic processes, correlation networks and conditional probabilities of default. For each country we consider three different economic sectors (sovereigns, corporates, banks) and we model each of them as a linear combination of two stochastic processes: a country-specific idiosyncratic component and a common systematic factor. Through correlation networks we derive conditional default probabilities, thus obtaining the CoRisk, which measures the variation in the probability of default due to contagion effects. Our model is applied to Eurozone countries, and the results show that the sovereign crisis has increased systemic risks more than the financial one: the two events together have caused a phase transition difficult to reverse, as risk propagation does not act as a mean for balancing inequalities across countries but, on the contrary, weakens the weakest and strengthens the strongest.

Keywords: Stochastic processes, Default probabilities, Partial correlation networks, Contagion effects.

JEL: C32, C58, E43, F36.

5.1 Introduction

The financial crisis and, more recently, the sovereign crisis, have led to an increasing research literature on systemic risk, with different definitions and measurement models.

According to ECB (2009) "Systemic risk is the risk of experiencing a strong systemic event, which adversely affects a number of systemically important intermediaries or markets". This definition introduces two key elements for the study of systemic risk: as emphasised by Borio and Drehmann (2009), financial instability firstly involves the system as a whole, and not only individual institutions; secondly, it does not consider the

[1]Joint work with P. Giudici (University of Pavia).

[2]First, major revision.

financial system in isolation, but as ultimately linked to the real economy. While systemic risk definitions share this broad view and differ on implementation details, such as the involved agents, the kind of shocks or the analysed dynamics, measurement models are still quite divergent.

A first distinction between systemic risk models derives from the use of a cross-sectional rather than a time-dynamic perspective: while the former mostly concentrates on the relationships between agents operating in the market, the latter focuses on cause-and-effect relationships over time. As a consequence, we can distinguish between models centred on the notion of contagion, and models that aim at predicting what will happen in the nearby future, in an early-warning perspective. In addition, while contagion models identify transmission channels, thus embracing the whole system but only for descriptive purposes, time-dependent models associate a specific risk measure to individual institutions.

A second distinction originates in the identification of the risk sources, thus setting endogenous against exogenous causes, as well as idiosyncratic against systematic shocks.

A third diversity concerns the economic environment as the context in which systemic risk arises and propagates: most models concentrate either on the financial or the sovereign sector, while others include both of them.

In this work we will overcome these classifications by combining different approaches. First of all, for each country we will consider three different economic sectors: sovereigns, corporates and financials. Secondly, we will model each of them as a spread measure, derived as a linear combination of two stochastic processes: an idiosyncratic and a systematic factor. Doing so, we can disentangle, in an endogenous way, different sources of risk. Third, the spread will be used for two purposes: (a) to derive correlation networks, thus identifying contagion channels in a cross-sectional perspective; (b) to calculate the default probabilities associated to each economic sector in each country, in an early warning perspective. Last, we will combine default probabilities and correlation networks by deriving $CoRisk_{in}$ and $CoRisk_{out}$, two time-dependent measures which explain to what extent the default probability of each economic sector is affected by ($CoRisk_{in}$) or affects ($CoRisk_{out}$) its neighbours when contagion is included.

The described strategy allows to simultaneously assign precise risk measures to the different agents operating in the system by considering, at the same time, the system as a whole. Differently from most related papers, we will allow for both positive and negative contagion, meaning that the default probability of each agent can be decreased or increased by its relationships with other nodes. Moreover, the distinction between incoming and outgoing effects enables to decouple the identification of vulnerable, rather than systemic important economic sectors. Finally, in order to overcome the micro- vs macro-based dualism recognisable in the literature, we will derive aggregate default probabilities at the country level using a bottom-up approach. As a consequence, we will obtain a synthetic risk measure for each country, that can be disentangled in all its components according to the source (economic sector) or the kind (sector-specific or contagion) of risk, and which varies along time reacting to both idiosyncratic and systematic shocks.

5.2 An overview of systemic risk measures

As previously introduced, the study of systemic risk is particularly problematic because of the high number of dimensions that can be included: according to this choice different perspectives have been adopted and, therefore, different statistical tools have been used and applied to a great variety of data in many geographical regions and periods. For simplicity we have chosen two main discriminant factors, thus dividing models on systemic risk into three main categories: bivariate models, causal models and cross-sectional models. While the first two explicitly deal with the time-dimension in an endogenous rather than an exogenous way, the latter focuses on the cross-sectional dimension.

Bivariate Models. From a chronological viewpoint, the first systemic risk measures have been proposed for the financial sector, in particular by Acharya et al. (2010), Adrian and Brunnermeier (2011), Brownlees and Engle (2012), Acharya et al. (2012), Dumitrescu and Banulescu (2014) and Hautsch et al. (2015). On the basis of market share prices, these models consider systemic risk as endogenously determined and calculate the quantiles of the estimated loss probability distribution of a bank conditional on an extreme event in the financial market. A similar approach has been applied by Popescu and Turcu (2014) to the sovereign sector, using bond interest rates.

The above described methodology is useful to identify the most *systemically important* institutions, since its bivariate nature allows the derivation of conditional default probabilities or losses during shock events in the reference market, possibly caused by other institutions. However, it does not address the issue of how risks are transmitted between different institutions in a multivariate framework.

Causal Models. A different stream of research considers systemic risks as exogenous factors and has been proposed, among others, by Chong et al. (2006), Longstaff (2010) and Shleifer and Vishny (2010), who examined the impact of monetary policies on default probabilities for the banking sector, with a particular focus on crisis periods. More general causal models, proposed by Duffie et al. (2000), Lando and Nielsen (2010), Koopman et al. (2012), Betz et al. (2014) and Duprey et al. (2015), explain whether the default probability of a bank, a country, or a company depends on a set of exogenous risk sources, thus combining idiosyncratic and systematic factors. A further evolution has been proposed, among others, by Bartram et al. (2007), Ang and Longstaff (2012) and Brownlees et al. (2014): they combine idiosyncratic and systematic sources of distress through endogenous models expressed in terms of univariate stochastic processes.

While powerful from an early warning perspective, causal models, similarly to bivariate ones, concentrate on single institutions rather than on the economic system as a whole and, therefore, underestimate systemic sources of risk arising from contagion effects within the system.

Cross-sectional Models. In order to address the multivariate nature of systemic risk, researchers have recently proposed correlation network models, able to combine the rich structure of financial networks (see, e.g., Lorenz et al., 2009; Battiston et al., 2012) with a parsimonious approach based on the dependence structure among market prices. The first contributions in this framework are Billio et al. (2012) and Diebold and Yilmaz (2014), who derive connectedness measures based on Granger-causality tests and variance decompositions. Barigozzi and Brownlees (2013), Ahelegbey et al. (2015) and Giudici and Spelta (2015) extend such methodology by introducing stochastic graphical models, while Das (2015) derives a systemic risk decomposition into individual and network contributions.

Correlation network models are very useful for identifying the most important contagion channels in a cross-sectional perspective, thus identifying the most *vulnerable* institutions. However, since they are built on cross-sectional data, they can not be used as predictive models in a time-varying context. Moreover, the importance of each institution only depends on its position in the graph, and not on its specific risk.

A new combined approach. Bivariate and causal models explain whether the risk of a bank, a company, or of a country, is affected by a market crisis event or by a set of exogenous risk factors; correlation network models explain whether the same risk depends on contagion effects. We improve all these three classes of models by introducing multivariate stochastic processes and by combining them with correlation networks: doing so, we merge the advantages of bivariate models (endogeneity and non-linearity), causal models (predictive capability) and correlation networks (contagion channels). To achieve our aim, we significantly extend the approach by Ang and Longstaff (2012) and Brownlees et al. (2014), by employing a correlated set of linear combinations of two stochastic processes (a systematic and an idiosyncratic one) rather than a single process, and by applying it to three rather than one economic sector.

In more detail, we first select three risk measures from publicly available data: (a) the spread between the cost of debt for countries (interest rates on 10-years maturity government bonds) and a benchmark rate, which gives a measure of sovereign risk; (b) the spread between the cost of debt for corporates (aggregate interest rates on bank lendings to non-financial corporates) and a benchmark rate, which gives a measure of corporate risk; (c) the spread between the funding cost of the banking system (aggregate interest rates on deposits of non-financial corporates and households) and a benchmark rate, which gives a measure of bank risk. We define three stochastic processes on the three risk measures so that, on the basis of the estimated parameters, a probability of default can be calculated, for each economic sector and within each country, independently from the others. We then estimate a correlation network model based on the estimated partial correlations between the risk measures: by so doing, we simultaneously consider both the cross-sectional and the time perspectives. In addition, we derive default probabilities conditionally on the estimated network. The difference between such conditional probabilities and the unconditional ones can be employed to assess the effect of systemic contagion: the resulting measure will be named *CoRisk*. We propose two different kinds of *CoRisk*: $CoRisk_{in}$, which measures how an economic sector is influenced by the default probability of its neighbours, thus providing a measure of its *vulnerability* (as in bivariate and econometric causal models); $CoRisk_{out}$, which measures to what extent each economic sector influences its neighbours, thus providing a measure of its *systemic importance* (as in cross-sectional models). Furthermore, since conditional default probabilities can be aggregated at the country level, we obtain a country specific default probability that can be disentangled according to all the dimensions introduced so far: time, economic sector (sovereign, corporate and bank), kind of risk (sector-specific and contagion), or source of risk (idiosyncratic and systematic).

We remark that a multivariate approach related to ours has been suggested by Gray et al. (2013), Ramsay and Sarlin (2015) and Schwaab et al. (2015). We extend these contributions by taking endogeneity into account as well as by using a proper probability metric, thus making explicit what suggested in Das (2015): a measure of systemic risk that can be decomposed in an individual node and a network component. Other similar approaches have been recently proposed by Mezei and Sarlin (2015) and Betz et al. (2016): the former define an aggregation operator in order to jointly estimate the importance of each single node as well as contagion effects deriving from links with other nodes; the latter develop a tail risk analysis of networks in order to build a robust set of regressors for defining systemic contributions. We improve both approaches by calculating node default probabilities for three different economic sectors in each country and by deriving link measures of contagion through partial correlations between linear combinations of stochastic processes: in such a way we can (a) allow for non-linear effects through stochastic differential equations, (b) allow for contagion effects not only between, but also within each country, and (c) disentangle the idiosyncratic and the systematic, as well as the sector-specific and the systemic components for the three economic sectors in each country. In addition, our *CoRisk* measure is allowed to be both positive or negative, meaning that the resulting default probability of each economic sector or country can be increased or decreased according to the sign of partial correlations. From an economic viewpoint, when a country is negatively related to troubled countries, its final default probability decreases because it is perceived as a flight-to-quality haven, meaning that it is positively affected by contagion effects. On the contrary, when countries are positively connected to troubled economies their default probability increases because they suffer negative contagion. Such a distinction between positive and negative contagion, to our knowledge, only appears in Grinis (2015).

Our proposed model will be applied to data from Eurozone countries in the recent time period. For descriptive purposes, we have identified four crucial time windows and we will show correlation networks and risk distributions in each of them: the pre-crisis period (2003-2006), the financial-crisis period (2007-2009), the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015).

Our main economic findings can be summarised according to three dimensions: (a) the economic sector dimension, (b) the country dimension and (c) the time dimension at the aggregate country level. Concerning (a), the corporate sector is strongly influenced by the systematic component, and this explains why it reacts to monetary policy changes more than sovereigns and banks. On the other hand, the sovereign and bank sectors have deeply suffered, respectively, the sovereign and the financial crisis, and they seem to behave quite similarly, thus confirming their "diabolic loop". In the last period, correlation networks show the creation of two distinct clusters, characterised by positive within and negative cross correlations, that clearly separate peripheral and core economies: such separation creates loop effects within each cluster, further alienating troubled and strong economies. Concerning (b), peripheral countries mostly behave as exporters, rather than importers of system risk: as a consequence, core economies are mostly affected by contagion risk, while peripheral countries strongly suffer high sector-specific default probabilities. Concerning (c), the sovereign crisis has had a larger impact on systemic risk with respect to the financial crisis. A possible explanation consists in different ways peripheral and core economies reacted to the financial crisis: peripheral countries, with high public debts, had little fiscal space to improve balance sheets and, therefore, the financial crisis triggered their imbalances to emerge in the subsequent sovereign crisis. However, the sequence of these two events has determined an irreversible phase transition, leading to a new non-stable and non-optimal equilibrium, where instability derives from peripheral-countries trajectories diverging from core ones. This conclusion is further confirmed by the time evolution of risk distributions across Eurozone countries and by the role of risk propagation, which does not act as a mean for balancing inequalities across countries but, on the contrary, weakens the weakest and strengthens the strongest ones.

The paper is structured as follows: Sections 5.3 and 5.4 describe the proposed models, with Section 5.3 introducing multivariate linear combinations of interest rate spreads and correlation networks and Section 5.4 defining default probabilities and $CoRisk$. Section 5.5 presents data and the empirical evidence obtained from multivariate stochastic processes and correlation networks, while Section 5.6 shows the obtained partial correlation networks (Section 5.6.1), default probabilities and $CoRisk$ (Section 5.6.2) and default probabilities aggregated at the country level (Section 5.6.3). The comparison between systemic importance and vulnerability is described in Section 5.6.4, while Sections 5.6.5 and 5.6.6 describe the comparison between $CoRisk$ and, respectively, other centrality measures and $\Delta CoVar$. Finally, Section 5.7 concludes with some closing remarks.

5.3 Multivariate spread processes

Consider $i = 1, ..., N$ countries which, in a first stage, have only one economic sector. We assume that the time dynamics of the liability side of each country is expressed by the evolution of the associated interest rate, which can be described by a linear combination of two stochastic processes: a common systematic component and an idiosyncratic factor. More formally, for each country $i = 1, \dots, N$:

$$Z_t^i = y_t^i - S_t, \tag{5.3.1}$$

where S_t stands for the systematic process, while y_t^i represents the idiosyncratic process referred to country i; the complete process Z_t^i describes the resulting time evolution of interest spreads. From an economic viewpoint, the above formulation expresses Z_t^i as the difference between the cost of a long term debt and the cost of liquidity.

Both the systematic and the idiosyncratic processes can be modelled as CIR processes (Cox et al., 1985), as follows:

$$\begin{cases} \mathrm{d}\, S_t = (a - vS_{t-1})\,\mathrm{d}\, t + b\sqrt{S_{t-1}}\,\mathrm{d}\, B_t, \\ \mathrm{d}\, y_t^i = (\theta_1^i - \theta_2^i y_{t-1}^i)\,\mathrm{d}\, t + \theta_3^i\sqrt{y_{t-1}^i}\,\mathrm{d}\, W_t, \end{cases} \tag{5.3.2}$$

where $\mathrm{d}\, B_t$ and $\mathrm{d}\, W_t$ are two independent Brownian motions.

We then assume the following correlation structure:

$$\begin{cases} \mathrm{Corr}[y_t^i, y_t^j] = \rho^{ij}, \\ \mathrm{Corr}[S_t, y_t^j] = \gamma^j. \end{cases} \tag{5.3.3}$$

Note that the first equation in (5.3.3) is consistent with the assumptions used in the formulation of multidimensional CIR processes (see e.g. Kalogeropoulos et al., 2011); the second one introduces a correlation between each idiosyncratic process and the systematic process S_t.

The model proposed in (5.3.1)-(5.3.3) defines a multivariate stochastic process able: (a) to capture both the systematic and the idiosyncratic components that may affect interest rate spread dynamics, using linear combinations of stochastic processes; (b) to model the correlation structure of interest rate spreads across different countries.

To exploit (b) we now derive the instantaneous covariance matrix corresponding to our proposed model. First define:

$$P = \begin{bmatrix} 1 & \rho^{12} & \dots & \rho^{1N} \\ \rho^{21} & 1 & \dots & \rho^{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{N1} & \rho^{N2} & \dots & 1 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} \gamma^1 \\ \vdots \\ \gamma^i \\ \vdots \\ \gamma^N \end{bmatrix}, \tag{5.3.4}$$

where each element in P is the correlation coefficient between the idiosyncratic processes of any two countries, while each element of Γ is the correlation coefficient between any idiosyncratic process and the systematic process, as defined in (5.3.3). Let A be the instantaneous covariance matrix of the spread vector $Z = (Z^1, \dots, Z^N)$. A can be shown to be as follows:

$$A = \Phi \cdot \Theta^T, \tag{5.3.5}$$

where each vector of the matrices Φ and Θ^T is equal to:

$$[\Phi]^i = \left[b\sqrt{S_0}, \quad 1, \quad \sqrt{S_0 y_0^i} b\theta_3^i [\Gamma]^i, \quad \sqrt{y_0^i}\theta_3^i \sqrt{[P]^i} \right],$$

$$[\Theta^T]^j = \begin{bmatrix} b\sqrt{S_0} \\ \\ \sqrt{S_0 y_0^j} b\theta_3^j [\Gamma]^j \\ \\ 1 \\ \\ \sqrt{y_0^j}\theta_3^j \sqrt{[P]^j} \end{bmatrix}.$$

The parameters of the proposed process can be estimated by extending results available for univariate stochastic processes (see e.g. Iacus, 2008), and based on the maximisation of the log-likelihood function. Through the application of the invariance principle of maximum likelihood estimators, we can compute the covariance matrix A on which networks are based. Before that, we extend the methodology proposed so far to the more realistic multi-sector situation.

For each country, let us consider the aggregate financial liabilities of sovereigns, (non-financial) corporates and banks as the idiosyncratic components in (5.3.1). Formally, by denoting the three economic sectors respectively with $\{1,2,3\}$, for each country $i = 1,\ldots,N$ equation (5.3.1) becomes the following system:

$$\begin{cases} Z^i_{t,1} = y^i_{t,1} - S_t, \\ Z^i_{t,2} = y^i_{t,2} - S_t, \\ Z^i_{t,3} = y^i_{t,3} - S_t. \end{cases} \tag{5.3.6}$$

In (5.3.6) the systematic component S_t as well as the idiosyncratic factors $y_{t,\{1,2,3\}}$ follow a CIR process:

$$\begin{cases} \mathrm{d}\, S_t = (a - vS_{t-1})\,\mathrm{d}\, t + b\sqrt{S_{t-1}}\,\mathrm{d}\, B_t, \\ \mathrm{d}\, y^i_{t,\{1,2,3\}} = [(\theta_1)^i_{\{1,2,3\}} - (\theta_2)^i_{\{1,2,3\}} y^i_{t-1,\{1,2,3\}}]\,\mathrm{d}\, t + (\theta_3)^i_{\{1,2,3\}}\sqrt{y^i_{t-1,\{1,2,3\}}}\,\mathrm{d}\, W_t. \end{cases} \tag{5.3.7}$$

We then assume the following correlation structure:

$$\begin{cases} \mathrm{Corr}[y^m_t; y^n_t] = \rho^{mn}, \\ \mathrm{Corr}[S_t; y^m_t;] = \gamma^m, \end{cases} \tag{5.3.8}$$

where $\{m,n\} \in (V \times W)$, with $V = \{1,...,N\}$ denoting countries and $W = \{1,2,3\}$ economic sectors.

The new model in (5.3.6)-(5.3.8) defines a general multivariate stochastic process able: (a) to capture both the systematic and the sector-specific idiosyncratic components that may affect interest rate spread dynamics, using linear combinations of stochastic processes; (b) to model the correlation structure of interest rate spreads across different countries and different sectors. Note that the instantaneous covariance matrix of the new process turns out to be the same as that in (5.3.5), albeit with a different dimensionality, being a $3N \times 3N$ rather than a $N \times N$ matrix.

Once the covariance matrix A has been estimated (as previously discussed), it can be employed to calculate correlation coefficients and, consistently, correlation networks between countries and economic sectors (following Billio, 2012; Ahelegbey et al., 2015; Giudici and Spelta, 2015). However, such correlations can be misleading because they take into account bivariate (marginal) relationships between interest spreads. For this reason we propose to employ conditional (partial) correlations, different from bivariate ones as they are adjusted by the presence of all the other variables in the system. Let A^{-1} be the inverse of the covariance matrix, with elements a^{mn}. The partial correlation coefficient ρ_{mnVW} between variables Z^m and Z^n, conditional on the remaining variables in $V \times W$, can be obtained as:

$$\rho_{mnVW} = \frac{-a^{mn}}{\sqrt{a^{mm}a^{nn}}}. \tag{5.3.9}$$

In order to better explain partial correlations and their differences with respect to marginal ones, we now report a useful and interesting property. For $\{m, n\} \in (V \times W)$, set $S = (V \times W) \setminus \{m, n\}$ and suppose to express the dependence between spread measures through multiple linear models in the following way:

$$\begin{cases} Z^m = a^m + \sum_{n \neq m} a_{mn|S} Z^n; \\ Z^n = a^n + \sum_{m \neq n} a_{nm|S} Z^m. \end{cases} \tag{5.3.10}$$

It can be shown that the partial correlation coefficient between Z^m and Z^n, given all the other $3N - 2$ spread measures, can be interpreted as the geometric average between the multiple linear coefficients in (5.3.10):

$$|\rho_{mn|S}| = |\rho_{nm|S}| = \sqrt{a_{mn|S} \cdot a_{nm|S}}. \tag{5.3.11}$$

Note that in case of only two components ($S = \varnothing$), equation (5.3.10) becomes:

$$\begin{cases} Z^m = a_m + a_{mn} Z^n \\ Z^n = a_n + a_{nm} Z^m, \end{cases} \tag{5.3.12}$$

from which the marginal correlation coefficient ρ_{mn} can be derived as the geometric average between the coefficients in (5.3.12):

$$|\rho_{mn}| = |\rho_{nm}| = \sqrt{a_{mn} \cdot a_{nm}}.$$

We propose to build a correlation network based on partial correlations rather than on marginal correlations. To achieve this aim we introduce an undirected graph $G = (P, E)$, with a vertex set $P = V \times W = \{1, ..., 3N\}$ and an edge set $E = P \times P$. Such edge set is defined by binary elements e_{mn} that describe whether pairs of vertices are (symmetrically) linked to each other ($e_{mn} = 1$) or not ($e_{mn} = 0$), depending on whether the partial correlation coefficient between the corresponding pair of variables is different from or equal to zero.

5.4 Default probabilities and CoRisk

For each node $m \in V \times W$, a sector-specific probability of default, PD_t^m, can be obtained by considering the expected dynamic of debt:

$$D_{t+1}^m = (1 - PD_t^m)e^{y_t^m} D_t^m, \tag{5.4.1}$$

where D_{t+1}^m (D_t^m) is the total debt at time $t+1$ (t). Note that the analogous dynamic of a risk-free debt is the following:

$$D_{t+1}^m = e^{S_t} D_t^m. \tag{5.4.2}$$

If we (reasonably) assume to be in an arbitrage-free context, we can equate (5.4.1) and (5.4.2), thus obtaining PD_t^m:

$$PD_t^m = 1 - e^{-Z_t^m}. \tag{5.4.3}$$

From (5.4.3), a decrease in Z_t^m implies a decrease in the probability of default, consistently with the definition of the process Z_t^m as an interest rate spread.

The probability of default derived in (5.4.3) is sector-specific, as it is assumed independent from the default probability of other institutions: in our view this is an unrealistic assumption, since economic sectors of different countries depend on each other, as well as the default probability of each country depends on all its three economic sectors.

We thus propose to evolve the PD into a total default probability, TPD, able to incorporate both sector-specific and contagion components. To ease the exposition, in this Section we will propose an economically intuitive approach: a complete mathematical treatment is provided in Appendix D.

Let us assume to have a "global" spread process $\tilde{Z}^m$, expressed as a linear function of the "baseline" spread $Z_t^m = -ln(1 - PD_t^m)$, which depends exclusively on m, and of a further component which depends on the spread measures Z_t^n of the other nodes $n \neq m$:

$$\tilde{Z}_{t+1}^m = Z_t^m + \sum_{n \neq m} a_{mn|S} Z_t^n. \tag{5.4.4}$$

Assuming that the total default probability TPD can be expressed as a function of $\tilde{Z}$ as in (5.4.3), by replacing Z_t^m in (5.4.3) with the right hand side of equation (5.4.4), and by substituting the coefficients $a_{mn|S}$ with their geometric averages $\rho_{mn|S}$ (obtained from the estimated process parameters) according to (5.3.11), we derive a new expression for the probability of default conditional on non-defaulted neighbours n in the previous time, that we name TPD:

$$TPD_{t+1}^{m} = 1 - (1 - PD_t^m) \cdot \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn|S}}. \tag{5.4.5}$$

From (5.4.5), we can define the incoming contagion effect ($CoRisk_{in}$), conditional on non-defaulted neighbours in the previous time, as the TPD component that strictly depends on neighbours $n \neq m$:

$$CoRisk_{in,t}^{m} = 1 - \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn|S}}. \tag{5.4.6}$$

For each agent m, $CoRisk_{in}$ is an increasing function of both PD^n (default probability of neighbours) and $\rho_{mn|S}$ (partial correlations). In other words, the worse the nodes to which m is more connected, the worse the default probability of m itself.

By combining (5.4.5) with (5.4.6) and by assuming $TPD_{t+1}^m > 0$ (a rather obvious request), $CoRisk_{in}$ can be interpreted as the percentage variation of the survival probability due to contagion:

$$CoRisk_{in,t}^{m} = \frac{(1 - PD_t^m) - (1 - TPD_{t+1}^m)}{1 - PD_t^m}. \tag{5.4.7}$$

Economically, $CoRisk_{in}$ measures the change in the survival probability of an agent m when potential contagion deriving from all other agents is included. According to equations (5.4.5) and (5.4.7), the total default probability TPD can be either greater or lower than PD depending on the sign of the $CoRisk_{in}$ measure: more precisely, if $CoRisk_{in} > (<) 0$, the default probability of node m increases (decreases) after the inclusion of contagion effects. This distinction comes from considering partial correlations as signed numbers rather than in absolute value, thus allowing for "beneficial" or "adverse" effects. As a consequence we will obtain *negative contagion* when an institution m is disadvantaged by positive links with its neighbours ($CoRisk_{in} > 0$ and $TPD > PD$), while *positive contagion* will occur if institution m takes advantage of negative links with its neighbours ($CoRisk_{in} < 0$ and $TPD < PD$).

In order to define outgoing contagion effects, we can calculate to what extent agent m affects its neighbours. Formally, we can define $CoRisk_{out}$, conditional on not having defaulted in the previous time, as follows:

$$CoRisk^m_{out,t} = 1 - \prod_{n \neq m} (1 - PD^m_t)^{\rho_{nm|S}} = 1 - (1 - PD^m_t)^{\sum_{n \neq m} \rho_{nm|S}}. \tag{5.4.8}$$

Note that the two definitions (5.4.6) and (5.4.8) introduce asymmetries in the model: even if the graph is undirected and, thus, symmetric, the incoming and outgoing contagion effects are different, since each node is associated to a different default probability and, consequently, its contagion effect towards its neighbours is different from the effect it receives from them.

This distinction allows us to disjointly calculate, for each agent, its vulnerability ($CoRisk_{in}$) and its systemic importance ($CoRisk_{out}$). If the two measures coincide, the default probability of node m is equal to the geometric average of the default probabilities of its neighbours: on the contrary, if $CoRisk^m_{out} > CoRisk^m_{in}$ ($<$), the default probability of node m is greater (lower) than the geometric average of the default probabilities of its neighbours, meaning that its systemic importance is greater (lower) than its vulnerability.

As an example, consider the graphs in Figure 5.4.1, where each node is associated to its sector-specific PD and each pair of nodes is associated to the corresponding partial correlation coefficient $\rho_{mn|S}$.

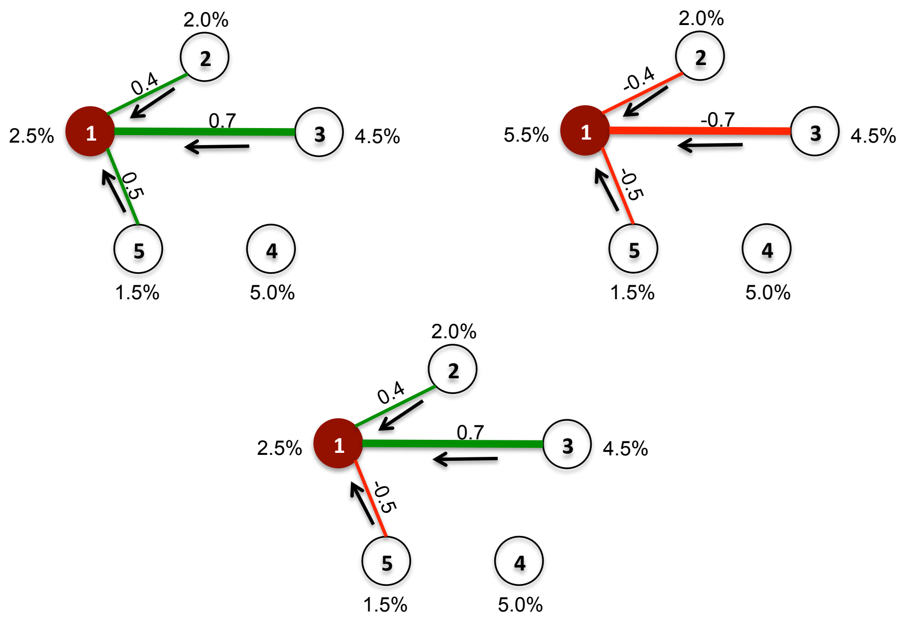


Figure 5.4.1: $CoRisk_{in}$, an illustrative example

In the first case (all positive correlations) the final $CoRisk_{in}$ value is 0.047, meaning that contagion has decreased the survival probability of node 1 by 4.7%, bringing its default probability from $PD^1 = 2.9\%$ to $TPD^1 = 7.2\%$ (negative contagion). In the second example, instead, all the correlation coefficients are negative, and the calculated $CoRisk_{in}$ becomes -0.049, meaning that contagion has increased the survival probability of node 1 by 4.9% (positive contagion). According to equation (5.4.5), the total TPD^1 has decreased, being equal to 0.87%. Note that in this second example the $CoRisk_{in}$ measure is not equal, in absolute value, to the one obtained in the previous example: this because the exponent ρ introduces non-linear effects in the relationship (5.4.6). In the last example, where both positive and negative correlations appear, the calculated $CoRisk_{in}$ measure is equal to 0.032, meaning that contagion has decreased the survival probability of node 1 by 3.2%, reaching a total default probability $TPD^1 = 5.6\%$ (overall negative contagion).

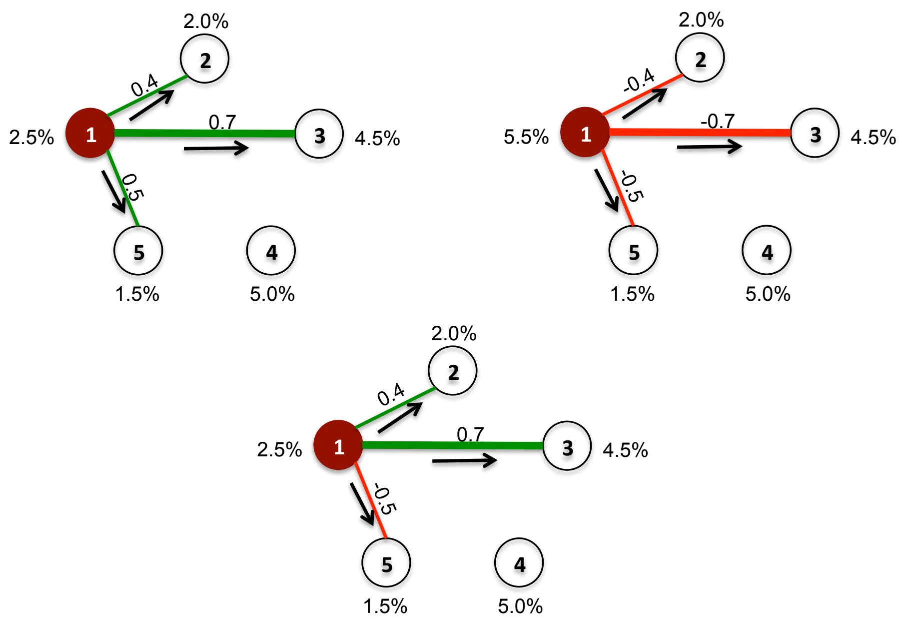


Figure 5.4.2: $CoRisk_{out}$, an illustrative example

Figure 5.4.2 reports the same graphs as in Figure 5.4.1, but we now concentrate on the outgoing effects in order to understand how node 1 affects its neighbours. In the first example, the overall $CoRisk_{out}$ is equal to 0.040: this result is lower with respect to the $CoRisk_{in}$ value because the incoming contagion is highly affected by the large default probability of node 3. Similarly, in the second situation the final $CoRisk_{out}$ is -0.095. This result is lower than the corresponding $CoRisk_{in}$ because, now, the default probability of node 1 is much bigger than the default probabilities of its neighbours: consequently, the contagion effect due to negative correlations is amplified, meaning that a negative relation with node 1 strongly decreases the default probability of the set $ne(1)$. In the last example the calculated $CoRisk_{out}$ measure is equal to 0.015, lower than $CoRisk_{in}$ as in the first example.

The total default probabilities introduced in (5.4.5) are defined for each economic sector within each country. However, it is important to calculate the total default probability of an entire country, obtained by aggregating the default probabilities of its economic sectors. To derive such probability we assume that a country will default if at least one of its economic sectors defaults.

Thus, denoting with $A^{3,i}$, $A^{1,i}$ and $A^{2,i}$ the sets of defaults for, respectively, the sovereign, corporate and bank sectors of country i, we are interested in deriving $P(\bigcup_{j\in W} A^{j,i}|S^i)$, where $S^i = \{A^m; \forall m \in V \times W, m \in ne(i,j), m \neq (i,j)\}$. It can be shown that such a probability, named $TPD^i_{country,t}$, is equal to:

$$TPD^i_{country,t} = 1 - \prod_{\substack{j,j'=1 \\ j'>j}}^{3} (1 - TPD^{j,i}_t|\bar{A}^{j'}), \tag{5.4.9}$$

where the three probabilities are the $TPDs$ derived through (5.4.5) by considering, respectively, all the other nodes, all the other nodes but the corporate sector of country i, all the other nodes but the corporate and bank sectors of country i.

5.5 Data

We focus on eleven european countries: Austria, Belgium, Finland, France, Germany and the Netherlands (core countries); Greece, Ireland, Italy, Portugal and Spain (peripheral countries). For each country and economic sector we consider the aggregate funding costs as idiosyncratic components for modelling sovereign, corporate and bank risk: (a) interest rates on 10-year maturity government bonds, (b) aggregate interest rates on bank loans to non-financial corporates, (c) aggregate interest rates on bank deposits from non-financial corporates and households. Concerning the common systematic component, there are many choices for a benchmark rate: we suggest a rate that reflects the impact of the European Central Bank monetary policy, such as the 3-months Euribor. All data are publicly available and have been selected with monthly frequencies.

Summary statistics are shown in Tables 5.10.1 and 5.10.2. In order to better describe the country-specific, sector-specific and time-evolution components of the resulting $N = 11 \times 3$ -dimensional system of interest rate spreads, data have been grouped in four different time windows: (a) the pre-crisis period (2003-2006), (b) the financial crisis period (2007-

2009), (c) the sovereign crisis period (2010-2012) and (d) the post-crisis period (2013-2015). For each of them means, standard deviations as well as correlations with Euribor interest rates are reported.

[Tables 5.10.1, 5.10.2]

From Tables 5.10.1 and 5.10.2 note that interest rates on loans have the highest correlation coefficients with Euribor interest rates, during all time-windows and in almost all countries. The same correlations referred to interest rates on government bonds vary over time: low during the pre-crisis period and higher afterwards (with the exception of Greece). The correlations of bank interest rates with the Euribor follow a similar pattern, being very low until 2012 in almost all countries, and strongly positive afterwards.

The time evolution of the interest rate processes for the sovereign sector can be observed in Figure 5.10.1.

[Figure 5.10.1]

Figure 5.10.1 shows that interest rates on government bonds were initially very similar, while in 2010 they started diverging: decreasing in core countries and increasing in peripheral countries. Greece, Ireland and Portugal present the highest volatility, especially during their sovereign crisis in 2010-2011, followed by Italy and Spain and, to a lesser extent, Belgium.

The time evolution of the interest rate processes for the corporate sector can be observed in Figure 5.10.2.

[Figure 5.10.2]

From Figure 5.10.2 note that interest rates on loans to non-financial corporates differ across the main european countries. In particular, Greece and Portugal have the highest values while Finland and Austria present the lowest ones. The interest curves of corporates do not show substantial overlaps: they all increase during the financial crisis of 2008 and, to a lesser extent, during the sovereign crisis of 2011. All rates show positive correlations with the Euribor dynamics. Overall, the scale of variation of corporate rates is much smaller than that of sovereign rates, especially in peripheral countries.

The time evolution of the interest rate processes for the bank sector can be observed in Figure 5.10.3.

[Figure 5.10.3]

Figure 5.10.3 shows an interest rate pattern substantially different with respect to sovereigns and non-financial corporates. The highest rates occur in France, Belgium and the Netherlands consistently through time, while the curves of the other countries do overlap: this is especially true for peripheral countries, affected not only by the financial crisis but also by the sovereign crisis. Overall, the scale of variation of bank rates is slightly lower than that observed for corporates.

5.6 Empirical Results

The first step in model estimation consists in deriving the coefficients of the stochastic processes in (5.3.2), for the two components (idiosyncratic and systematic) of each economic sector and country: such results are reported in Tables 5.10.3, 5.10.4 and 5.10.5. Tables 5.10.4 and 5.10.5 show that, during the two crisis periods, all the parameters (drift and volatility) are sensibly higher in peripheral countries than in core countries. In the post-crisis period, however, drift terms return to their initial values (with the exception of Greece), while volatilities remain quite high.

5.6.1 Correlation Networks

Our aim is now to derive the correlation network models obtained by calculating partial correlations as in (5.3.9), for sovereigns, corporates and banks. To achieve this aim it is necessary to calculate, within each sector j, the 11×11 inverse correlation matrix of the spreads $Z^i_{t,j}$ for each time period t. To better interpret the results, we only show the most significant correlations: in particular, a connection between two countries will be kept or dropped on the basis of a correlation *t-tests* based on $\alpha = 0.10$. Moreover, for explanatory purposes we will just show networks referred to the four time windows previously identified. Such correlations are depicted in Figure 5.10.4: green lines stand for positive partial correlations, while red lines indicate negative partial correlations; the ticker the line, the stronger the connection.

[Figure 5.10.4]

Comparing the sovereign correlation networks in Figure 5.10.4, note that their pattern has substantially changed over the years: in the pre-crisis period the overall number of significant partial correlations is quite high; during the financial crisis they decrease; during the sovereign crisis they further decrease and a "clustering effect" that separates core and peripheral economies in two quite distinct subgraphs emerges. Last, in the post-crisis period the partial correlation pattern returns to the pre-crisis situation, however with a persisting clustering effect, emphasised not only by positive within subgraph correlations, but also by negative ones across the two subgraphs.

Bank correlation networks, similarly to sovereign ones, are quite connected in the first two periods, and become sparser afterwards. In this case, the clustering effect becomes evident in the last, rather than in the third period. This time delay may also be due to the different kind of data used for banks with respect to sovereigns: the latter are market-based data, characterised by quick reactions to the economic perspectives of a country; the former, instead, depend upon banks' decisions and are characterised by a degree of viscosity with respect to the external environment.

By analysing the corporate correlation networks in Figure 5.10.4, a substantial change over time in the partial correlation pattern emerges again, further underlining the importance of the dynamic perspective. During the pre-crisis period the overall number of significant correlations is quite high, similarly to sovereign and bank ones. During the financial crisis such number substantially decreases; during the sovereign crisis significant correlations increase again, and they drop in the last period, characterised by low growth and close-to-zero Euribor interest rates. Differently from what observed in the other two economic sectors, a clustering effect between core and peripheral countries is not evident: a possible explanation is that corporate interest rates are highly and constantly correlated with Euribor rates across time and, thus, clustering effects become less significant while the systematic component, affected by monetary policies, becomes the most important risk driver.

5.6.2 Default probabilities and contagion

Having estimated all the parameters, as well as partial correlations, we are now able to calculate the sector-specific and time-dependent probabilities of default of each sovereign ($PD^i_{t,1}$), corporate ($PD^i_{t,2}$) and bank ($PD^i_{t,3}$) sector in each country i, based respectively on the spread measures $Z^i_{t,1}$, $Z^i_{t,2}$ and $Z^i_{t,3}$ according to equation (4.2.20). Using such PDs and the estimated partial correlations, we can thus calculate the total default probability of each economic sector in each country $TPD^i_{t,\{1,2,3\}}$ as in (5.4.5) and, by comparing them with the sector specific default probabilities, we can obtain the $CoRisk$ measures.

Summary statistics of $CoRisk_{in}$ during the four different time windows are shown in Tables 5.10.6 and 5.10.7. Their corresponding time evolutions, together with default probabilities PD and TPD, are shown in Figures 5.10.5 (sovereign sector), 5.10.6 (corporate sector) and 5.10.7 (banking sector).

[Tables 5.10.6, 5.10.7]
[Figures 5.10.5, 5.10.6, 5.10.7]

Let us first consider the results referred to the sovereign sector, obtained by jointly reading Tables 5.10.6 and 5.10.7 together with Figure 5.10.5. By looking at the single sector-specific PDs (top graphs), it is clear that Greece presents the most critical situation, with the highest PD values. Portugal has similar, but lower results. Ireland presents an anticipated increase in its default probability because of its deep sovereign crisis in 2011, but in the following years it starts performing quite well until reaching very low PD values in 2015. Italy and Spain show similar intermediate values, while core countries behave quite similarly to each other, with the lowest PDs across time.

The $CoRisk_{in}$ pattern (middle graphs) can be understood by looking at the networks in Figure 5.10.4: countries with high positive correlations with peripheral economies, characterised by high PDs, have a high $CoRisk_{in}$: this is the case, for example, of France and Belgium in the second period, strongly connected, respectively, to Italy and Portugal, and to Italy and Spain. Similarly, Spain presents a high $CoRisk_{in}$ during the sovereign crisis period, due to its strong positive link with Ireland, a particularly troubled country in such years. On the other hand, countries which are negatively or not connected with peripheral ones (such as, for instance, Germany in the second period and Finland and Austria in the latest years) have close to zero or negative $CoRisk_{in}$ measures. The clustering effect observed in Figure 5.10.4 in recent times implies that large peripheral countries (such as Italy and Spain) are negatively affected by positive correlations with each other (negative contagion: $CoRisk_{in} > 0$), while smaller core countries (such as Austria and Finland) take advantage of negative correlations with peripheral economies (positive contagion: $CoRisk_{in} < 0$), thus decreasing their own default probability. This result can be explained thinking at capital flows: when a country i is facing a crisis period, investors tend to shift their portfolio towards "safer" places in order to reduce risk, and such places are the countries negatively related to i, which, therefore, show an improvement in their survival probability. This mechanism justifies the difference between *positive* and *negative contagion* derived in Section 5.4.

Moving to the TPD time-evolution, it appears to be a mix between the sector-specific PD and the $CoRisk_{in}$ contribution, with the former prevailing. In peripheral economies, characterised by high sector-specific PDs, the $CoRisk_{in}$ contribution should be very low; however, the rise of two distinct clusters originates a sort of "diabolic loop", by which peripheral countries become positively connected between each other and negatively connected to core ones. For this reason their total default probability TPD is strongly influenced not only by its corresponding sector-specific PD, but also by high $CoRisk_{in}$ values. Following the same (but reversed) mechanism, core economies preserve low TPD even after the inclusion of contagion effects: the only exception is France, which presents an extremely high $CoRisk_{in}$ during the financial crisis due to a positive connection with Italy. Germany lies in an intermediate situation, with its $CoRisk_{in}$ growing in the recent years, along with positive connections with the periphery, in the light of its increasing leading role in the Euro area.

The results referred to corporates show sector-specific PDs less volatile than sovereign ones, both across countries and time. They all peak during the financial crisis and decrease afterwards, remaining almost constant during the following years. In recent times, the ranking of countries reflects what has been observed for sovereign risk, with Greece presenting much higher values than all the other countries, and core economies having the lowest ones. This means that, in the Euro area, sovereign risk has become the driving risk source. The $CoRisk_{in}$ pattern shows that almost all countries suffered contagion effects during the financial crisis and, to a lesser extent, during the sovereign crisis, thus highlighting an overall negative contagion across corporates. More precisely, Italy presents the highest $CoRisk_{in}$ values because of its strong positive relationships with Portugal and Spain, as shown in Figure 5.10.4. Differently from what has been observed for sovereigns, $CoRisk_{in}$ is the prevailing effect in the calculation of the total default probability for the corporate sector (with the exception of Germany in the last two periods, because of its very low sector-specific PD values): such a conclusion is supported by Figure 5.10.4, which shows that partial correlations are much higher both in number and value with respect to the sovereign sector, and that a clustering effect is not evident.

The results for the banks reveal that the sector-specific PDs of all countries have only been influenced by the financial crisis. Consistently with Figure 5.10.3, France, Belgium and the Netherlands present the highest levels, because of their high values of interest rates on deposits. The $CoRisk_{in}$ pattern shows both negative and positive contagion effects during the second time-period, with the former regarding core countries and the latter peripheral ones. However, from 2010-2012 (when two distinct clusters start emerging, as for the sovereign sector) $CoRisk_{in}$ starts increasing both in core and peripheral economies because of highly positive partial correlations within each cluster; this effect is further amplified in peripheral countries because of their higher sector-specific default probabilities, thus generating again the self-reinforcing and "diabolic" loop previously observed. Similarly to corporates, $CoRisk_{in}$ is the prevailing component in the composition of the total default probability of banks, both across time and countries.

5.6.3 From economic sectors to countries

In order to understand to what extent a whole country is influenced by the others, the aggregate total default probability proposed in (5.4.9) can be employed to summarise contagion effects into a unique default probability at the country level. Such aggregate $TPDs$ for the euro area countries are shown in Figure 5.10.8.

[Figure 5.10.8]

From Figure 5.10.8 two main considerations emerge. First, the financial crisis has had a more homogenous impact across countries than the sovereign one: all the aggregated TPDs strongly increased during 2008, while in the following time-window a clear distinction between peripheral and core countries appears, with the former having higher values than the latter. Two notable exceptions to the general pattern are: (a) France, which presents high values mainly because of its positive correlations with peripheral countries during both the financial and the sovereign crisis; (b) Ireland, characterised by a deep sovereign and bank crisis in 2011 (worsened by positive links with peripheral countries), followed by strong reforms and, recently, good economic results (increased by positive relations with core economies). Second, the pre- and post- crisis periods appear to be substantially different: during the pre-crisis years, in fact, default probabilities were almost constant and stable across time, and very homogenous across countries; but after the sovereign crisis the situation has become more heterogenous, with high volatilities in all countries and a clear distinction between peripheral and core economies, with Ireland joining the latter. This effect, consistently with Figures 5.10.5, 5.10.6 and 5.10.7, means that the sovereign crisis has had the strongest impact on the Euro area, an impact that still persists through diverging clusters.

The total PD of a country is a function of the total default probabilities of its three economic sectors which, in turn, are functions of two contributions: their sector-specific PD and the $CoRisk_{in}$ measure. Moreover, previous results confirm that the hypothesis of Corollary A.1.7 (see Appendix A) are verified. We are thus allowed to disentangle the final default probability of a country into six percentage components. The normalised results are shown in Figures 5.10.9 (core countries) and 5.10.10 (peripheral countries).

[Figures 5.10.9, 5.10.10]

From Figures 5.10.9 and 5.10.10 note that the sovereign contribution in peripheral countries is larger than in core ones across time; in the latter, the main component of sovereign risk is due to contagion effects, while in the former the sector-specific PD component is higher. In almost all countries the corporate contribution is stronger during "normal" times, such as before the financial crisis and in the latest period, depending on sector-specific PDs in peripheral economies and on contagion effects in core economies. Last, core economies suffered a substantial improvement in contagion effects for the bank sector during the sovereign crisis through their exposition to peripheral banks, while peripheral economies witnessed an increase in their sector-specific bank default probabilities.

Overall, combining cross-sectional and time comparisons, the distribution of risk in its six components looks quite homogenous across countries before the financial crisis while, recently, the situation has not returned back to equilibrium: strong contagion risks persist in core economies, while sector-specific default probabilities are still high, and worsened by intra-clustering effects, in peripheral countries.

5.6.4 Vulnerability vs Systemic Importance

While $CoRisk_{in}$ incorporates incoming effects and thus measures the *vulnerability* of each economic sector in a country, $CoRisk_{out}$ can be applied to obtain an estimation of the *systemic importance* of each economic sector in each country. A comparison between vulnerability ($CoRisk_{in}$) and systemic importance ($CoRisk_{out}$) across time and countries is shown in Figures 5.10.11 (sovereign sector), 5.10.12 (corporate sector) and 5.10.13 (banking sector).

[Figure 5.10.11, 5.10.12, 5.10.13]

By comparing $CoRisk_{out}$ and $CoRisk_{in}$ contributions for the sovereign sector, different conclusions can be deduced. First, during the pre-crisis and the financial crisis periods the two measures look very similar, meaning that sector-specific default probabilities were homogenous across countries. Important differences start emerging during the sovereign crisis period: in such years Greece is clearly more an exporter rather than an importer of risk, while the situation is reversed for Portugal and Spain. In most recent years, all peripheral countries have the highest, even if decreasing, $CoRisk_{out}$ contributions, since their sector-specific PD is significantly higher than in core economies. It is interesting to observe that there are not negative $CoRisk_{out}$ measures for the sovereign sector, meaning that all Euro countries contribute to increase the default probability of their neighbours.

The incoming and outgoing contributions for the corporate sector emphasise, once again, the difference between core and peripheral countries, with the latter characterised by higher $CoRisk_{out}$ and the former by higher $CoRisk_{in}$. Similarly to sovereigns, the two $CoRisk$ contributions are very close during the first two time-periods, while they start diverging afterwards. Similar results can be observed for the bank sector.

Overall, peripheral (core) countries appear to be more exporters (importers) rather than importers (exporters) of systemic risk, especially after the sovereign crisis. This result can be once more explained by the emerging of clustering effects in the third period, and it is a further confirmation of the persisting, and difficult to reverse consequences of the sovereign crisis on Eurozone countries.

5.6.5 CoRisk as a new centrality measure

We have considered both the incoming and outgoing $CoRisk$ as risk measures, able to calculate the vulnerability or the systemic importance of an economic sector in different countries and across time. However, $CoRisk$ has been derived employing network features, and can thus be applied in more general frameworks. More precisely, differently from other centrality measures, we assign two weights to a network: (a) $\rho_{mn} \in [-1, 1]$, which measures the weight of the link between each pair of nodes; (b) $PD^m \in [0, 1]$, which measures the dimension of each node. We can thus derive two centrality measures, both based on these two weights but different for the meaning they attribute to centrality:

Incoming centrality: how much a node is affected by its neighbours, according to (a) the number and weight of links, and (b) the importance (dimension) of neighbours.

Outgoing centrality: how much a node affects its neighbours, according to (a) the number and weight of links, and (b) the importance (dimension) of the node itself.

In order to better understand the meaning of these new network centrality definitions, we have decided to compare them to other two measures, commonly used especially in the systemic risk field: the eigenvector centrality (see e.g. Furfine, 2003; Billio et al.,

2012) and the weighted degree, calculated as the sum of all partial correlations (see e.g. Giudici and Spelta, 2015). We have applied them to our data, and the corresponding results are shown in Tables 5.10.8 and 5.10.9 (centrality measures) and in Tables5.10.10 and 5.10.11 (centrality ranks). In order to summarise the comparison between rankings, the Spearman correlation coefficient has been calculated: the results are shown in Table 5.10.12.

[Table 5.10.12]

Table 5.10.12 reveals that, overall, both $CoRisk_{in}$ and $CoRisk_{out}$ orderings are quite similar to the one obtained with the weighted degree of centrality. As previously underlined, the difference between the two lies in the inclusion of two rather than one weight in $CoRisk$: more precisely, $CoRisk_{in}$ and $CoRisk_{out}$ depend on both partial correlations and, respectively, the default probabilities (or, more generally, the dimensions) of neighbours or the default probability (dimension) of the node itself.

On the other hand, eigenvector centrality measures the importance of each node in the graph by looking at its relations with other central nodes, so that a node becomes much more important if it is connected to important ones. This mechanism, applied without considering the impact of each node on the basis of its dimension, amplifies the distance between $CoRisk$ and the eigenvector centrality measure. This effect is particularly evident during crisis periods, for both incoming and outgoing effects.

5.6.6 $CoRisk$ as a new systemic risk measure

The incoming and outgoing $CoRisk$, as derived in (5.4.6) and (5.4.8), can be considered not only as centrality measures, but also as new early warning indicators of systemic risk, and can thus be compared to other systemic risk measures. In particular, we can analyse their differences with respect to $\Delta CoVaR_{in}$ and $\Delta CoVaR_{out}$, calculated, respectively, as the part of the of the m-th economic sector's systemic risk that can be attributed to the system, or as the part of the system's systemic risk that can be attributed to an economic sector m

$$\begin{cases} \Delta CoVaR^m_{in} = CoVaR_q^{m|VaR_q^{system}} - CoVaR^{m|VaR_{50}^{system}}, \\ \Delta CoVaR^m_{out} = CoVaR_q^{system|VaR_q^m} - CoVaR^{system|VaR_{50}^m}, \end{cases} \tag{5.6.1}$$

with $CoVaR_q^{m|VaR_q^{system}}$ defined as the q-th Value at Risk of the economic sector m, conditional on an extreme event in the system; $CoVaR_q^{system|VaR_q^m}$ defined as the q-th

Value at Risk of the system, conditional on an extreme event in the sector m. Values at Risk and, consequently, $CoVaR$ have been calculated for the sector-specific default probability distributions (PD^m).

In order to compute $CoVar$, we have computed, In each time-period, a quantile regression of each economic sector on the system ($CoVaR_{in}$), or of the system on each economic sector ($CoVaR_{out}$): with such coefficients and the Value at Risks, we then obtained six $CoVar$ time-series: for incoming and outgoing effects, and for the three economic sectors.

The differences between $\Delta CoVar$ and $CoRisk$ are shown in Figures 5.10.14 (sovereign sector), 5.10.15 (corporate sector) and 5.10.16 (banking sector).

[Figures 5.10.14, 5.10.15, 5.10.16]

The graphs show that, in general, $\Delta CoVaR$ underestimates risk with respect to our measure *CoRisk* (both *in* and *out*) during the first two periods. This difference may be due to the fact that $CoVaR$ is based on the correlations between extreme, and possibly unfrequent events (tails of the distributions), and does not consider the effects of an homogenous increase in the default probabilities across countries.

For this reason, $\Delta CoVaR$ does not distinguish between the pre-crisis and the sovereign-crisis periods, being almost equal to zero in both of them: *CoRisk*, on the contrary, is very low until 2006 while it increases afterwards, because it takes into account not only correlations, but also the increasing levels of the PDs. Furthermore, $\Delta CoVaR$ overestimates both incoming and outgoing risk effects for those economic sectors whose distributions considerably differ from the others: this is, for example, the case of Portuguese and Greek sovereigns during the last years. Finally, $\Delta CoVaR$ does not really differentiate incoming and outgoing contagion effects: both of them are extremely high for Greece since 2012, even if it is reasonable to assume that Greece is more an exporter, rather than an importer, of systemic risk.

In this framework, we believe that *CoRisk* better captures contagion effects, consistently across time, across countries and economic sectors, and between incoming and outgoing effects.

5.7 Conclusions

In this work we have proposed a new systemic risk measurement model, based on multivariate stochastic processes, default probabilities and correlation networks. The model has been applied to the economies of the European monetary union in the recent time period. For each country we have considered three economic sectors (sovereigns, corporates and banks), and we have modelled each of them as a linear combination of two stochastic processes: a country-specific idiosyncratic component and a common systematic factor. We have built a partial correlation network within each sector, thus deriving a statistical representation of the transmission mechanisms of systemic risk that correctly takes into account interdependence effects. We have then derived the default probability of each economic sector in each country, both unconditionally and conditionally on the network structure: the comparison between them allows the definition of a novel risk indicator, $CoRisk$, that explicitly measures the contagion effect on the probability of default of each economic sector of a country.

The proposed methodology seems quite effective and efficient, particularly when compared to alternative network based measures, such as the node degree and the eigenvector centrality, and to classical systemic risk measures such as $CoVaR$.

From an applied viewpoint, our main economic findings for the Euro area can be summarised according to three dimensions: (a) the economic sector dimension, (b) the country dimension and (c) the time dimension at the aggregate country level.

Concerning (a), the corporate sector is strongly influenced by the systematic component, and this implies that it responds to monetary policy changes more than sovereigns and banks. On the other hand, the sovereign and bank sectors deeply suffered, respectively, the sovereign and the financial crisis. In both sectors, the sovereign crisis has generated two distinct clusters, characterised by positive within and negative cross correlations, clearly separating peripheral and core economies. Such separation creates loop effects within each cluster, further alienating troubled and strong economies. In a situation in which core economies benefit from positive contagion while peripheral countries suffer negative contagion, risk propagation does not act as a mean for balancing inequalities across countries; on the contrary, it weakens the weakest and strengthens the strongest countries.

Concerning (b), core countries mostly behave as importers, rather than exporters of system risk. As a consequence, core economies are mostly affected by contagion risk and are rather vulnerable than systemic important; peripheral countries, instead, strongly suffer high sector-specific default probabilities and high contagion deriving from cluster effects, so they are both vulnerable and systemic important.

Concerning (c) the sovereign crisis has had a larger impact on systemic risk with respect to the financial crisis. A possible explanation consists in different ways peripheral and core economies reacted to the financial crisis: peripheral countries, with high public debts, had little fiscal space to improve balance sheets and, therefore, the financial crisis triggered their imbalances to emerge in the subsequent sovereign crisis. The time sequence of these two events has determined an irreversible phase change, leading to a new non-stable equilibrium, where instability derives from peripheral-countries trajectories diverging from core ones.

5.8 References

Acharya, V.V., Pedersen, L.H., Philippon, T., Richardson, M., 2010. Measuring Systemic Risk. Technical Report. NYU Stern School of Business.

Acharya, V.V., Engle, R., Richardson, M., 2012. Capital Shortfall: A New Approach to Ranking and Regulating Systemic Risks. American Economic Review: Papers and Proceedings, 102(3), 59-64.

Adrian, T., Brunnermeier, M.K., 2011. CoVar. NBER Working Paper No. 17454. National Bureau of Economic Research.

Ahelegbey, D.F., Billio, M., Casarin, R., 2016. Bayesian Graphical Models for Structural Vector Autoregressive Processes. Journal of Applied Econometrics, 31(2), 357-386.

Allen, F., Gale, D., 2000. Financial Contagion. Journal of Political Economy, 108(1), 1-33.

Ang, A., Longstaff, F.A. 2012. Systemic sovereign credit risk: lessons from the U.S. and Europe. NBER Working Paper No. 16982. National Bureau of Economic Research.

Barigozzi, M., Brownlees, C., 2013. Nets: Network Estimation for Time Series. Technical Report.

Bartram, S.M., Brown, G.W., Hund, J.E., 2007. Estimating systemic risk in the international financial system. Journal of Financial Economics, 86(3), 835-869.

Battiston, S., Delli Gatti, D., Gallegati, M., Greenwald, B., Stiglitz, J.E., 2012. Liasons dangereuses: Increasing connectivity risk sharing, and systemic risk. Journal of Economic Dynamics and Control, 36(8), 1121-1141.

Betz, F., Oprica, S., Peltonen, T.A., Sarlin, P., 2014. Predicting Distress in European Banks. Journal of Banking and Finance, 45(C), 225-241.

Betz, F., Hautsch, N., Peltonen, T.A., Schienle, M., 2016. Systemic risk spillovers in the European banking and sovereign network. Technical Report. KIT.

Billio, M., Getmansky, M., Lo, A.W., Pelizzon, L., 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of Financial Economics, 104, 535-559.

Borio, C., Drehmann, M., 2009. Assessing the risk of banking crises. Technical Report. BIS.

Brownlees, C., Engle, R., 2012. Volatility, Correlation and Tails for Systemic Risk Measurement. Technical Report. NYU Stern School of Business.

Brownlees, C., Hans, C., Nualart, E., 2014. Bank Credit Risk Networks: Evidence from the Eurozone Crisis. Technical Report. Universitat Pompeu Fabra.

Chong, B.S., Liu, M., Shrestha, K., 2006. Monetary transmission via the administered interest rates channel. Journal of Banking and Finance, 30(5), 1467-1484.

Cox, J.C., Ingersoll, J.E.Jr, Ross, S.A., 1985. A Theory of Term Structure of Interest Rates. Econometrica, 53(2), 385-408.

Das, S.R., 2015. Matrix Metrics: Network-Based Systemic Risk Scoring. Technical Report. Santa Clara University.

Diebold, F.X., Yilmaz, K., 2014. On the network topology of variance decompositions: Measuring the connectedness of financial firms. Journal of Econometrics, 182, 119-134.

Duffie, D., Saita, L., Wang, K., 2007. Multi-Period Corporate Default Prediction with Stochastic Covariates. Journal of Financial Economics, 88(3), 635-665.

Dumitrescu, E., Banulescu, D.G., 2014. Which are the SIFIs? A Component Expected Shortfall (CES) Approach to Systemic Risk. Journal of Banking and Finance, 50, 575-588.

Duprey, T., Klaus, B., Peltonen, T.A., 2015. Dating systemic financial stress episodes in the EU countries. Technical Report. ECB.

ECB, 2009. Financial Stability Review. Technical Report. ECB.

Giudici, P., Spelta, A., 2015. Graphical Network Models for International Financial Flows. Journal of Business and Economic Statistics, to appear.

Gray, D., Gross, M., Paredes, J., Sydow, M., 2013. Modeling Banking, Sovereign and Macro Risk in a CCA Global VAR. Technical Report. International Monetary Fund.

Grinis, I., 2015. Credit Risk Spillovers, Systemic Importance and Vulnerability in Financial Networks. Technical Report. SRC.

Hautsch, N., Schaumburg, J., Schienle, M., 2015. Financial Network Systemic Risk Contributions. Review of Finance, 19(2), 685-738.

Iacus, S.M., 2008. Simulation and Inference for Stochastic Differential Equations. Springer.

Kalogeropoulos, K., Dellaportas, P., Roberts, G.O., 2011. Likelihood-based inference for correlated diffusions. The Canadian Journal of Statistics, 39(1), 52-72.

Koopman, S.J., Lucas, A., Schwaab, B., 2012. Dynamic factor models with macro, frailty, and industry effects for U.S. default counts: the credit crisis of 2008. Journal of Business and Economic Statistics, 30(4), 521-532.

Lando, D., Nielsen, M.S., 2010. Correlation in corporate defaults: contagion or conditional independence. Journal of Financial Intermediation, 19(3), 355-372.

Longstaff, F.A., 2010. The subprime credit crisis and contagion in financial markets. Journal of Financial Economics, 97(3), 436-450.

Lorenz, J., Battiston, S., Schweitzer, F., 2009. Systemic risk in a unifying framework for cascading processes on networks. The European Physical Journal B - Condensed Matter and Complex Systems, 71(4), 441-460.

Mezei, J., Sarlin, P., 2015. RiskRank: Measuring interconnected risk. Technical Report. RiskLab Finland.

Popescu, A., Turcu, C., 2014. Systemic Sovereign Risk in Europe: an MES and CES Approach. Revue d'economie politique, 124(6), 899-925.

Ramsay, B.A., Sarlin, P., 2015. Ending over-lending: assessing systemic risk with debt to cash flow. International Journal of Finance and Economics, to appear.

Schwaab, B., Koopman, S.J., Lucas, A., 2015. Global Credit Risk: World, Country and Industry Factors. Technical Report. Duisenberg School of Finance - Tinbergen Institute.

Shleifer, A., Vishny, R.W., 2010. Unstable banking. Journal of Financial Economics, 97(3), 306-318.

5.9 Appendix D: Definitions, Lemmas and Proofs

5.9.1 TPD and $CoRisk$ derivation

Since we want to consider the default probability of each economic sector as a function of both its sector-specific PD and contagion effects coming from neighbours, the first step consists in deriving the functional form for TPD_t^m as in (5.4.5).

Lemma 5.9.1.1. *Given an undirected graph $G = (P, E)$ with vertex set $P = V \times W$ and edge set $E = P \times P$, given the weights PD_t^m of each node $m \in V \times W$ at time t and a matrix P_t of partial correlation coefficients, $\rho_{mn|S}$, measuring the strength of each edge $e \in E$ at time t; the total default probability of each node m, conditional on non-defaulted neighbours in the previous time, can be expressed as a function if its weight PD_t^m, its neighbours' weights PD_t^n $(n \neq m)$ and their partial correlations $\rho_{mn|S}$ as follows:*

$$TPD_{t+1}^m = 1 - (1 - PD_t^m) \cdot \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn|S}}. \tag{5.9.1}$$

Proof. Let m be the node for which we want to measure the contagion effect, and n be any other node in the graph G which may have an effect on m because of their partial correlation $\rho_{mn|S}$. According to (4.2.20), let us define the functional form $f(x^m, t) = 1 - e^{-x_t^m}$, so that $PD_t^n = f(Z^n, t)$. We can now introduce a random variable $\tilde{Z}_{t+1}^m$ such that $TPD_{t+1}^m = f(\tilde{Z}^m, t+1)$. Without loss of generality, the linear combination in (5.3.10) can be rewritten by substituting Z^m with $\tilde{Z}_{t+1}^m = f^{-1}(TPD^m, t+1)$ and Z^n with $f^{-1}(PD^n, t)$. Furthermore, we can consider the sector-specific contribution $a^m = f^{-1}(PD^m, t)$ as a fixed effect (baseline Z^m), and we can approximate the regression coefficients $a_{mn|S}$ with their geometric average $\rho_{mn|S}$ as a consequence of the partial correlation property (5.3.11). Doing so, we obtain the following system:

$$\begin{cases} \tilde{Z}_{t+1}^m = Z_t^m + \sum_{n \neq m} \rho_{mn|S} Z_t^n \\ TPD_{t+1}^m = 1 - e^{-\tilde{Z}_{t+1}^m} \\ Z_t^m = -\ln(1 - PD_t^m), \end{cases}$$

and, consequently,

$$TPD_{t+1}^m = 1 - e^{-(Z_t^m + \sum_{n \neq m} \rho_{mn|S} Z_t^n)} = 1 - (1 - PD_t^m) \cdot \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn|S}}.$$

□

After having derived the total default probability associated to each economic sector in each country, we are now interested in extracting the contagion component from it. More precisely, we want to identify a variable able to measure to what extent the total default probability of node m depends on all the other nodes $n \neq m$. This variable, named incoming contagion risk ($CoRisk_{in,t}^m$), needs to have some properties: (a) it has to depend on the neighbours $n \neq m$ and not on m itself; (b) it has to be related to TPD^m since it is part of it; (c) it has to measure contagion risk and, therefore, it has to be an increasing function of both PD^n and $\rho_{mn|S}$. As a consequence, we propose the following.

Definition 5.9.1.2. Given the assumptions in Lemma A.1.1 and $\forall m \in V \times W$, the incoming contagion risk conditional on defaulted neighbours in the previous time, $CoRisk^m_{in,t}$, is defined as the TPD^m component which measures to what extent the default probabilities of the other nodes are transmitted to node m:

$$CoRisk^m_{in,t} = 1 - \prod_{n \neq m} (1 - PD^n_t)^{\rho_{mn|S}}. \tag{5.9.2}$$

The following Lemma derives the relationship between incoming contagion risk and the total default probability.

Lemma 5.9.1.3. *The incoming contagion risk, $CoRisk^m_{in,t}$, can be interpreted as the percentage variation in the survival probability of node m due to contagion effects:*

$$CoRisk^m_{in,t} = \frac{(1 - PD^m_t) - (1 - TPD^m_{t+1})}{1 - PD^m_t}. \tag{5.9.3}$$

Proof. From (5.9.2) the following can be derived:

$$\prod_{n \neq m} (1 - PD^n_t)^{\rho_{mn|S}} = 1 - CoRisk^m_{in,t}.$$

By substituting it into (5.9.1) the result is

$$TPD^m_{t+1} = 1 - (1 - PD^m_t) \cdot (1 - CoRisk^m_{in,t}),$$

and, after having rearranged terms, equation (5.9.3) can be obtained and the Lemma is proven. □

Similarly to what has been proposed for measuring incoming contagion effects, we now focus on outgoing contagion: the objective is to jointly estimate not only to what extent node m is affected by, but also affects the other nodes in the network. For this reason we define outgoing contagion as follows.

Definition 5.9.1.4. Given the assumptions in Lemma A.1.1 and $\forall m \in V \times W$, the outgoing contagion risk conditional on not having defaulted in the previous time, $CoRisk^m_{out,t}$, measures to what extent the default probability of node m is transmitted to the other nodes $n \neq m$:

$$CoRisk^m_{out,t} = 1 - \prod_{n \neq m} (1 - PD^m_t)^{\rho_{nm|S}} = 1 - (1 - PD^m_t)^{\sum_{n \neq m} \rho_{nm|S}}. \tag{5.9.4}$$

Incoming and outgoing contagion risks, as well as total default probabilities, have been derived for each economic sector, without considering correlations between different economic sectors of the same country. Our aim is to build a total default probability, aggregated over different sectors and able to measure the total default probability of an entire country. Since it has to be based on contagion risk, sector-specific risk and intra-country correlations, the following can be shown.

Lemma 5.9.1.5. *Given the sets of default events for the corporate* ($A_t^{1,i}$)*, bank* ($A_t^{2,i}$) *and sovereign* ($A_t^{3,i}$) *sector for each country i at time t, and given the system set* $S_t^i = \{A_t^m; \forall m \in V \times W, m \in ne(i,j), m \neq (i,j)\}$*, the total default probability aggregated at the country level is:*

$$TPD^i_{country,t} = 1 - \prod_{\substack{j,j'=1\\ j'>j}}^{3} (1 - TPD_t^{j,i} | \bar{A}^{j'}), \tag{5.9.5}$$

where the three probabilities $TPD_t^{j,i}$ are calculated through (5.9.1) *conditional on, respectively, all the other nodes, all the other nodes but the corporate sector of the same country i, all the other nodes but the corporate and bank sectors of the same country i.*

Proof. First, the $TPDs$ of a sector can be treated as conditional probabilities on default events of other sectors. Second, in order to consider correlations within different economic sectors belonging to the same country, we assume that a country will default if at least one of its economic sectors is in default. By combining these two issues, our objective function becomes the following probability:

$$Pr(A_t^{1,i} \cup A_t^{2,i} \cup A_t^{3,i} | S_t^i) = 1 - Pr(\bar{A}_t^{1,i} \cap \bar{A}_t^{2,i} \cap \bar{A}_t^{3,i} | S_t^i). \tag{5.9.6}$$

By exploiting conditional probability definition, the following holds:

$$\begin{aligned} Pr(\bar{A}_t^{1,i} \cap \bar{A}_t^{2,i} \cap \bar{A}_t^{3,i} | S_t^i) &= Pr(\bar{A}_t^{1,i} \cap \bar{A}_t^{2,i} | \bar{A}_t^{3,i}, S_t^i) \cdot Pr(\bar{A}_t^{3,i} | S_t^i) = \\ &= Pr(\bar{A}_t^{1,i} | \bar{A}_t^{2,i}, \bar{A}_t^{3,i}, S_t^i) \cdot Pr(\bar{A}_t^{2,i} | \bar{A}_t^{3,i}, S_t^i) \cdot Pr(\bar{A}_t^{3,i} | S_t^i). \end{aligned}$$

Recalling that A^i identifies defaults and $\bar{A}^i$ is its complementary set, (5.9.6) can be rewritten as follows:

$$\begin{aligned} TPD^i_{country} &= Pr(A_t^{1,i} \cup A_t^{2,i} \cup A_t^{3,i} | S_t^i) = \\ &= 1 - [(1 - Pr(A_t^{1,i} | \bar{A}_t^{2,i}, \bar{A}_t^{3,i}, S_t^i)) \cdot (1 - Pr(A_t^{2,i} | \bar{A}_t^{3,i}, S_t^i)) \cdot (1 - Pr(A_t^{3,i} | S_t^i))], \end{aligned} \tag{5.9.7}$$

where the conditional default probabilities can be substituted with the total default probabilities $TPD_t^{j,i}$ which are, by definition, conditional on the system S^i.

□

Lemma 5.9.1.6. *The total survival probability aggregated at the country level,* $1 - TPD^i_{country,t+1}$*, can be disentangled in its components, according to the reference economic sector j and to the source of risk (sector-specific or deriving from contagion), as follows:*

$$\ln(1 - TPD^i_{country,t+1}) = \sum_{j=1}^{3} \ln(1 - PD_t^{j,i}) + \sum_{\substack{j,j'=1\\ j'>j}}^{3} \ln(1 - CoRisk_t^{j,i} | \bar{A}^{j'}) \tag{5.9.8}$$

Proof. Lemma A.1.3 and Lemma A.1.5 provide the following system:

$$\begin{cases} TPD_{t+1}^{j,i} = 1 - (1 - PD_t^{j,i}) \cdot (1 - CoRisk_{in,t}^{j,i}), \\ TPD^i_{country,t+1} = 1 - [1 - (TPD_{t+1}^{1,i} | \bar{A}_t^{2,i}, \bar{A}_t^{3,i}, S_t^i)] \cdot [1 - (TPD_{t+1}^{2,i} | \bar{A}_t^{3,i}, S_t^i)] \cdot [1 - TPD_{t+1}^{3,i} | S_t^i)], \end{cases} \tag{5.9.9}$$

Remembering that $PD_t^{j,i}$ are sector-specific default probabilities and are thus independent from other sectors or countries, the solution of the system is:

$$\begin{aligned}1 - TPD^i_{country,t+1} =& (1 - PD_t^{1,i})(1 - CoRisk^{1,i}_{in,t}|\bar{A}_t^{2,i}, \bar{A}_t^{3,i}, S_t^i)\cdot \\ &\cdot (1 - PD_t^{2,i})(1 - CoRisk^{2,i}_{in,t}|\bar{A}_t^{3,i}, S_t^i) \cdot (1 - PD_t^{3,i})(1 - CoRisk^{3,i}_{in,t}).\end{aligned} \tag{5.9.10}$$

By applying a logarithmic transformation the result is:

$$\begin{aligned}\ln(1 - TPD^i_{country,t+1}) =& \ln(1 - PD_t^{1,i}) + \ln(1 - PD_t^{2,i}) + \ln(1 - PD_t^{3,i}) + \\ &+ \ln(1 - CoRisk^{1,i}_{in,t}|\bar{A}_t^{2,i}, \bar{A}_t^{3,i}, S_t^i) + \ln(1 - CoRisk^{2,i}_{in,t}|\bar{A}_t^{3,i}, S_t^i) + \\ &+ \ln(1 - CoRisk^{3,i}_{in,t}),\end{aligned} \tag{5.9.11}$$

and the Lemma is proven.

□

Corollary 5.9.1.7. *If a country i is not in default and if $|CoRisk_t^{i,j}| < 1$, the total default probability aggregated at the country level, $TPD^i_{country,t+1}$, can be disentangled in its components, according to the reference economic sector j and to the source of risk (sector-specific or deriving from contagion), as follows:*

$$TPD^i_{country,t+1} \approx \sum_{j=1}^{3} PD_t^{j,i} + \sum_{\substack{j,j'=1 \\ j'>j}}^{3} (CoRisk_t^{j,i}|\bar{A}^{j'}) \tag{5.9.12}$$

Proof. The result can be directly derived by applying a first-order Taylor expansion to the logarithmic function in (5.9.8). Since, by definition, default probabilities have always values in [0,1], the following constraints must be added in order to approximate logarithms with linear functions:

$$\begin{cases} TPD^i_{country,t+1} \neq 1, \\ PD_t^{j,i} \neq 1, \\ |CoRisk_t^{j,i}| < 1 \qquad \forall t, \forall i \in V, \forall j \in W. \end{cases} \tag{5.9.13}$$

□

Remark: Consistently with the application of this paper, Lemma A.1.5, A.1.6 and Corollary A.1.7 have been proposed for a system composed by N countries and 3 economic sectors. However, they can be easily generalised for a $N \times M$ system, with economic sectors $j \in W = \{1, ..., M\}$.

5.9.2 *CoRisk* properties

From a mathematical viewpoint, *CoRisk* (both *in* and *out*) is a non-linear and asymmetric function of partial correlations and default probabilities. Remembering that $\rho_{mn|S} \in [-1, 1]$ and $PD \in [0, 1]$, *CoRisk* is a function $f : \Re^2 \to \Re$ and, in particular, $CoRisk = f(x, y) : [-1, 1] \times [0, 1] \to (-\infty, 1]$.

In order to better interpret the CoRisk measure, it is important to study its limit conditions. More precisely, $CoRisk_{in}$ is equal to zero when, one of the two following conditions holds:

$$\begin{cases} PD^n = 0, & \forall n \in ne(m); \\ \rho_{mn|S} = 0, & \forall n \neq m. \end{cases} \tag{5.9.14}$$

This is consistent with the definition of $CoRisk_{in}$, meaning that the contribution to the default probability of a country m that derives from contagion effects is null (a) if all its neighbours have zero default probability, or (b) if country m is not partially related to any other country. Secondly, $CoRisk_{in}$ reaches its highest value 1 if $\exists\ n \in ne(m)\ s.t.\ PD^n = 1$, meaning that the highest contribution to the vulnerability of node m occurs when at least one of the other nodes n is in default. Finally, it is interesting to observe that $CoRisk_{in}$ is negative (the so-called *positive contagion*) when negative partial correlations prevail: in particular, $CoRisk_{in} \to -\infty$ if $\exists\ n \in ne(m)\ s.t.$ the two following conditions simultaneously hold:

$$\begin{cases} PD^n \to 1, \\ \rho_{mn|S} \to -1. \end{cases} \tag{5.9.15}$$

Similarly, the systemic importance of node m is null ($CoRisk^m_{out} = 0$) when m is not connected to any node or when its sector-specific default probability is equal to zero. Under the hypothesis that $\sum_{n \in ne(m)} \rho_{nm|S} \neq 0$, $CoRisk^m_{out}$ reaches its maximum point when $PD^m = 1$, meaning that the highest systemic importance of node m occurs when m itself is in default and is not an isolated point. On the other hand, when negative correlations prevail node m positively affects its neighbours, overall decreasing their default probability.

5.10 Appendix E: Tables and Figures

Table 5.10.1: Interest rates: pre-crisis and financial-crisis periods

Pre-crisis Period									
	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
Country	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	3.866	0.368	-0.033	4.096	0.289	0.371	3.248	0.189	-0.463
Bel	3.894	0.366	-0.041	4.525	0.225	0.171	4.117	0.251	-0.415
Fin	3.845	0.381	0.009	3.640	0.312	0.791	2.664	0.225	0.202
Fra	3.859	0.352	-0.020	4.351	0.159	0.399	3.669	0.102	-0.389
Ger	3.806	0.352	0.008	4.982	0.189	-0.065	3.142	0.274	-0.540
Gre	4.045	0.343	0.109	5.659	0.264	0.880	0.402	0.117	0.606
Ire	3.826	0.378	-0.002	4.675	0.372	0.634	2.564	0.202	0.806
Ita	4.027	0.349	0.096	4.538	0.307	0.654	3.131	0.258	0.140
Net	3.843	0.362	-0.015	4.693	0.207	0.272	3.971	0.269	-0.074
Por	3.919	0.358	0.071	4.548	0.321	0.929	3.033	0.252	0.301
Spa	3.850	0.362	-0.019	3.619	0.324	0.780	2.487	0.174	0.144

Financial-crisis Period									
	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
Country	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	4.198	0.298	0.752	4.531	0.943	0.967	3.352	0.179	0.383
Bel	4.216	0.322	0.834	4.754	0.671	0.986	4.009	0.156	0.427
Fin	4.107	0.350	0.847	4.378	1.103	0.980	2.968	0.355	0.838
Fra	4.063	0.370	0.852	4.710	0.587	0.956	3.565	0.059	0.353
Ger	3.808	0.502	0.840	4.961	0.579	0.986	2.688	0.047	0.726
Gre	4.826	0.437	-0.417	6.326	0.777	0.972	1.386	0.523	-0.175
Ire	4.686	0.481	-0.668	5.321	1.274	0.990	2.427	0.457	0.748
Ita	4.494	0.268	0.644	5.208	1.062	0.968	3.182	0.533	0.972
Net	4.067	0.352	0.846	4.751	0.797	0.994	3.758	0.053	0.153
Por	4.385	0.292	0.605	5.416	1.031	0.949	3.058	0.407	0.930
Spa	4.218	0.278	0.756	4.873	0.789	0.904	2.717	0.234	0.528

Notes: summary statistics for interest rates on government bonds ($y^i_{t,1}$), interest rates on loans to non-financial corporates ($y^i_{t,2}$) and interest rates on deposits to families and non-financial corporates ($y^i_{t,3}$) during the pre-crisis period (2003-2006) and the financial-crisis period (2007-2009), for 11 Eurozone countries. Means, standard deviations and correlations with Euribor interest rates have been reported.

Table 5.10.2: Interest rates: sovereign-crisis and post-crisis periods

Sovereign-crisis Period

	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
Country	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	2.972	0.587	0.586	2.845	0.232	0.934	2.297	0.104	0.256
Bel	3.565	0.660	0.877	3.460	0.169	0.920	3.182	0.177	-0.136
Fin	2.634	0.642	0.499	2.450	0.269	0.971	2.138	0.129	-0.232
Fra	2.992	0.482	0.618	3.318	0.144	0.823	3.150	0.080	0.023
Ger	2.282	0.697	0.383	3.837	0.191	0.933	2.564	0.095	0.500
Gre	15.780	6.526	0.011	5.649	0.553	0.313	2.491	0.312	-0.265
Ire	7.171	2.136	0.832	3.323	0.281	0.869	1.939	0.399	-0.182
Ita	4.984	0.891	0.305	3.505	0.332	0.221	2.784	0.441	-0.470
Net	2.638	0.622	0.469	3.436	0.192	0.991	3.801	0.139	0.026
Por	8.728	2.953	0.426	4.264	0.676	0.203	2.511	0.456	-0.172
Spa	5.179	0.815	-0.008	3.532	0.260	0.404	2.486	0.282	0.051

Post-crisis Period

	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
Country	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	1.430	0.608	0.829	2.323	0.097	0.969	1.681	0.206	0.837
Bel	1.676	0.740	0.854	2.851	0.190	0.950	2.697	0.222	0.847
Fin	1.357	0.564	0.864	1.870	0.110	0.964	1.587	0.330	0.786
Fra	1.589	0.650	0.856	2.725	0.194	0.856	2.864	0.124	0.835
Ger	1.091	0.521	0.857	3.085	0.204	0.911	2.015	0.192	0.833
Gre	8.943	1.881	-0.293	5.442	0.366	0.921	2.161	0.971	0.891
Ire	2.485	1.158	0.816	3.095	0.071	-0.496	1.892	0.280	0.630
Ita	3.014	1.133	0.801	3.511	0.237	0.947	2.896	0.368	0.578
Net	1.387	0.612	0.850	2.908	0.174	0.953	3.560	0.170	0.820
Por	4.205	1.708	0.718	4.108	0.338	0.919	2.730	0.434	0.952
Spa	3.045	1.266	0.728	3.129	0.360	0.935	2.258	0.348	0.800

Notes: summary statistics for interest rates on government bonds ($y^i_{t,1}$), interest rates on loans to non-financial corporates ($y^i_{t,2}$) and interest rates on deposits to families and non-financial corporates ($y^i_{t,3}$) during the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015), for 11 Eurozone countries. Means, standard deviations and correlations with Euribor interest rates have been reported.

Table 5.10.3: Estimated parameters for the systematic process

	a	v	b
2003-2006	0.014	0.001	0.056
2007-2009	0.899	0.355	0.569
2010-2012	0.405	0.262	0.149
2013-2015	0.008	0.002	0.090

Notes: estimated parameters of the systematic process S_t (3-months Euribor), for the pre-crisis (2003-2006), financial-crisis (2007-2009), sovereign-crisis (2010-2012) and post-crisis (2013-2015) periods.

Table 5.10.4: Estimated parameters for the idiosyncratic processes: pre-crisis and financial-crisis periods

Pre-crisis Period

	$y_{t,1}$ (Sov)			$y_{t,2}$ (Corp)			$y_{t,3}$ (Bank)		
Country	$(\theta_1)_1$	$(\theta_2)_1$	$(\theta_3)_1$	$(\theta_1)_2$	$(\theta_2)_2$	$(\theta_3)_2$	$(\theta_1)_3$	$(\theta_2)_3$	$(\theta_3)_3$
Aus	0.319	0.085	0.073	0.365	0.092	0.029	0.101	0.035	0.009
Bel	0.351	0.093	0.075	0.481	0.108	0.034	0.143	0.039	0.015
Fin	0.348	0.093	0.079	0.053	0.014	0.032	0.237	0.093	0.018
Fra	0.388	0.103	0.077	0.828	0.191	0.041	0.998	0.285	0.033
Ger	0.390	0.105	0.079	0.305	0.063	0.012	0.173	0.065	0.021
Gre	0.433	0.109	0.076	0.006	0.001	0.028	0.012	0.001	0.035
Ire	0.333	0.090	0.077	0.190	0.041	0.036	0.011	0.001	0.042
Ita	0.392	0.099	0.074	0.252	0.057	0.030	0.075	0.041	0.039
Net	0.379	0.101	0.080	0.366	0.079	0.018	0.500	0.075	0.032
Por	0.374	0.097	0.076	0.004	0.001	0.035	0.728	0.248	0.034
Spa	0.361	0.096	0.076	0.056	0.015	0.032	0.033	0.015	0.030

Financial-crisis Period

	$y_{t,1}$ (Sov)			$y_{t,2}$ (Corp)			$y_{t,3}$ (Bank)		
Country	$(\theta_1)_1$	$(\theta_2)_1$	$(\theta_3)_1$	$(\theta_1)_2$	$(\theta_2)_2$	$(\theta_3)_2$	$(\theta_1)_3$	$(\theta_2)_3$	$(\theta_3)_3$
Aus	0.448	0.110	0.083	1.500	0.342	0.001	0.069	0.021	0.027
Bel	0.333	0.082	0.080	1.497	0.324	0.001	0.070	0.019	0.025
Fin	0.235	0.061	0.081	1.487	0.356	0.001	1.507	0.514	0.001
Fra	0.238	0.062	0.081	1.500	0.216	0.001	0.522	0.022	0.515
Ger	0.145	0.045	0.093	1.243	0.319	0.001	0.401	0.012	0.523
Gre	0.760	0.150	0.108	1.518	0.320	0.002	1.096	0.022	1.480
Ire	0.755	0.157	0.109	1.494	0.318	0.001	0.023	0.009	0.906
Ita	0.635	0.143	0.074	1.507	0.347	0.001	0.001	0.011	0.568
Net	0.231	0.061	0.080	1.483	0.351	0.001	0.001	0.006	0.620
Por	0.796	0.183	0.091	1.507	0.288	0.001	1.497	0.099	0.001
Spa	0.708	0.169	0.083	1.521	0.311	0.001	0.033	0.005	0.756

Notes: estimated parameters of the idiosyncratic processes for sovereigns $y^m_{t,1}$ (interest rates on 10-years maturity government bonds), corporates $y^m_{t,2}$ (interest rates on loans to non-financial corporates) and banks $y^m_{t,3}$ (interest rates on deposits), during the pre-crisis period (2003-2006) and the financial-crisis period (2007-2009).

Table 5.10.5: Estimated parameters for the idiosyncratic processes: sovereign-crisis and post-crisis periods

Sovereign-crisis Period									
	$y_{t,1}$ (Sov)			$y_{t,2}$ (Corp)			$y_{t,3}$ (Bank)		
Country	$(\theta_1)_1$	$(\theta_2)_1$	$(\theta_3)_1$	$(\theta_1)_2$	$(\theta_2)_2$	$(\theta_3)_2$	$(\theta_1)_3$	$(\theta_2)_3$	$(\theta_3)_3$
Aus	1.507	0.529	0.001	0.592	0.126	0.060	1.487	0.670	0.001
Bel	1.492	0.435	0.001	0.349	0.106	0.034	0.356	0.118	0.012
Fin	0.030	0.032	0.112	0.232	0.106	0.047	0.099	0.047	0.025
Fra	0.073	0.039	0.115	0.691	0.073	0.345	1.271	0.406	0.037
Ger	0.040	0.042	0.121	0.150	0.045	0.023	0.001	0.003	0.021
Gre	1.835	0.102	0.593	0.362	0.058	0.046	0.407	0.148	0.094
Ire	0.407	0.057	0.262	0.067	0.021	0.037	0.157	0.069	0.108
Ita	0.489	0.095	0.158	0.093	0.023	0.030	0.035	0.001	0.043
Net	1.477	0.597	0.001	0.001	0.035	0.027	0.212	0.054	0.021
Por	0.624	0.061	0.234	0.156	0.029	0.037	0.027	0.001	0.027
Spa	0.710	0.129	0.162	0.115	0.031	0.028	0.038	0.008	0.019

Post-crisis Period									
	$y_{t,1}$ (Sov)			$y_{t,2}$ (Corp)			$y_{t,3}$ (Bank)		
Country	$(\theta_1)_1$	$(\theta_2)_1$	$(\theta_3)_1$	$(\theta_1)_2$	$(\theta_2)_2$	$(\theta_3)_2$	$(\theta_1)_3$	$(\theta_2)_3$	$(\theta_3)_3$
Aus	0.050	0.057	0.168	0.001	0.034	0.015	0.001	0.116	0.007
Bel	0.038	0.047	0.157	0.001	0.058	0.021	0.001	0.078	0.014
Fin	0.057	0.061	0.170	0.001	0.040	0.012	0.019	0.029	0.021
Fra	0.047	0.053	0.154	0.057	0.028	0.036	0.250	0.091	0.035
Ger	0.047	0.069	0.191	1.483	0.497	0.001	0.985	0.274	0.001
Gre	1.023	0.124	0.286	0.020	0.009	0.024	1.493	0.653	0.001
Ire	0.038	0.052	0.159	0.484	0.155	0.018	0.057	0.042	0.030
Ita	0.014	0.030	0.134	0.001	0.002	0.022	0.104	0.048	0.034
Net	0.039	0.049	0.165	0.001	0.046	0.029	1.486	0.430	0.001
Por	0.104	0.051	0.188	0.001	0.070	0.015	0.001	0.107	0.026
Spa	0.066	0.054	0.135	1.511	0.493	0.001	0.001	0.142	0.014

Notes: estimated parameters of the idiosyncratic processes for sovereigns $y^m_{t,1}$ (interest rates on 10-years maturity government bonds), corporates $y^m_{t,2}$ (interest rates on loans to non-financial corporates) and banks $y^m_{t,3}$ (interest rates on deposits), during the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015).

Table 5.10.6: $CoRisk_{in}$: pre-crisis and financial-crisis periods

Pre-crisis Period

	$CoRisk_{sov}$ (%)				$CoRisk_{corp}$(%)				$CoRisk_{bank}$(%)			
Country	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	1.36	0.30	1.09	2.23	1.91	0.09	1.78	2.11	1.90	0.18	1.68	2.31
Bel	1.42	0.32	1.14	2.34	0.49	0.09	0.26	0.56	0.78	0.28	0.44	1.49
Fin	-0.01	0.01	-0.03	0.01	4.28	0.32	3.97	5.16	0.52	0.13	0.39	0.84
Fra	0.60	0.14	0.47	1.01	0.92	0.09	0.73	1.07	1.11	0.31	0.84	2.01
Ger	0.44	0.13	0.32	0.79	2.01	0.23	1.56	2.33	2.20	0.55	1.75	3.87
Gre	0.96	0.17	0.82	1.45	0.99	0.18	0.83	1.51	-0.37	0.08	-0.64	-0.30
Ire	0.89	0.20	0.71	1.46	1.21	0.13	1.08	1.54	2.09	0.25	1.65	2.56
Ita	0.91	0.16	0.78	1.36	0.92	0.07	0.83	1.10	1.90	0.15	1.78	2.30
Net	0.95	0.22	0.75	1.57	1.97	0.17	1.67	2.23	0.60	0.09	0.52	0.85
Por	0.58	0.14	0.44	0.98	1.15	0.26	0.94	1.87	-0.04	0.15	-0.21	0.30
Spa	0.51	0.12	0.40	0.86	1.25	0.12	1.10	1.54	1.52	0.30	1.25	2.34

Financial-crisis Period

	$CoRisk_{sov}$ (%)				$CoRisk_{corp}$(%)				$CoRisk_{bank}$(%)			
Country	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	1.86	0.78	0.97	3.52	-0.04	0.16	-0.54	0.13	6.86	1.56	4.08	8.98
Bel	4.82	1.73	2.46	8.19	-0.20	0.36	-0.75	0.14	4.17	0.55	3.07	4.98
Fin	3.98	1.40	2.02	6.62	2.05	1.61	0.11	5.35	3.60	0.91	1.98	4.83
Fra	6.66	2.65	3.76	12.13	1.26	0.52	0.43	2.10	5.32	1.40	2.84	7.11
Ger	-2.18	0.97	-4.11	-1.00	5.94	2.49	2.02	9.40	7.79	1.76	4.32	10.06
Gre	3.19	1.34	1.71	6.00	2.12	1.28	0.58	4.63	3.19	1.15	1.24	4.58
Ire	1.28	0.69	0.34	2.29	5.44	2.34	2.56	10.83	-1.78	0.63	-2.52	-0.62
Ita	-0.06	0.67	-0.92	0.94	7.37	3.43	2.38	13.57	2.86	0.51	2.13	4.01
Net	3.10	1.10	1.32	4.40	2.06	1.30	-0.01	3.85	3.77	1.18	1.77	5.27
Por	1.96	0.71	0.74	2.80	2.70	1.25	0.75	4.79	-1.80	0.34	-2.36	-1.28
Spa	1.79	0.71	0.94	3.25	2.14	1.14	0.35	3.65	-1.07	1.05	-2.19	0.59

Notes: summary statistics of $CoRisk_{in}$ for the three economic sectors (sovereigns, corporates and banks) and during the pre-crisis period (2003-2006) and the financial-crisis period (2007-2009). Means and standard deviations have been reported.

Table 5.10.7: $CoRisk_{in}$: sovereign-crisis and post-crisis periods

Sovereign-crisis Period												
	$CoRisk_{sov}$ (%)				$CoRisk_{corp}$(%)				$CoRisk_{bank}$(%)			
Country	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	2.86	0.53	1.73	3.62	3.21	0.48	2.40	3.98	2.45	0.62	1.35	3.44
Bel	2.59	0.28	2.17	3.15	0.44	0.30	-0.16	0.87	-1.63	0.34	-2.19	-1.11
Fin	1.47	0.39	0.77	2.09	5.04	0.86	3.60	6.42	4.29	0.88	2.86	5.74
Fra	4.02	0.89	2.65	6.08	0.66	0.21	0.23	0.95	1.95	0.30	1.43	2.45
Ger	1.78	0.45	0.97	2.44	-0.40	0.28	-0.97	-0.04	-2.02	0.21	-2.40	-1.74
Gre	3.61	1.22	1.71	5.71	1.55	0.31	1.05	2.00	-1.41	0.23	-1.76	-0.97
Ire	4.99	1.29	2.78	6.72	1.70	0.28	1.32	2.17	-0.48	0.03	-0.53	-0.42
Ita	2.82	1.19	1.45	4.83	1.47	0.31	1.05	2.00	0.43	0.17	0.18	0.71
Net	1.64	0.36	0.73	2.35	2.90	0.50	2.04	3.68	0.57	0.12	0.43	0.77
Por	10.80	3.06	5.70	16.10	4.36	0.53	3.51	5.12	0.07	0.07	-0.05	0.18
Spa	8.81	1.71	6.46	12.69	3.72	0.68	2.68	4.86	1.84	0.37	1.36	2.45
Post-crisis Period												
	$CoRisk_{sov}$ (%)				$CoRisk_{corp}$(%)				$CoRisk_{bank}$(%)			
Country	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	-4.60	0.82	-5.88	-3.18	2.78	0.11	2.58	2.99	0.64	0.06	0.52	0.74
Bel	6.78	1.08	4.87	8.42	0.31	0.02	0.26	0.34	2.60	0.21	2.21	2.90
Fin	-3.75	1.31	-6.79	-1.81	6.14	0.39	5.40	6.52	0.65	0.19	0.42	1.04
Fra	0.43	0.20	0.05	0.76	0.51	0.06	0.41	0.59	0.07	0.00	0.07	0.08
Ger	5.23	0.95	3.64	6.60	7.20	0.40	6.52	7.76	5.62	0.78	4.29	6.80
Gre	-0.94	0.21	-1.21	-0.53	-0.72	0.07	-0.81	-0.59	0.89	0.16	0.55	1.12
Ire	1.64	0.67	0.68	2.81	-1.26	0.05	-1.37	-1.20	0.93	0.19	0.69	1.28
Ita	1.80	0.64	0.85	2.87	-1.57	0.07	-1.72	-1.47	0.52	0.13	0.35	0.78
Net	0.82	0.24	0.46	1.19	0.62	0.11	0.45	0.79	0.18	0.24	-0.11	0.62
Por	-4.01	0.98	-5.70	-2.44	1.61	0.08	1.44	1.72	1.42	0.61	0.25	2.10
Spa	2.63	1.15	0.91	4.24	1.78	0.07	1.65	1.85	4.15	0.46	3.43	4.90

Notes: summary statistics of $CoRisk_{in}$ for the three economic sectors (sovereigns, corporates and banks) and during the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015). Means and standard deviations have been reported.

Table 5.10.8: Network centrality measures: pre-crisis and financial-crisis periods

Pre-crisis Period						
	Sovereign		Corporate		Bank	
Country	DC	Eigen.	DC	Eigen.	DC	Eigen.
Aus	0.701	0.227	0.931	0.277	1.448	0.990
Bel	0.971	0.158	0.161	0.000	0.995	0.356
Fin	0.015	0.000	2.026	1.000	0.186	0.121
Fra	0.916	0.196	0.227	0.000	0.761	0.000
Ger	0.915	0.211	1.100	0.000	1.236	1.000
Gre	1.161	1.000	0.597	0.520	-0.308	0.781
Ire	0.877	0.231	0.791	0.378	0.707	0.000
Ita	1.061	0.954	0.516	0.325	1.324	0.000
Net	1.219	0.125	1.149	0.226	0.273	0.000
Por	0.979	0.578	0.717	0.710	0.240	0.000
Spa	0.873	0.340	0.573	0.694	0.739	0.000

Financial-crisis Period						
	Sovereign		Corporate		Bank	
Country	DC	Eigen.	DC	Eigen.	DC	Eigen.
Aus	1.064	0.210	-0.106	0.099	1.610	0.894
Bel	0.721	0.010	-0.044	0.050	0.821	0.806
Fin	0.916	0.307	0.712	0.298	0.962	0.244
Fra	2.402	0.973	0.435	0.135	1.185	0.548
Ger	-0.185	1.000	1.439	0.710	1.921	0.466
Gre	1.019	0.189	0.691	0.794	1.038	0.000
Ire	0.512	0.182	1.512	0.758	0.021	0.000
Ita	0.156	0.000	2.383	0.938	0.784	0.000
Net	0.887	0.585	0.242	0.118	1.093	0.927
Por	0.451	0.277	0.639	1.000	-0.012	0.000
Spa	1.309	0.423	0.276	0.210	0.154	1.000

Notes: degree of connectivity (DC) and eigenvector centrality (Eigen.) measures referred to the three economic sectors (sovereigns, corporates and banks) during the pre-crisis period (2003-2006) and the financial-crisis period (2007-2009).

Table 5.10.9: Network centrality measures: sovereign-crisis and post-crisis periods

Sovereign-crisis Period						
	Sovereign		Corporate		Bank	
Country	DC	Eigen.	DC	Eigen.	DC	Eigen.
Aus	1.170	0.388	1.379	0.000	1.379	0.000
Bel	0.872	0.220	-1.070	0.174	-1.070	0.000
Fin	0.651	1.000	2.592	0.698	2.592	0.000
Fra	1.663	0.000	0.911	0.000	0.911	0.000
Ger	1.009	0.862	-0.794	0.000	-0.794	0.000
Gre	0.676	0.000	-0.632	0.836	-0.632	0.217
Ire	0.514	0.000	-0.100	0.880	-0.100	0.000
Ita	0.765	0.000	0.174	0.000	0.174	0.356
Net	0.940	0.612	0.357	0.301	0.357	0.332
Por	0.813	0.000	0.047	1.000	0.047	1.000
Spa	1.614	0.000	1.204	0.516	1.204	0.482

Post-crisis Period						
	Sovereign		Corporate		Bank	
Country	DC	Eigen.	DC	Eigen.	DC	Eigen.
Aus	0.069	0.000	1.381	0.650	0.125	0.728
Bel	2.242	0.565	0.025	0.192	1.023	0.150
Fin	-0.147	0.619	2.011	1.000	0.316	0.268
Fra	0.784	0.000	0.213	0.000	-0.012	0.059
Ger	1.883	0.000	1.872	0.000	2.773	1.000
Gre	-0.705	0.000	-0.192	0.019	0.391	0.676
Ire	0.571	0.068	-0.183	0.126	0.513	0.094
Ita	1.286	0.312	0.036	0.216	0.189	0.353
Net	-0.200	0.769	0.383	0.616	-0.232	0.002
Por	0.540	1.000	0.479	0.000	0.878	0.635
Spa	0.584	0.000	0.707	0.348	1.805	0.719

Notes: degree of connectivity (DC) and eigenvector centrality (Eigen.) measures referred to the three economic sectors (sovereigns, corporates and banks) during the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015).

Table 5.10.10: Rankings comparison: pre-crisis and financial-crisis periods

Pre-crisis Period											
Sovereign				Corporate				Bank			
CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$
Bel	Net	Net	Gre	Fin	Ger	Fin	Fin	Ger	Ita	Aus	Ger
Aus	Ita	Gre	Ita	Ger	Net	Net	Por	Ire	Spa	Ita	Aus
Gre	Bel	Ita	Por	Net	Fin	Ger	Spa	Ita	Ire	Ger	Gre
Net	Fra	Por	Spa	Aus	Gre	Aus	Gre	Aus	Ger	Bel	Bel
Ita	Spa	Bel	Ire	Spa	Ire	Ire	Ire	Spa	Fra	Fra	Fin
Ire	Ger	Fra	Aus	Ire	Ita	Por	Ita	Fra	Net	Spa	Ita
Fra	Aus	Ger	Ger	Por	Aus	Gre	Aus	Bel	Aus	Ire	Fra
Por	Por	Ire	Fra	Gre	Por	Spa	Net	Net	Bel	Net	Spa
Spa	Gre	Spa	Bel	Ita	Bel	Ita	Ger	Fin	Por	Por	Ire
Ger	Ire	Aus	Net	Fra	Spa	Fra	Fra	Por	Fin	Fin	Net
Fin	Fin	Fin	Fin	Bel	Fra	Bel	Bel	Gre	Gre	Gre	Por

Financial-crisis Period											
Sovereign				Corporate				Bank			
CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$
Fra	Fra	Fra	Ger	Ita	Ita	Ita	Por	Ger	Aus	Ger	Spa
Bel	Bel	Spa	Fra	Ger	Ire	Ire	Ita	Aus	Ger	Aus	Net
Fin	Aus	Aus	Net	Ire	Ger	Ger	Gre	Fra	Bel	Fra	Aus
Gre	Spa	Gre	Spa	Por	Fin	Fin	Ire	Bel	Net	Net	Bel
Net	Net	Fin	Fin	Spa	Gre	Gre	Ger	Net	Fin	Gre	Fra
Por	Fin	Net	Por	Gre	Fra	Por	Fin	Fin	Gre	Fin	Ger
Aus	Gre	Bel	Aus	Net	Por	Fra	Spa	Gre	Ita	Bel	Fin
Spa	Por	Ire	Gre	Fin	Spa	Spa	Fra	Ita	Fra	Ita	Gre
Ire	Ita	Por	Ire	Fra	Bel	Net	Net	Spa	Por	Spa	Ita
Ita	Ire	Ita	Bel	Aus	Net	Bel	Aus	Ire	Ire	Ire	Ire
Ger	Ger	Ger	Ita	Bel	Aus	Aus	Bel	Por	Spa	Por	Por

Notes: rankings obtained with $CoRisk_{in}$, $CoRisk_{out}$, degree of connectivity and eigenvector centrality measures, ordered from the highest to the lowest and referred to the three economic sectors (sovereigns, corporates, banks) and to the pre-crisis period (2003-2006) and the financial-crisis period (2007-2009).

Table 5.10.11: Rankings comparison: sovereign-crisis and post-crisis periods

Sovereign-crisis Period											
Sovereign				Corporate				Bank			
CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$
Por	Gre	Fra	Fin	Fin	Spa	Fin	Por	Fin	Fin	Fin	Por
Spa	Spa	Spa	Ger	Por	Por	Aus	Ire	Aus	Spa	Aus	Spa
Ire	Por	Aus	Net	Spa	Fin	Spa	Gre	Fra	Fra	Spa	Ita
Fra	Ire	Ger	Aus	Aus	Gre	Fra	Fin	Spa	Aus	Fra	Net
Gre	Fra	Net	Bel	Net	Net	Net	Spa	Net	Net	Net	Gre
Aus	Ita	Bel	Fra	Ire	Aus	Ita	Net	Ita	Ita	Ita	Fin
Ita	Aus	Por	Spa	Gre	Ita	Por	Bel	Por	Por	Por	Aus
Bel	Bel	Ita	Por	Ita	Bel	Ire	Aus	Ire	Ire	Ire	Fra
Ger	Fin	Gre	Ita	Fra	Ire	Gre	Fra	Gre	Gre	Gre	Ire
Net	Ger	Fin	Gre	Bel	Fra	Ger	Ita	Bel	Ger	Ger	Ger
Fin	Net	Ire	Ire	Ger	Ger	Bel	Ger	Ger	Bel	Bel	Bel

Post-crisis Period											
Sovereign				Corporate				Bank			
CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$
Bel	Ita	Bel	Por	Ger	Ger	Fin	Fin	Ger	Ger	Ger	Ger
Ger	Bel	Ger	Net	Fin	Fin	Ger	Aus	Spa	Spa	Spa	Aus
Spa	Ire	Ita	Fin	Aus	Aus	Aus	Net	Bel	Por	Bel	Spa
Ita	Ger	Fra	Bel	Spa	Por	Spa	Spa	Por	Bel	Por	Gre
Ire	Spa	Spa	Ita	Por	Spa	Por	Ita	Ire	Ire	Ire	Por
Net	Por	Ire	Ire	Net	Bel	Net	Bel	Gre	Fra	Gre	Ita
Fra	Fra	Por	Ger	Fra	Net	Fra	Ire	Fin	Aus	Fin	Fin
Gre	Aus	Aus	Fra	Bel	Fra	Ita	Gre	Aus	Ita	Ita	Bel
Fin	Net	Fin	Spa	Gre	Ita	Bel	Ger	Ita	Fin	Aus	Ire
Por	Fin	Net	Aus	Ire	Ire	Ire	Por	Net	Gre	Fra	Fra
Aus	Gre	Gre	Gre	Ita	Gre	Gre	Fra	Fra	Net	Net	Net

Notes: rankings obtained with $CoRisk_{in}$, $CoRisk_{out}$, degree of connectivity and eigenvector centrality measures, ordered from the highest to the lowest and referred to the three economic sectors (sovereigns, corporates, banks) and to the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015).

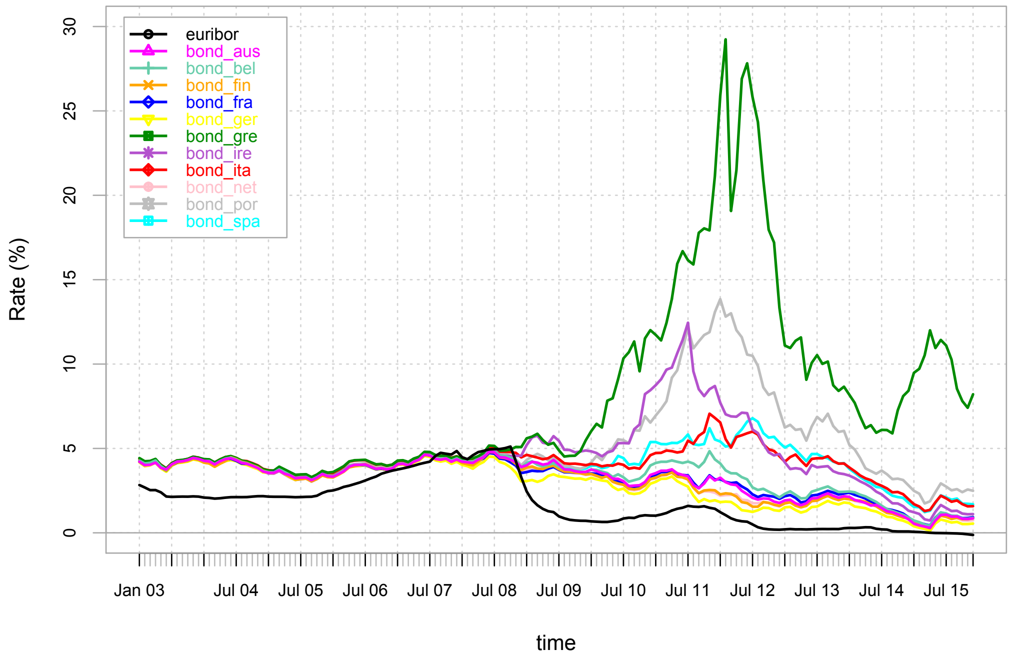
Table 5.10.12: Correlation coefficients between rankings

$CoRisk_{in}$						
	Sovereign		Corporate		Bank	
Period	*DC*	*Eigen.*	*DC*	*Eigen.*	*DC*	*Eigen.*
2003-2006	0.436	0.136	0.936	0.373	0.764	0.245
2007-2009	0.582	0.064	0.809	0.811	0.936	0.518
2010-2012	0.136	-0.736	0.691	0.655	0.982	0.345
2013-2015	0.736	0.018	0.927	0.245	0.982	0.573

$CoRisk_{out}$						
	Sovereign		Corporate		Bank	
Period	*DC*	*Eigen.*	*DC*	*Eigen.*	*DC*	*Eigen.*
2003-2006	0.527	-0.136	0.782	0.118	0.591	-0.182
2007-2009	0.791	0.109	0.982	0.709	0.755	0.318
2010-2012	-0.100	-0.818	0.445	0.618	0.973	0.445
2013-2015	0.864	0.173	0.927	0.264	0.809	0.445

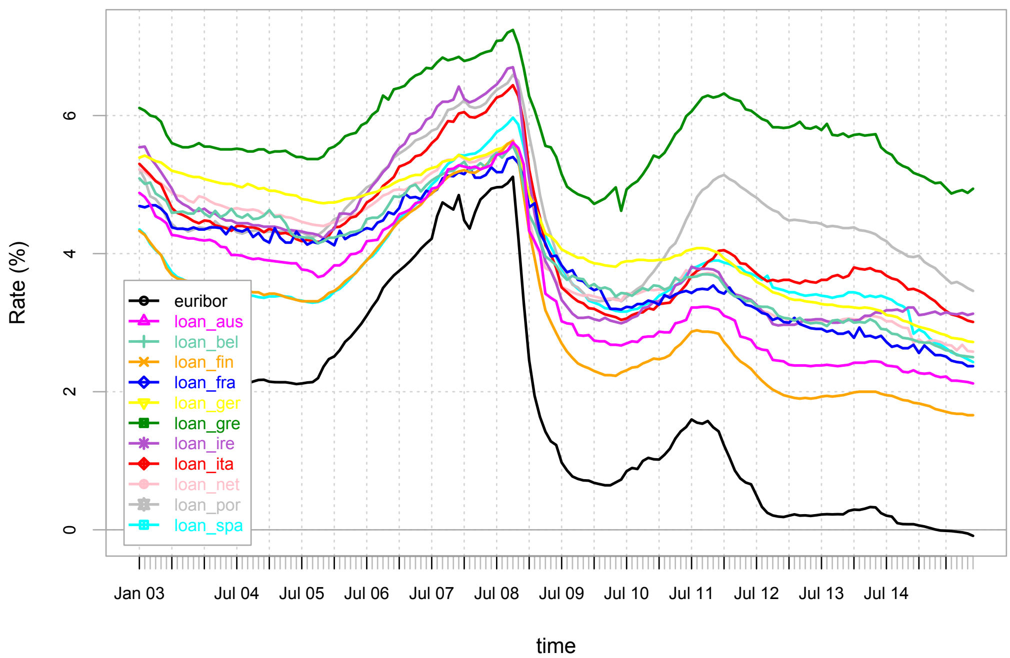
Notes: Spearman correlation coefficients between $CoRisk_{in}$ and $CoRisk_{out}$ rankings and rankings based on, respectively, degree centrality (DC) and eigenvector centrality (Eigen.), referred to the three economic sectors (sovereigns, corporates, banks) and to the four time-periods (pre-crisis, financial-crisis, sovereign-crisis, post-crisis).

Figure 5.10.1: Sovereigns interest rates



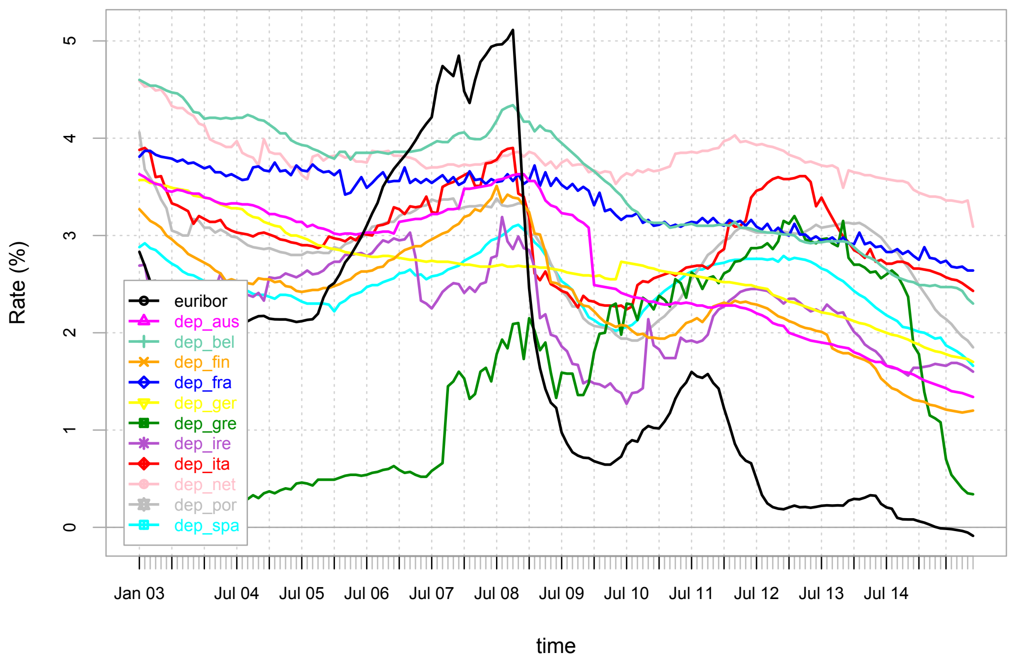
Notes: Monthly time evolution of 10-years maturity bond interest rates and 3-months Euribor, from January 2003 until December 2015 and referred to 11 Eurozone countries. Interest rates on government bonds were initially very similar, while in 2010 they started diverging: decreasing in core countries and increasing in peripheral countries.

Figure 5.10.2: Corporates interest rates



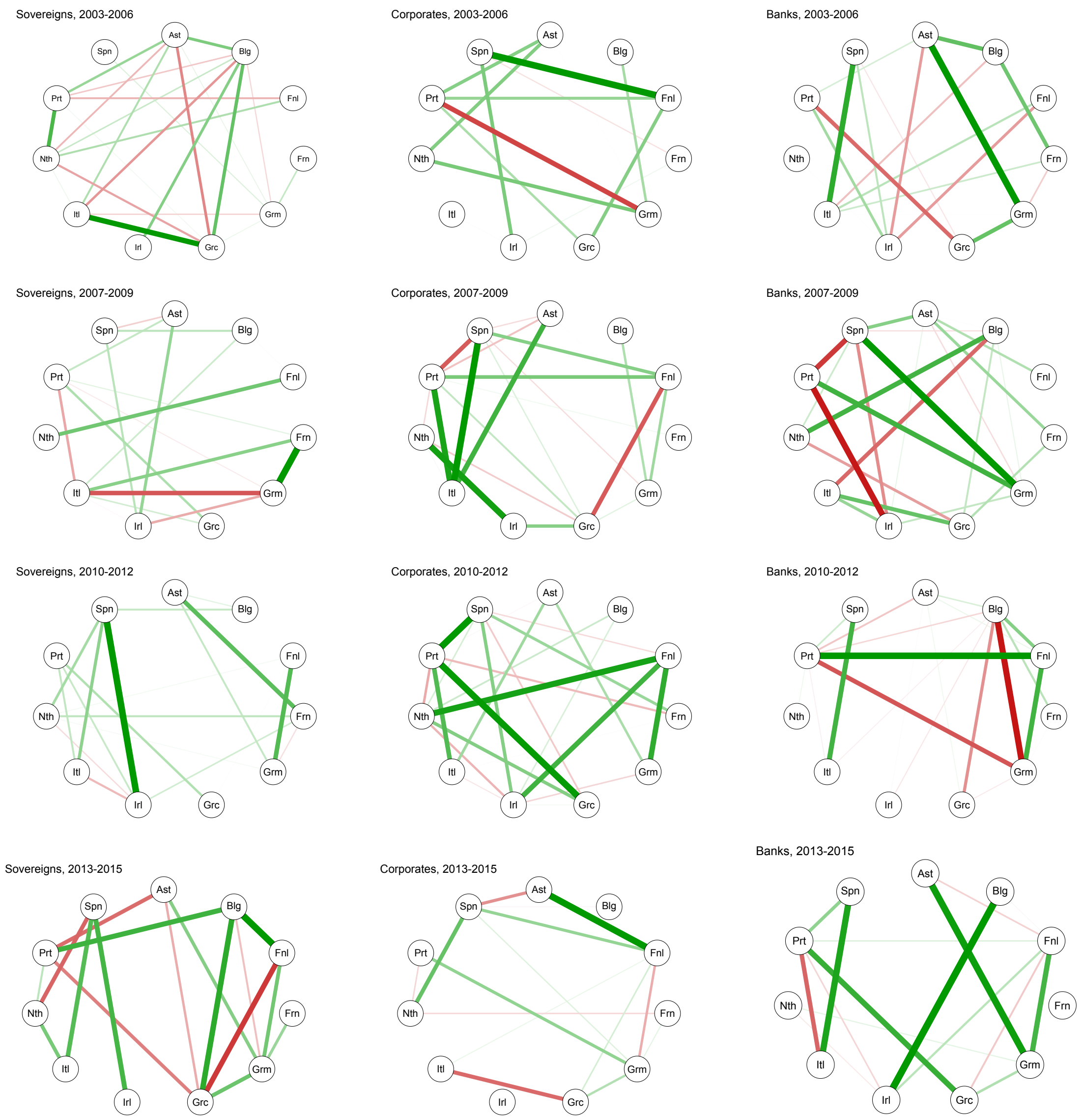
Notes: Monthly time evolution of aggregate interest rates on loans to non-financial corporates and 3-months Euribor, from January 2003 until December 2015 and referred to 11 Eurozone countries. Interest rates on loans to non-financial corporates differ across the main european countries, being the highest ones in Greece and Portugal and the lowest ones in Finland and Austria; however they all show positive correlations with the Euribor dynamics.

Figure 5.10.3: Banks interest rates



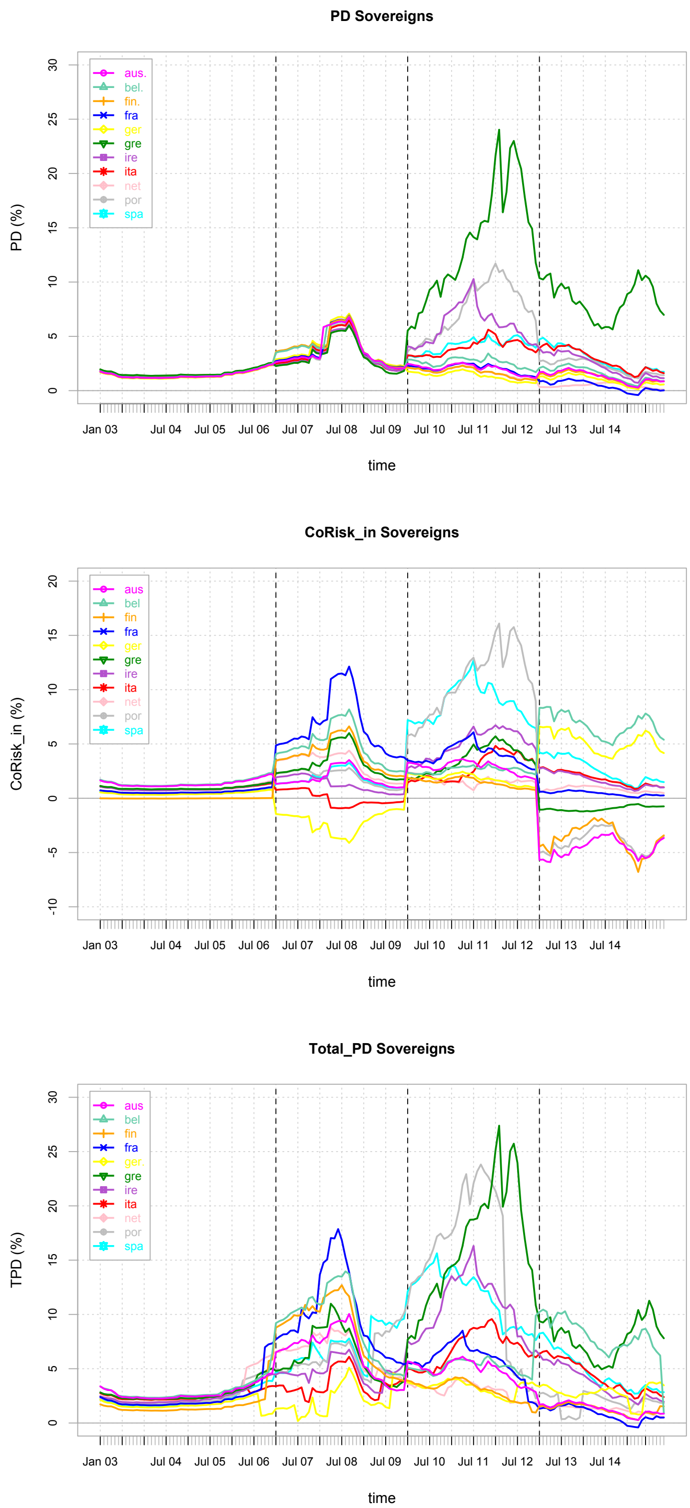
Notes: Monthly time evolution of aggregate interest rates on deposits to both families and non-financial corporates, and 3-months Euribor, from January 2003 until December 2015 and referred to 11 Eurozone countries. The highest rates occur in France, Belgium and the Netherlands consistently through time, while the curves of the other countries do overlap, especially in peripheral countries.

Figure 5.10.4: Partial correlation networks



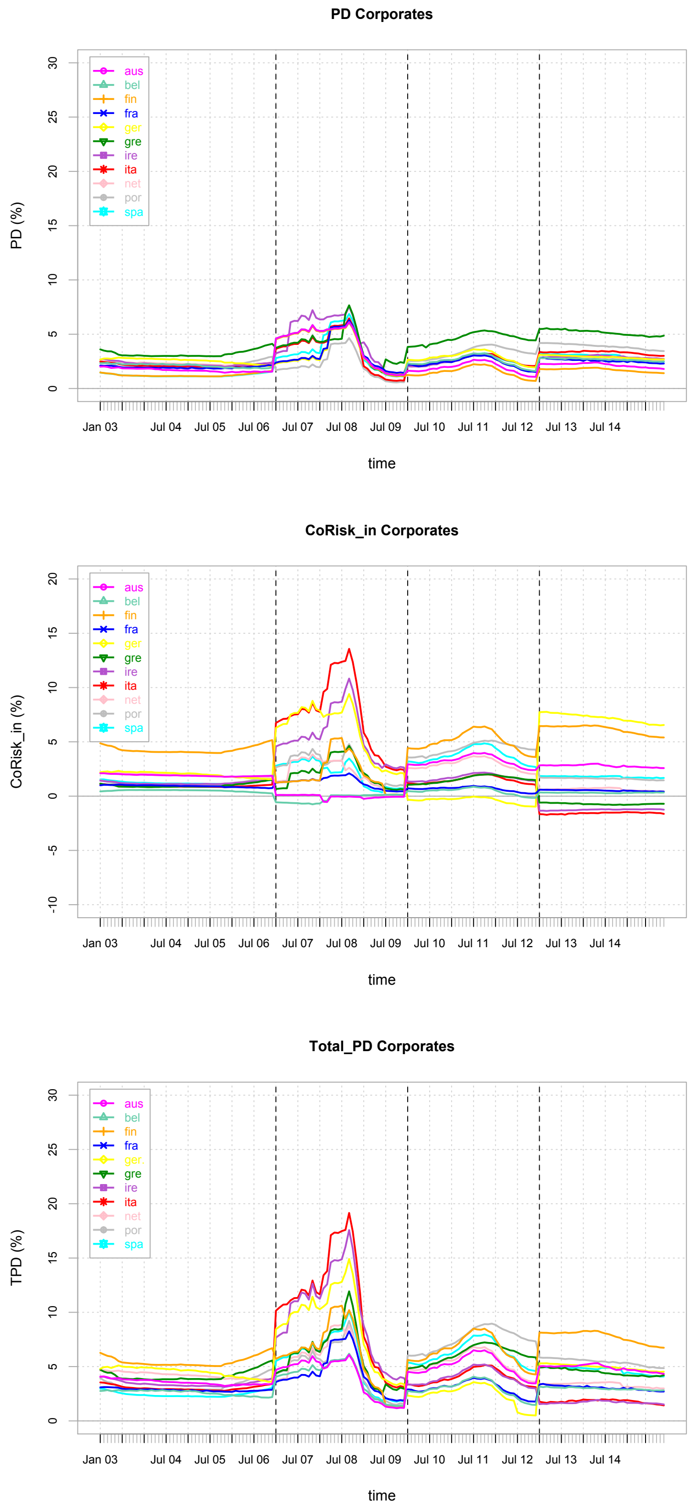
Notes: partial correlation networks for the 11 european countries considered in the sample, based on the spread measures for sovereigns $Z^i_{t,1}$ *(left), corporates* $Z^i_{t,2}$ *(middle) and banks* $Z^i_{t,3}$ *(right), during the pre-crisis (first row), financial-crisis (second row), sovereign-crisis (third row) and post-crisis (fourth row) periods. Green lines stand for positive partial correlations, red lines for negative correlations; the ticker the line, the stronger the connection.*

Figure 5.10.5: Sector-specific PD, $CoRisk_{in}$ and TPD, Sovereigns



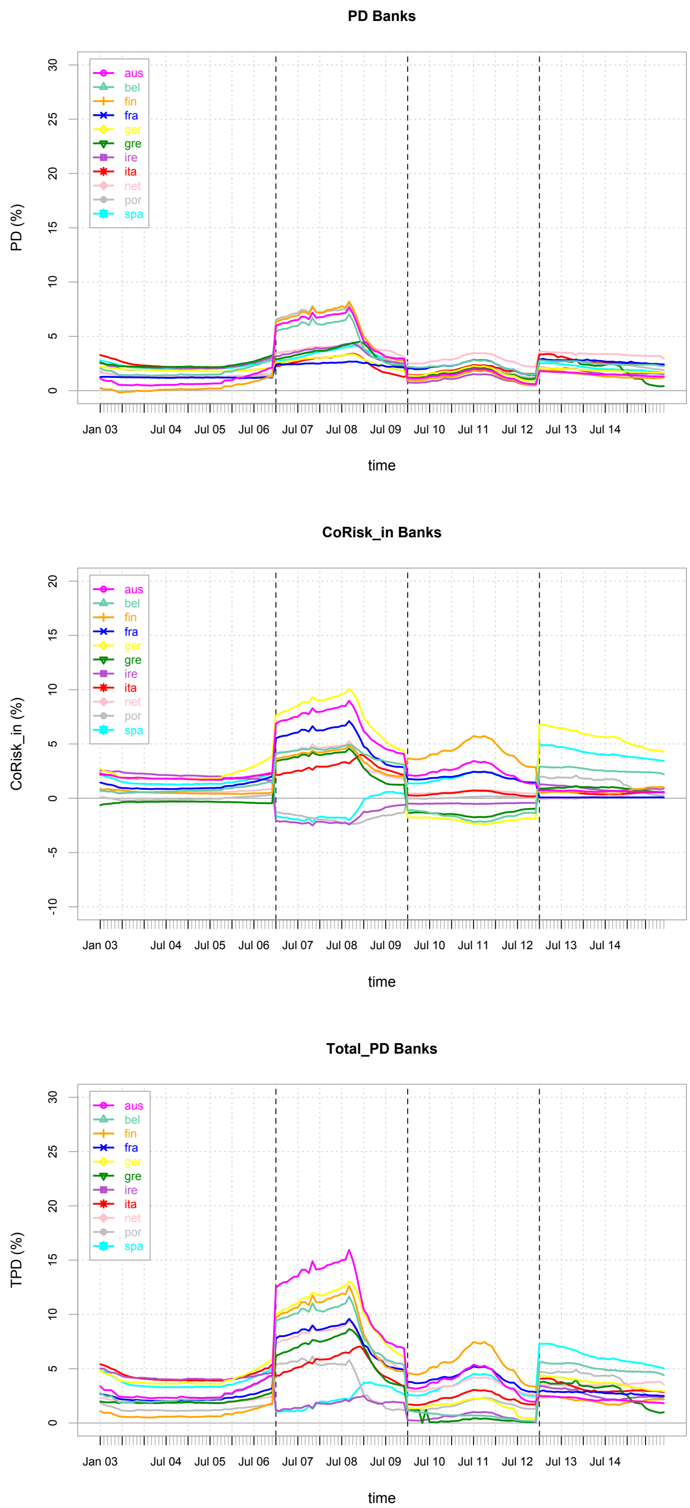
Notes: Sector-specific default probabilities $PD^i_{t,1}$ (top), $CoRisk_{in}$ measures (middle) and total default probabilities $TPD^i_{t,1}$ (bottom) from 2003 until 2015 for the sovereign sector, referred to 11 Eurozone countries. The clustering effect implies that large peripheral countries (such as Italy and Spain) are negatively affected by positive correlations with each other (negative contagion: $CoRisk_{in} > 0$), while smaller core countries (such as Austria and Finland) take advantage of negative correlations with peripheral economies (positive contagion: $CoRisk_{in} < 0$, interpreted as a flight to quality), thus decreasing their own default probability.

Figure 5.10.6: Sector-specific PD, $CoRisk_{in}$ and TPD, Corporates



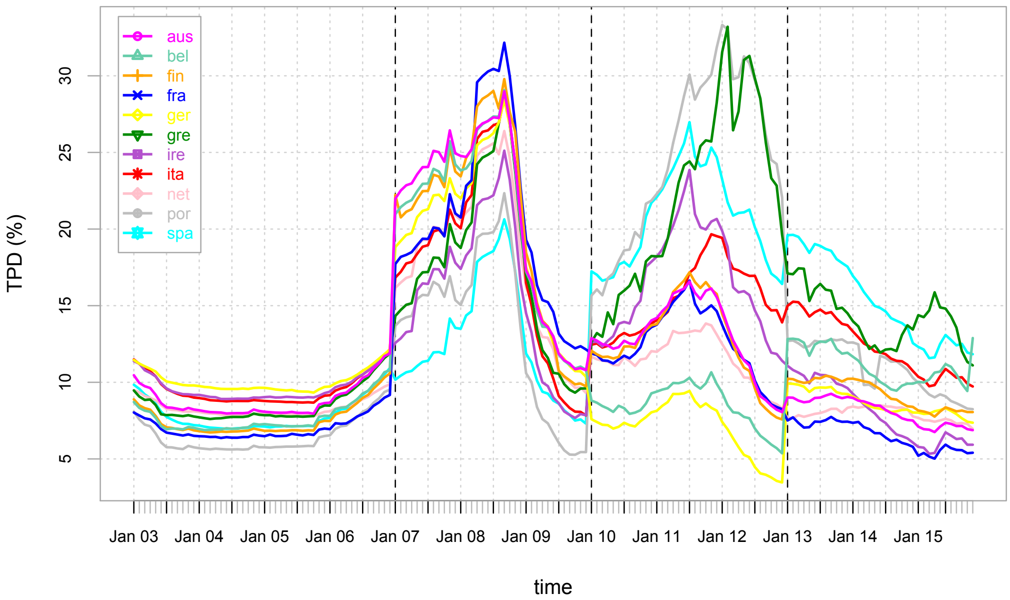
Notes: Sector-specific default probabilities $PD^i_{t,2}$ *(top),* $CoRisk_{in}$ *measures (middle) and total default probabilities* $TPD^i_{t,2}$ *(bottom) from 2003 until 2015 for the corporate sector, referred to 11 Eurozone countries. The* $CoRisk_{in}$ *pattern shows that almost all countries suffered contagion effects during the financial crisis and, to a lesser extent, during the sovereign crisis.*

Figure 5.10.7: Sector-specific PD, $CoRisk_{in}$ and TPD, Banks



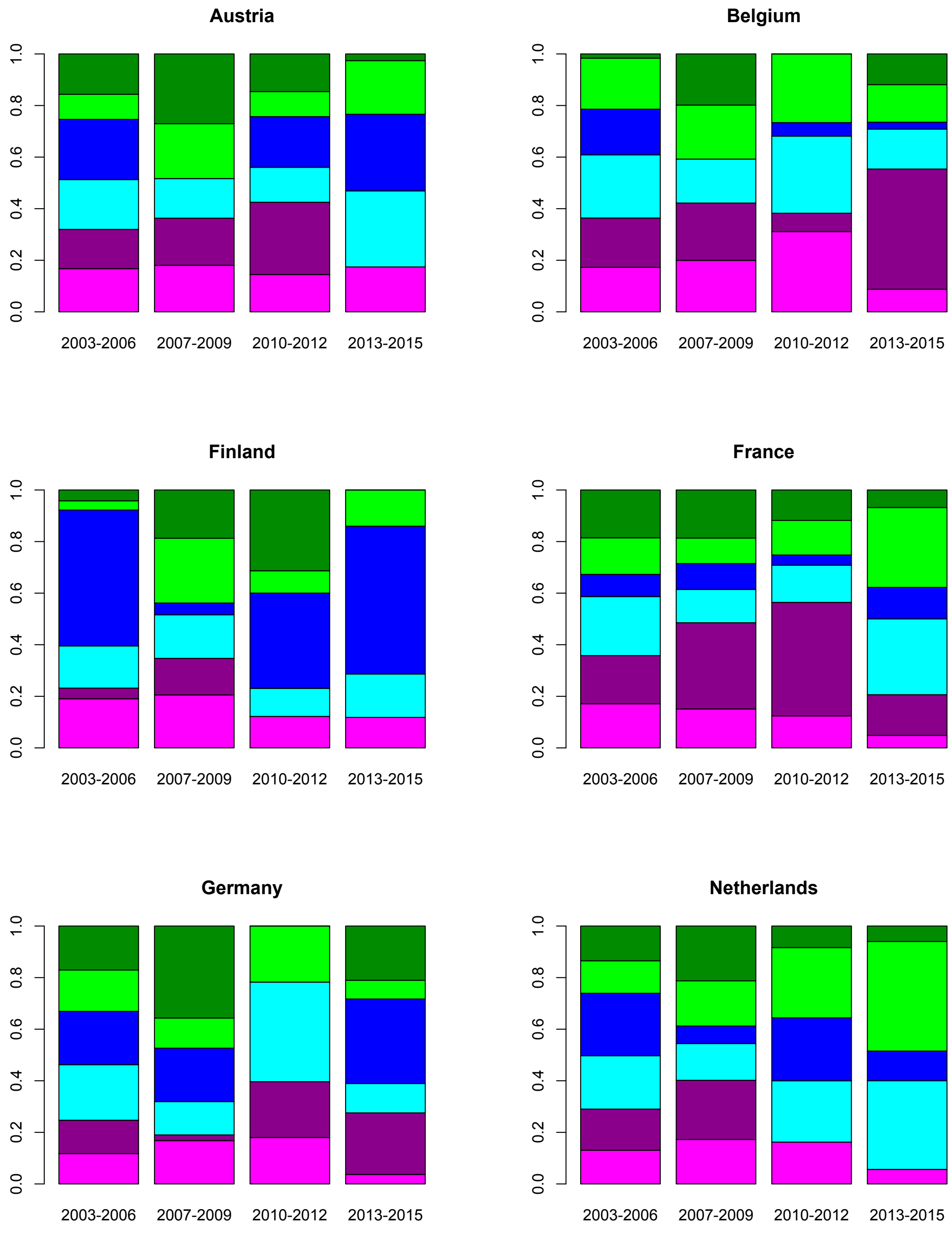
Notes: Sector-specific default probabilities $PD^i_{t,3}$ (top), $CoRisk_{in}$ measures (middle) and total default probabilities $TPD^i_{t,3}$ (bottom) from 2003 until 2015 for the banking sector, referred to 11 Eurozone countries. The $CoRisk_{in}$ pattern shows both negative and positive contagion effects during the second time-period, with the former regarding core countries and the latter peripheral ones; in 2012 it started increasing both in core and peripheral economies because of highly positive partial correlations within each cluster.

Figure 5.10.8: Aggregate total default probabilities $TPD_{country}$



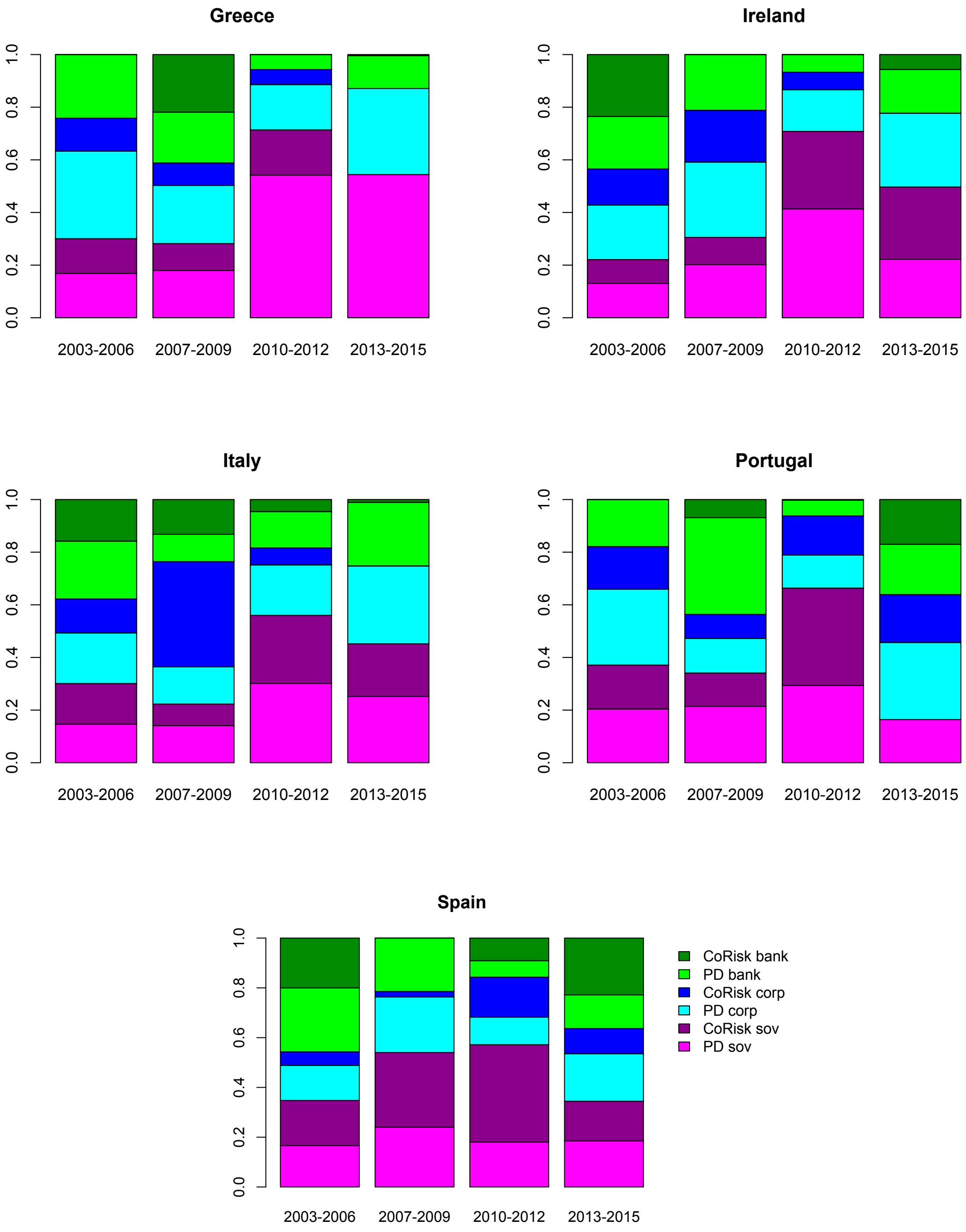
Notes: total default probabilities aggregated at the country level $TPD^{i}_{country}$*, from 2003 until 2015 and referred to 11 Eurozone countries. Th results show that (a) the financial crisis has had a more homogenous impact across countries than the sovereign one; (b) during the pre-crisis years default probabilities were almost constant and stable across time, and very homogenous across countries, but after the sovereign crisis the situation has become more heterogenous and volatile, with a clear distinction between peripheral and core economies and Ireland joining the latter.*

Figure 5.10.9: Risk contributions, Core countries



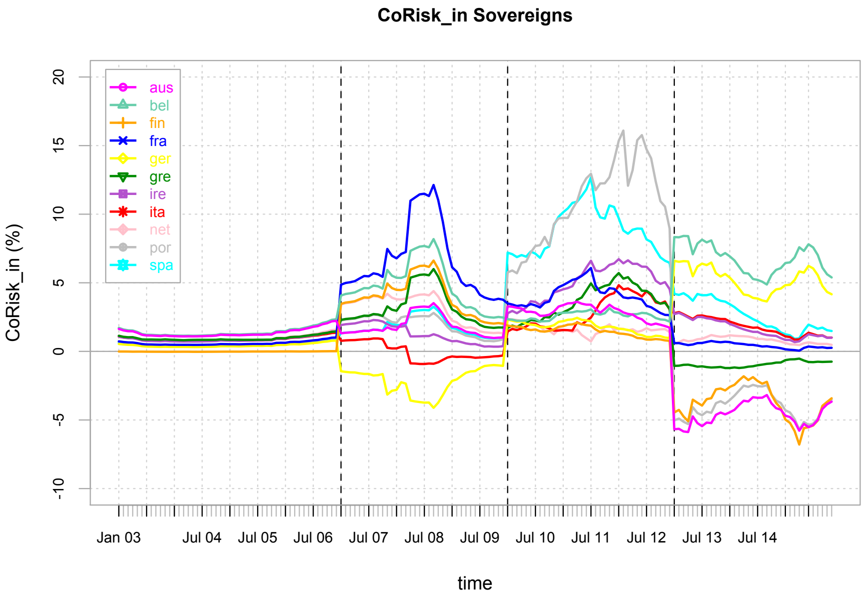
Notes: aggregate total default probabilities contributions: $CoRisk_{in}$ *and* PD *percentage components for the three economic sectors, averaged over the four time periods (pre-crisis, financial-crisis, sovereign-crisis, post-crisis) and referred to the 6 Eurozone core countries (Austria, Belgium, Finland, France, Germany, the Netherlands). The main component of sovereign risk is due to contagion effects; similarly, core economies suffered a substantial improvement in contagion effects for the bank sector during the sovereign crisis through their exposition to peripheral banks.*

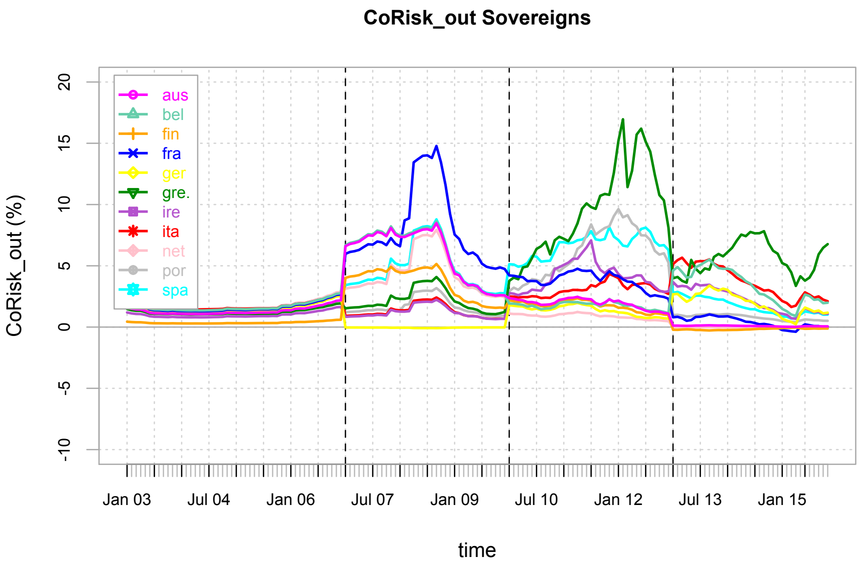
Figure 5.10.10: Risk contributions, Peripheral countries



Notes: aggregate total default probabilities contributions: $CoRisk_{in}$ *and* PD *percentage components for the three economic sectors, averaged over the four time periods (pre-crisis, financial-crisis, sovereign-crisis, post-crisis) and referred to the 5 Eurozone peripheral countries (Greece, Ireland, Italy, Portugal, Spain). The sovereign contribution in peripheral countries is larger than in core ones across time, and is mainly due to its higher sector-specific* PD *component.*

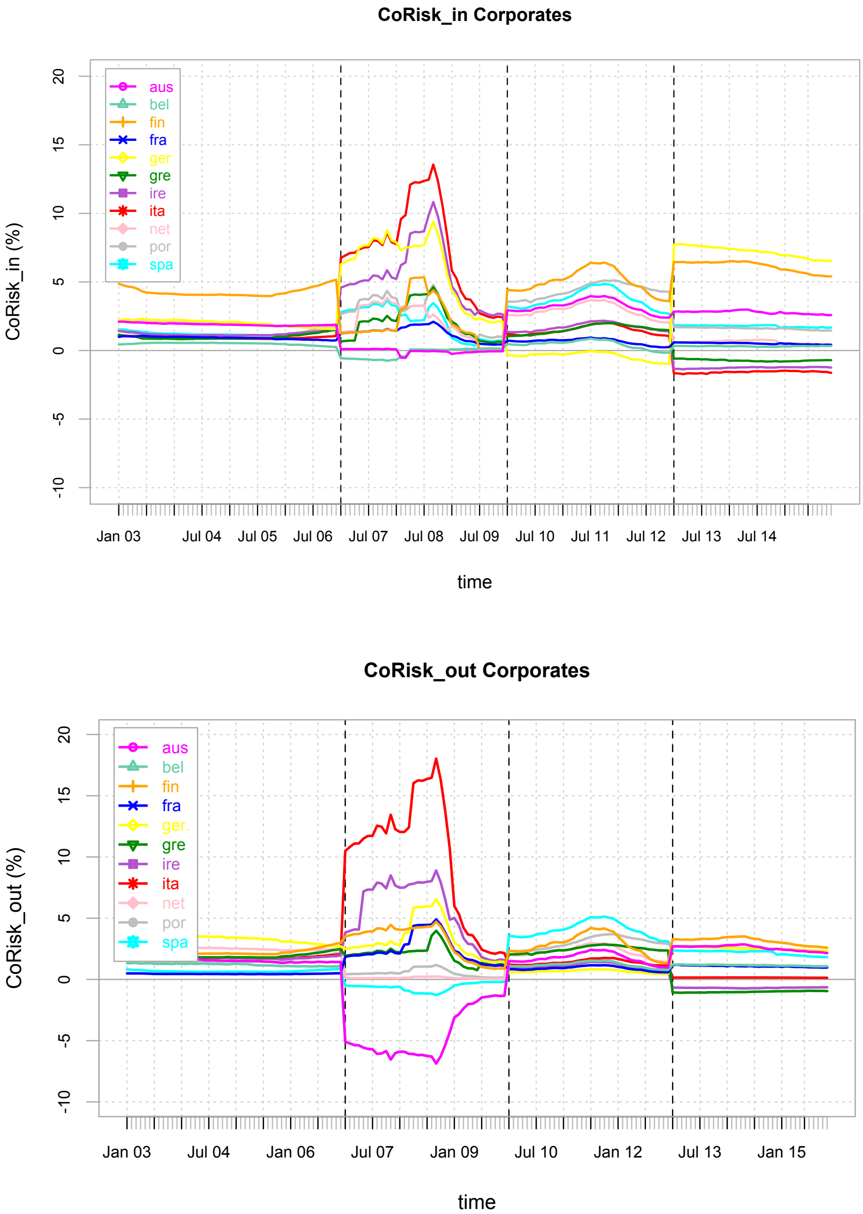
Figure 5.10.11: $CoRisk_{in}$ vs $CoRisk_{out}$, Sovereigns





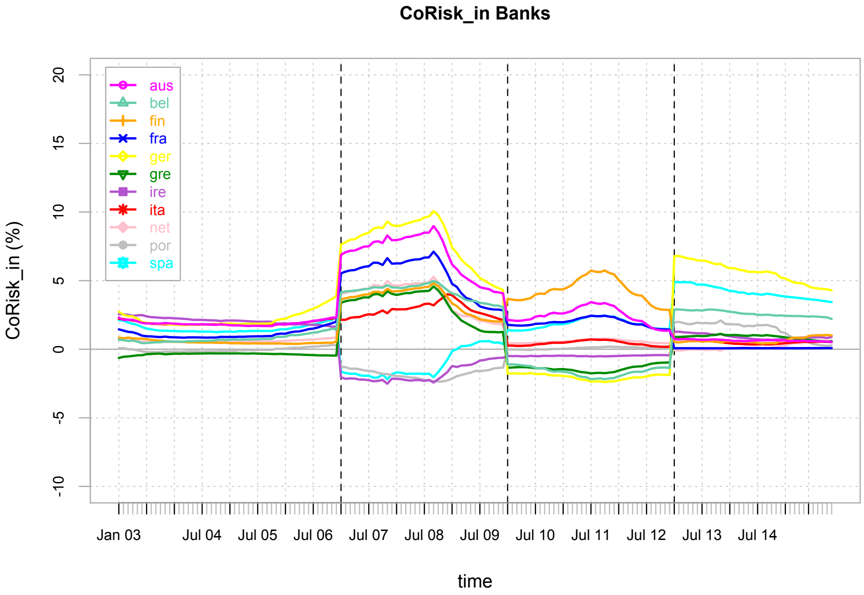
Notes: comparison between $CoRisk_{in}$ (top) and $CoRisk_{out}$ (bottom), from 2003 until 2015 for the sovereign sector and referred to 11 Eurozone countries. The results show that (a) during the pre-crisis and the financial crisis periods the two measures look very similar, meaning that sector-specific default probabilities were homogenous across countries; (b) important differences, distinguishing between core and peripheral countries, started emerging during the sovereign crisis; (c) there are not negative $CoRisk_{out}$ measures for the sovereign sector, meaning that all Euro countries contribute to increase the default probability of their neighbours.

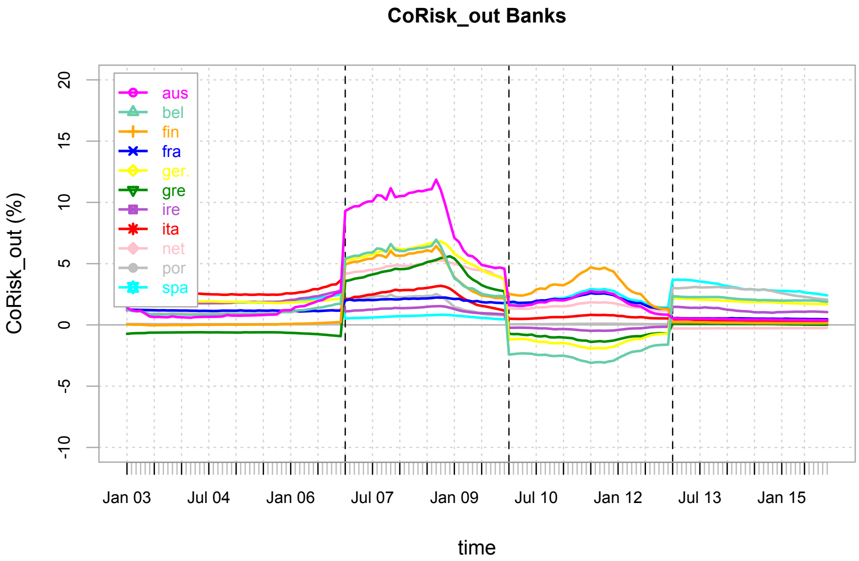
Figure 5.10.12: $CoRisk_{in}$ vs $CoRisk_{out}$, Corporates



Notes: comparison between $CoRisk_{in}$ (top) and $CoRisk_{out}$ (bottom), from 2003 until 2015 for the corporate (middle) sector and referred to 11 Eurozone countries. The results show that peripheral countries are characterised by higher $CoRisk_{out}$, while core economies have higher $CoRisk_{in}$.

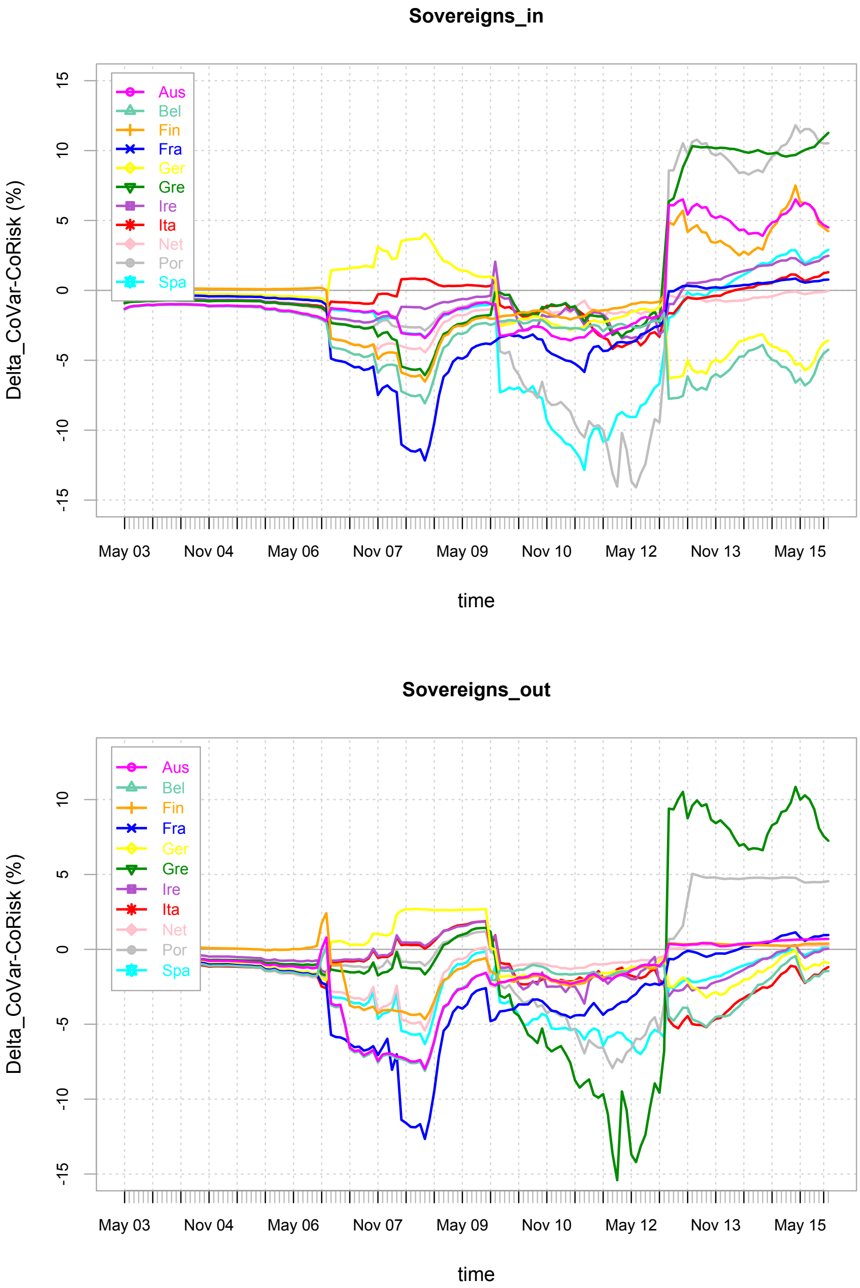
Figure 5.10.13: $CoRisk_{in}$ vs $CoRisk_{out}$, Banks





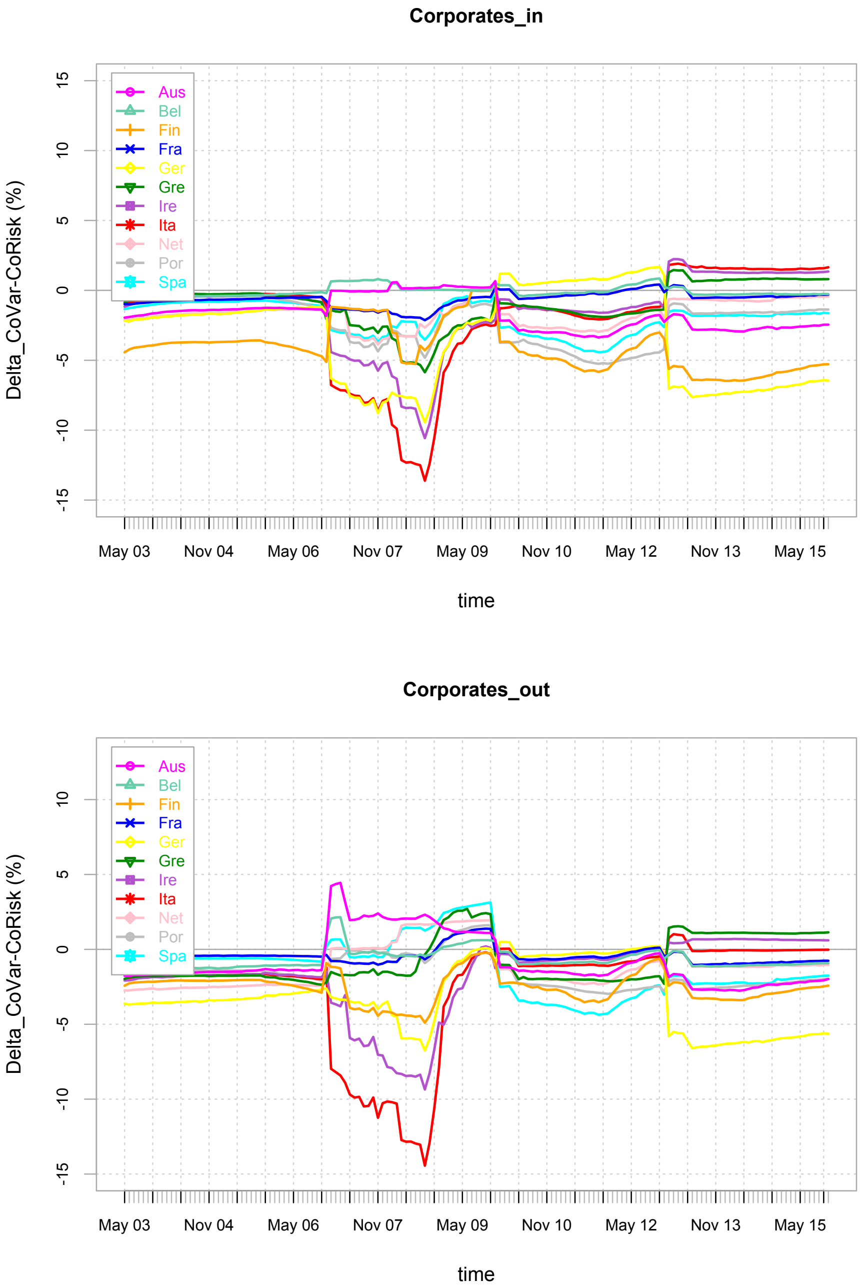
Notes: comparison between $CoRisk_{in}$ *(top) and* $CoRisk_{out}$ *(bottom), from 2003 until 2015 for the banking sector and referred to 11 Eurozone countries.*

Figure 5.10.14: $\Delta CoVar$ vs $CoRisk$, Sovereigns



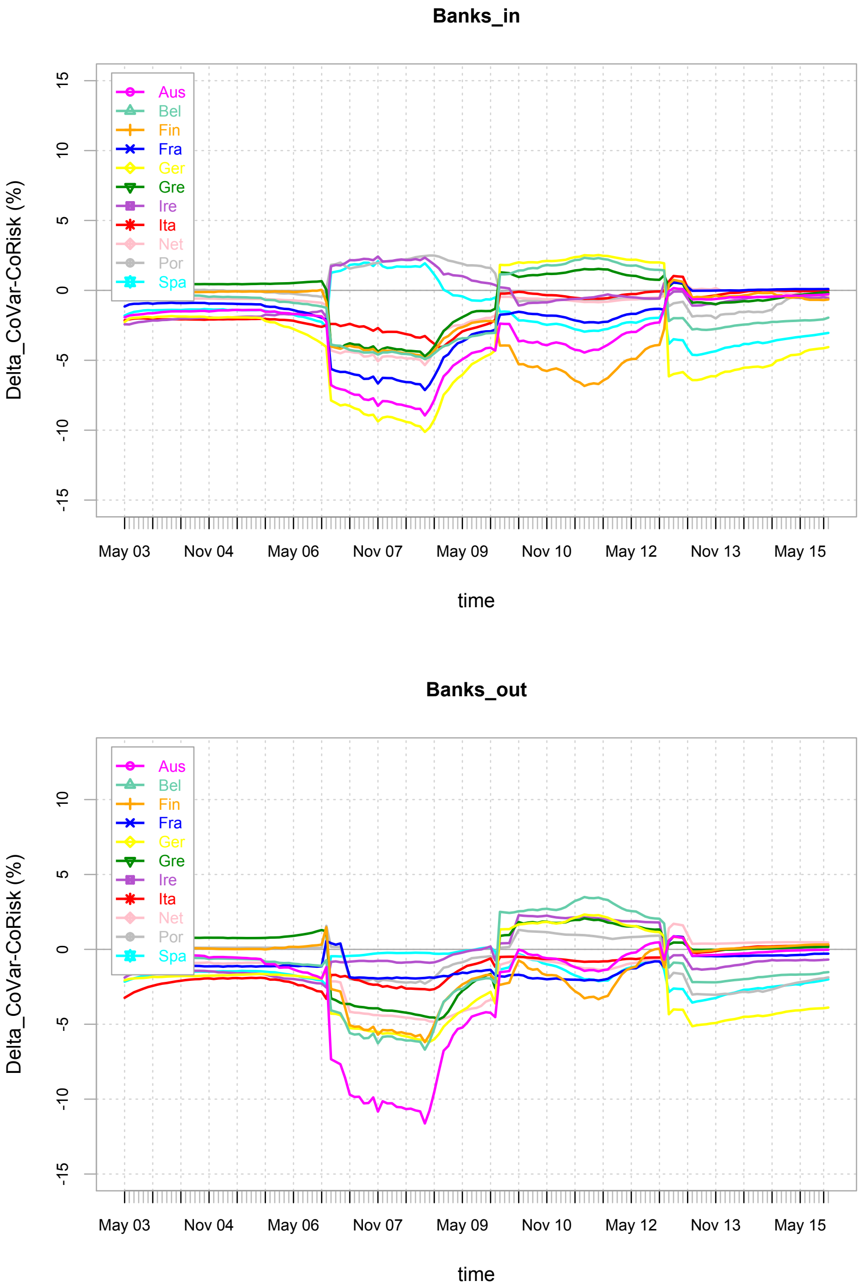
Notes: difference between $\Delta CoVar_in$ and $CoRisk_in$ (top) and $\Delta CoVaR_out$ and $CoRisk_out$ (bottom), from 2003 until 2015 for the sovereign sector and referred to 11 Eurozone countries ($q = 0.95$).

Figure 5.10.15: $\Delta CoVar$ vs $CoRisk$, Corporates



Notes: difference between $\Delta CoVaR_in$ and $CoRisk_in$ (top) and $\Delta CoVaR_out$ and $CoRisk_out$ (bottom), from 2003 until 2015 for the corporate sector and referred to 11 Eurozone countries ($q = 0.95$).

Figure 5.10.16: $\Delta CoVar$ vs $CoRisk$, Banks



Notes: difference between $\Delta CoVaR_in$ and $CoRisk_in$ (top) and $\Delta CoVaR_out$ and $CoRisk_out$ (bottom), from 2003 until 2015 for the banking sector and referred to 11 Eurozone countries ($q = 0.95$).

Chapter 6

Bail-in: a systemic risk perspective[1]

***European Financial Management Journal*[2]**

Abstract

Systemic risk can be measured as a probabilistic "add-on" to the probability of default of an institution, due to contagion effects from the neighbours. Here we focus on financial institutions in order to investigate the relative advantage, in terms of systemic risk, of a bail-in versus a bail-out decision, using both the bank's and the system's perspective . Our proposed methodology will be applied to banks for which CDS spreads are available. In particular, we will focus on the Italian banking system. The results show that the bail-out of a troubled bank is more convenient for the smaller, safer and highly correlated banks. From a system's viewpoint, the failure of a big rather than a small bank considerably increases the total expected losses of a banking system in case of bail-in.

Keywords: Default probabilities, Bail-in, Bail-out, Banks, CDS spreads, Systemic risk.

JEL: C21, C58, E44, G21.

6.1 Introduction

The study of systemic risk is particularly problematic, because of the high number of dimensions that can be included: accordingly, different perspectives have been adopted and, therefore, different econometric measurement models have been used and applied to a variety of data, in different geographical regions and periods. One of the main actors contributing to systemic risk is the banking sector. Financial institutions represent a peculiar case within the economic system, because they can be considered as the main driver, or the most important source of systemic risk; as a consequence, they are subject to special regulations, so that the study of systemic risk applied to the banking sector has to deal with macroprudential policy issues. For this reason, we will firstly provide a brief and general introduction on systemic risk; we will then discuss the new bail-in rules introduced in Europe in January 2016, which explain the choice of concentrating on

[1]Joint work with P. Giudici (University of Pavia).

[2]First, major revision.

the consequences of bail-in resolutions and the methodology that will be applied in the following Sections.

6.1.1 Systemic Risk

For simplicity, we have chosen two discriminant factors (cross-sectional or time-dependent) in order to divide systemic risk models into three main categories: bivariate models, causal models and network models. While the first two explicitly deal with the time-dimension, in an endogenous or in an exogenous way, the latter focuses on the cross-sectional dimension.

Bivariate Models. From a chronological viewpoint, the first systemic risk measures have been proposed for the financial sector, in particular by Acharya et al. (2010), Adrian and Brunnermeier (2011), Brownlees and Engle (2012), Acharya et al. (2012), Dumitrescu and Banulescu (2014) and Hautsch et al. (2015). On the basis of market share prices, these models consider systemic risk as endogenously determined and calculate, as in the classical market risk approach, appropriate percentiles of the estimated loss probability distribution of a bank, conditional on an extreme event in the financial market.

The above described methodology is useful to identify the most *systemically important* institutions, since its bivariate nature allows the derivation of conditional default probabilities or losses during shock events in the reference market, possibly caused by other institutions. However, it does not address the issue of how risks are transmitted between different institutions in a multivariate framework.

Causal Models. A different stream of research considers systemic risks as exogenous factors and has been proposed, among others, by Chong et al. (2006), Longstaff (2010) and Shleifer and Vishny (2010), who examined the impact of monetary policies on default probabilities for the banking sector, with a particular focus on crisis periods. More general causal models, proposed by Duffie and Lando (2001), Lando and Nielsen (2010), Koopman et al. (2012), Betz et al. (2014) and Duprey et al. (2015), explain whether the default probability of a bank, a country, or a company depends on a set of exogenous risk sources, thus combining idiosyncratic with systematic factors.

While powerful from an early warning perspective, causal models, similarly to bivariate ones, concentrate on single institutions rather than on the economic system as a whole and, therefore, underestimate systemic sources of risk arising from contagion effects within the system.

Network Models. In order to address the multivariate nature of systemic risk, researchers have recently proposed financial network models, able to combine the rich structure of network models (see, e.g., Lorenz et al., 2009; Battiston et al., 2012) with a parsimonious approach based on the dependence pattern among market prices. The first contributions in this framework are Billio et al. (2012) and Diebold and Yilmaz (2014), who derive connectedness measures based on Granger-causality tests and variance decompositions. Barigozzi and Brownlees (2013), Ahelegbey et al. (2015) and Giudici and Spelta (2016) extend such methodology introducing correlation network models, while Das (2015) derives a systemic risk decomposition into individual and network contributions.

Financial network models are very useful for understanding the most important contagion channels in a cross-sectional perspective, thus identifying the most *vulnerable* institutions. However, since they are built on cross-sectional data, they can not be used as predictive models in a time-varying context. Moreover, the importance of each institution only depends on its position in the graph, and not on its specific risk.

Combined Approach. Bivariate and causal models explain whether the risk of a bank, a company, or a country is affected by a market crisis event or by a set of exogenous risk factors; financial network models explain whether the same risk depends on contagion effects. Giudici and Parisi (2016) improve all these three categories in the context of country risk, introducing multivariate stochastic processes and combining them with correlation network models, thus idiosyncratic default probabilities into a total default probability that takes contagion into account. Doing so, they merge the advantages of bivariate models (endogeneity and non-linearity), causal models (predictive capability) and correlation networks (contagion channels).

6.1.2 Single Resolution Mechanism

The Single Resolution Mechanism (SRM), developed by the Single Resolution Board (SRB) in conjunction with national resolution authorities, became fully operational on 1 January 2016. It consists in a bail-in tool that has to be applied in all the 19 members of the Eurozone, and it enables the resolution authority to write down the claims of a broad range of creditors according to a predetermined hierarchy. More precisely (see SRM Regulation, 2014, for further details), the SRM Regulation prescribes that all the liabilities of a bank can be bailed-in, a part from: (a) secured or collateralised liabilities (including covered bonds); (b) deposits covered by deposit insurance; (c) interbank liabilities with an original maturity of less than seven days. In addition, the bail-in hierarchy follows a waterfall process, where junior liabilities are bailed-in first, followed by the more senior ones.

The bail-in tool can be used in a resolution mechanism if one bank in the system has been assessed by the supervisor or the resolution authority as likely to fail. In this case, before proceeding with the bail-in resolution two conditions must hold: (a) a private resolution is not possible, and (b) a bail-in action is necessary from a public interest viewpoint.

The SRM has been introduced with the following main objectives: to swiftness the resolution process, thus minimising the impacts on the economy; to privatise losses in order to break up the linkage and the feedback loop between bank risk and sovereign risk; to mitigate moral hazard incentives associated with too-big-to-fail financial institutions. Regarding this last issue, in fact, we should underline that, in the past, authorities had the only option of a public bail-out for systemically important financial institutions, usually associated with huge costs for taxpayers and negative consequences for the economy. As pointed out by Halaj et al. (2016), bail-outs create wrong incentives for internal risk managers and moral hazard problems, since large banks are perceived as more likely to be bailed-out, and can thus fund themselves more cheaply than smaller banks. In this sense, a common resolution framework can shift the costs of a bank failures from taxpayers to shareholders and creditors.

The objective of this paper is not to investigate whether the SRM Regulation is good or not for banks and countries belonging to the Eurozone: we take it as the context in which we develop our analysis. Consistently with the rules explained so far, we will limit ourselves in comparing the systemic effects of a bail-in resolution rather than a private bail-out, as will better shown in the next Section.

6.1.3 Proposal

In this work we propose to extend the *combined approach* described in Section 6.1.1 at the micro level: in particular, we combine expected losses of financial institutions with correlation networks, thus deriving time-dependent measures able to explain to what extent the expected loss of each financial institution is affected by contagion effects that come from the variation in the expected losses of the others.

The proposed methodology will be employed to analyse the main differences between bail-in and bail-out scenarios, that may occur in case one financial institution is close to its default point. In particular, we will simulate two alternatives: (a) the "troubled" institution defaults, thus affecting the other banks in the system through contagion propagation; (b) the "troubled" institution is helped by the other banks in the system through a capital-injection operation. In the first situation, that we will call the bail-in scenario, the troubled bank's default affects its neighbours through a shock in their expected losses as a consequence of contagion effects: however, after a while the bank system will reach a new equilibrium, without the defaulted bank and, thus, affected by less contagion risk. In the second situation, that we will call the private bail-out scenario, the troubled bank does not default and, consequently, does not affect the others through a shock in their default probabilities; however, it continues being part of the banking system, so that all the other banks in the network will still be affected by the high contagion risk coming from its persistence in the system.

The design of these two scenarios allows to establish which banks in the system would benefit from a bail-out, rather than from a bail-in scenario. More precisely, each bank can choose the scenario that leads to its lowest total probability of default. In addition to this bank perspective, we will derive a measure for the total default probability of the entire banking system: in particular, we will model contagion between financial institutions as a cascade effect, thus deriving how strong is the impact of a bail-in or a bail-out scenario on the system.

The above described research design will be firstly applied to the stylised case of three banks: two safe banks (one much bigger than the other) and one troubled bank. We will then extend the application to the Italian banking system: since we need CDS spreads for deriving default probabilities, we will focus on the eight larger banks for which such data are available. We will then focus on the Italian banking system. This is a particularly interesting case study since in early 2016 Italian banks have organised themselves by supporting an equity fund, called Atlante, which has, among its main aims, the recapitalisation of "troubled" financial institutions. Each bank has decided, on voluntary basis, whether to allocate capital in the Atlante fund: as a result, a medium size lender, Banca Popolare di Vicenza, that had been found strongly under capitalised by the European Central Bank regulatory supervisor, has been recapitalised with the help of most of the banks in the system, thus avoiding bail-in.

Through simulation exercises, we will examine wether the choice of each bank (to take part in a bail-out or not) can be considered as the best one from a systemic risk perspective; in particular, we will examine whether and by how much the advantage of choosing a bail-out rather than a bail-in scenario depends on (a) the default probability and the size of the safe banks and of the troubled bank; (b) correlations with the troubled bank; (c) correlations between safe banks. We will then examine these relationships from the system perspective. All simulations will be performed considering as troubled bank respectively a large (Monte dei Paschi di Siena) and a small (Banca Carige) bank. Finally, we will compare our results with those obtained for the Atlante bail-out.

Our results can be summarised as follows. First, in the stylised setting of three banks, the simulation results reveal that the smaller or the safer a bank is, the larger the advantage of choosing a bail-out scenario. The advantage increases with the correlation with the troubled bank; it decreases with the correlation between the safe banks and it decreases with the default probability of the troubled bank. Second, the application to the Italian banking system reveals that, on the long-run, the bail-out should always be preferred to the bail-in resolution from a system's perspective. Such preference, moreover, strongly increases as the size of the troubled bank increases. Regarding the banks perspective, the correlation pattern is the main driver for the choice of a bail-in or a bail-out.

The paper is structured as follows: Section 6.2 provides the methodological framework, with Section 6.2.2 describing the contagion effects on individual banks and Section 6.2.3 describing the systemic effects of bail-in and bail-out. Section 6.3 presents the comparison between bail-in and bail-out scenarios and proposes the simulation results deriving from the application of our model to a stylised system of three banks. Section 6.4 describes the results obtained for the Italian banking system, while Section 6.5 provides the application of the proposed methodology to the real case of the Atlante equity fund. Finally, Section 6.6 concludes with some final remarks.

6.2 Methodology

6.2.1 Systemic risk measurement

Consider a set V of N banks, with elements $m \in V = \{1, ..., N\}$. For each bank m we introduce a measure for its expected losses, derived as the product between its assets A^m and its default probability PD^m, in the worst case situation of a null recovery rate:

$$EL^m = A^m \cdot PD^m. \tag{6.2.1}$$

In the following, we show that such expected losses can be used to build correlation networks that transmit contagion between different banks. The final result will be a total default probability, TPD^m, able to incorporate bank-specific PDs and further contagion components.

Let C be the marginal correlation matrix between the expected losses of the N banks in the system, based on the following structure:

$$\mathrm{Corr}[EL^m, EL^n] = \rho_{mn}. \tag{6.2.2}$$

The correlation matrix C can be employed to derive correlation networks between banks (following Billio, 2012; Ahelegbey et al., 2015; Giudici and Spelta, 2016). However, such correlations can be misleading because they take into account bivariate (marginal) relationships which may be spurious. For this reason we propose to employ conditional (partial) correlations, different from bivariate ones as they are adjusted by the presence of all the other institutions in the system. Let C^{-1} be the inverse of the correlation matrix, with elements c^{mn}. The partial correlation coefficient $\rho_{mn|S}$ between variables EL^m and EL^n, conditional on the remaining variables in V: S, can be obtained as:

$$\rho_{mn|S} = \frac{-c^{mn}}{\sqrt{c^{mm}c^{nn}}}. \tag{6.2.3}$$

In order to better explain partial correlations and their differences with respect to marginal ones, we now report a useful and interesting property. For any two elements $\{m, n\} \in V$, set $S = V \setminus \{m, n\}$ and suppose, similarly as in Giudici and Parisi (2016), to express the dependence between expected losses through multiple linear models in the following way:

$$\begin{cases} EL^m = c^m + \sum_{n \neq m} c_{mn|S} EL^n; \\ EL^n = c^n + \sum_{m \neq n} c_{nm|S} EL^m. \end{cases} \tag{6.2.4}$$

It can be shown that the partial correlation coefficient between EL^m and EL^n, given all the other $N-2$ measures, can be interpreted as the geometric average between the multiple linear coefficients in (6.2.4):

$$|\rho_{mn|S}| = |\rho_{nm|S}| = \sqrt{c_{mn|S} \cdot c_{nm|S}}. \tag{6.2.5}$$

Note that in case of only two components ($S = \varnothing$), equation (6.2.4) becomes:

$$\begin{cases} EL^m = c_m + c_{mn} EL^n \\ EL^n = c_n + c_{nm} EL^m, \end{cases} \tag{6.2.6}$$

from which the marginal correlation coefficient ρ_{mn} can be derived as the geometric average between the coefficients in (6.2.6):

$$|\rho_{mn}| = |\rho_{nm}| = \sqrt{c_{mn} \cdot c_{nm}}.$$

We propose to build a correlation network based on partial correlations rather than on marginal correlations. To achieve this aim we introduce an undirected graph $G = (P, E)$, with a vertex set $P = V = \{1, ..., N\}$ and an edge set $E = P \times P$. Such edge set is defined by binary elements e_{mn} that describe whether pairs of vertices are (symmetrically) linked to each other ($e_{mn} = 1$) or not ($e_{mn} = 0$), depending on whether the partial correlation coefficient between the corresponding pair of variables is equal to zero or not.

6.2.2 Contagion effects on individual banks

The probability of default derived in (6.2.1) is bank-specific, as it is assumed independent from the default probability of other institutions: in our view this is an unrealistic assumption, since different banks are interrelated and depend on each other, as can be easily found looking at co-movements between the CDS spreads from which the default probabilities can be obtained. We thus propose to evolve the PD into a total default probability, TPD, able to incorporate both sector-specific and contagion components. For each bank m we define a "total" expected loss TEL^m, expressed as a linear function of a "baseline" loss EL^m, which depends exclusively on the bank m, and of a further component, which depends on the loss measures EL^n of the other banks $n \neq m$:

$$TEL^m = EL^m + \sum_{n \neq m} a_{mn|S} EL^n. \tag{6.2.7}$$

By substituting the coefficients $c_{mn|S}$ with their geometric averages $\rho_{mn|S}$ (obtained from the inverse of the correlation matrix C^{-1} consistently with (6.2.5)), we obtain that:

$$TEL^m = EL^m + \sum_{n \neq m} \rho_{mn|S} EL^n, \tag{6.2.8}$$

on which we place the following economic constraints:

$$\begin{cases} TEL^m = \min(A^m, TEL^m) & \text{if } TEL^m > 0, \\ TEL^m = \max(0, TEL^m) & \text{if } TEL^m < 0. \end{cases} \tag{6.2.9}$$

Note that, in analogy with the baseline expected loss, the total expected losses can be expressed as the product between the total assets A^m and a default probability, that we name TPD^m (total default probability). Equation (6.2.8) thus becomes:

$$TPD^m = PD^m + \sum_{n \neq m} \rho_{mn|S} \cdot PD^n \cdot \frac{A^n}{A^m}, \qquad (6.2.10)$$

which shows that TPD^ms add a further component to the standard PD: such spillover effects derive from the propagation of the PDs of the other banks, and are "mediated" by partial correlation coefficients and relative capitalisation sizes.

We can now introduce a time dimension. Each bank, in fact, should be able to evaluate systemic risk in a long-term perspective. We can think of a discrete timeline, made up by a number M of key events: $T = \{t_1, ..., t_M\}$. Each bank can evaluate which is its default probability after the occurrence of those events, by aggregating its TPD over time as follows:

$$TPD^m_{t_j} = 1 - \prod_{\substack{t_i \in T \\ i \leq j}} (1 - TPD^m_{t_i}). \qquad (6.2.11)$$

6.2.3 Contagion effects on the banking system

In the previous section we have derived the total default probabilities and expected losses of each institution operating in a given market. In other words, we have considered how much the institution-specific default probability can be affected by the propagation of its neighbours' PDs through the network. Such an approach is bank-based, meaning it allows each single bank to understand how much it is vulnerable in the system, and how great can its expected losses be.

An alternative approach consists in considering the entire system: more precisely, instead of focusing on single institutions, we can derive a total default probability and a total expected loss for the banking system considered as a whole. Let us suppose our banking system is composed by N institutions, each of them characterised by a market-implied default probability PD^i. At each time t, such probabilities can propagate through the network in a sort of "cascade effect", thus affecting each other and, consequently, the entire system. The total default probability of the system can thus be considered as the joint default probability of all the institutions composing it: as a consequence, the total expected loss of the system turns out to be the product between this joint default probability and the sum of the capitalisations of each bank.

More formally, we define the total loss of the system as:

$$TEL^{system} = \left[\sum_{i=1}^{N} A^i\right] \cdot \left[Pr\left(\bigcap_{i=1}^{N} D^i\right)\right], \tag{6.2.12}$$

where D^i represents the default event of bank i. Since default events are not independent but can propagate to each other, the previous equation can be rewritten as follows:

$$TEL^{system} = \left[\sum_{i=1}^{N} A^i\right] \cdot \left[Pr(D^1) \cdot Pr(D^2|D^1) \cdot ... \cdot Pr(D^N|D^1, D^2, ..., D^{(N-1)})\right]. \tag{6.2.13}$$

The conditional probabilities in equation (6.2.13) can be calculated as in (6.2.10), with the conditioning set composed by, respectively, 1, 2, ..., (N-1) institutions. Consistently, the sums in equation (6.2.10) will be, respectively, $\sum_{n=1}$, $\sum_{n=1,2}$, ..., $\sum_{n=1}^{N-1}$. Since the product in (6.2.13) depends on the choice of the order between institutions, let us introduce the following sets of indexes: $I = \{i, l, o, ..., v\}$, $J = \{j, m, p, ..., w\}$ such that the following ordering conditions hold:

$$\begin{cases} i \leq l \leq o \leq ... \leq v; \\ j \leq m \leq p \leq ... \leq w. \end{cases} \tag{6.2.14}$$

Consequently, we will obtain a set of $N-1$ ordered couples of indexes: $\{(i,j), (l,m), (o,p), ..., (v,w)\}$. After some calculations, (6.2.13) becomes:

$$\begin{aligned} TEL^{system} = & \left[\sum_{i=1}^{N} A^i\right] \cdot \left[\prod_{i=1}^{N} PD^N + \right. \\ & + \sum_{i,j} \rho_{ij} (PD^i)^2 \left(\prod_{k \neq i,j} PD^k\right) \frac{A^i}{A^j} + \\ & + \sum_{i,j,l,m} \rho_{ij}\rho_{lm} PD^i PD^l \left(\prod_{k \neq j,m} PD^k\right) \frac{A^i A^l}{A^j A^m} + \\ & + \sum_{i,j,l,m,o,p} \rho_{ij}\rho_{lm}\rho_{op} PD^i PD^l PD^o \left(\prod_{k \neq j,m,p} PD^k\right) \frac{A^i A^l A^o}{A^j A^m A^p} + \\ & + ... \\ & \left. + \sum_{i,j,l,m,o,p,...,v,w} (\rho_{ij}\rho_{lm}\rho_{op}...\rho_{vw})(PD^i PD^l PD^o ... PD^v) \left(\prod_{k \neq (j,m,p,...,w)} PD^k\right) \frac{A^i A^l A^o ... A^v}{A^j A^m A^p ... A^w}\right]. \end{aligned} \tag{6.2.15}$$

In order to rewrite the previous equation in a compact form, let us consider the following matrix of indexes:

$$I = \begin{pmatrix} 11 & 12 & 13 & ... & 1N \\ 21 & 22 & 23 & ... & 2N \\ 31 & 32 & 33 & ... & 3N \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ N1 & N2 & N3 & ... & NN \end{pmatrix},$$

where the highlighted part indicates the coefficients consistent with the conditions in (6.2.14). Let us call $L(I)$ the lower triangular part of matrix I, $L(I[i,1])$ the first column of matrix I and $L(I[1,j]$ the first row of matrix I. We can thus define the following indexes:

$$\begin{cases} \omega \in vec(L(I)), \\ \omega_1 \in vec(L(I[i,1]), \\ \omega_2 \in vec(L(I[1,j]). \end{cases} \tag{6.2.16}$$

It can be shown that (6.2.15) can be rewritten as:

$$\begin{aligned} TEL^{system} = &\left[\sum_{i=1}^{N} A^i\right] \cdot \left[\prod_{i=1}^{N} PD^N + \right. \\ &\left. + \sum_{\omega,\omega_1,\omega_2} \left(\prod_{\omega=1}^{N(N+1)/2} \rho_\omega \prod_{\omega_1=1}^{N} PD^{\omega_1} \prod_{k \neq \omega_2} PD^k \prod_{\omega_1,\omega_2=1}^{N} \frac{A^{\omega_1}}{A^{\omega_2}} \right) \right]. \end{aligned} \tag{6.2.17}$$

Equation (6.2.17) defines the total expected loss of the entire system as the product between the sum of the capitalisations of all the banks and a factor composed by two parts: the first one represents the product between the default probabilities of banks; the second one adds a further component deriving from the propagation of default probabilities through the system. By developing the product, equation (6.2.17) becomes:

$$\begin{aligned} TEL^{system} = &\left[\sum_{i=1}^{N} A^i \cdot \prod_{i=1}^{N} PD^N\right] + \\ &+ \left[\sum_{i=1}^{N} A^i \cdot \sum_{\omega,\omega_1,\omega_2} \left(\prod_{\omega=1}^{N(N+1)/2} \rho_\omega \prod_{\omega_1=1}^{N} PD^{\omega_1} \prod_{k \neq \omega_2} PD^k \prod_{\omega_1,\omega_2=1}^{N} \frac{A^{\omega_1}}{A^{\omega_2}} \right) \right] = \\ &= TEL^{system,1} + TEL^{system,2}. \end{aligned} \tag{6.2.18}$$

In the above equation, $TEL^{system,1}$ calculates the total expected losses of the system in case of independent default probabilities; $TEL^{system,2}$ adds a further component, which represents the expected losses of the system due to contagion effects. Consistently, $TEL^{system,2}$ is equal to zero when all the partial correlation coefficients are null. We remark that $TEL^{system,2}$ is composed by a series of sums, with each term corresponding to a different degree of propagation: the first element represents the PD propagation from the first to the second bank; the second element represents the contagion effect from the first to the second and the third bank, and so on.

6.3 Application to a stylised banking system

In this section we compare, by means of a simulation study, the total default probabilities of each bank under two hypothesis: (a) one bank in the system defaults (bail-in scenario); (b) one bank in the system is in a troubled situation because of its high bank-specific PD, but at some point it is saved by the other banks through a process of capital injection (bail-out scenario). For each bank, the best scenario will be the one with the lowest TPD.

As we have introduced in the previous Section, each total probability of default depends on a set of variables: its default probability, the default probabilities of the other banks, the correlation structure between the banks, and the relative capital sizes. To better understand the dependence of TPD on all these variables, we first propose a stylised simulation exercise. Let us consider a system composed by three banks B^1, B^2 and B^3, with the last one being in a troubled situation, as shown in Figure 6.8.1.

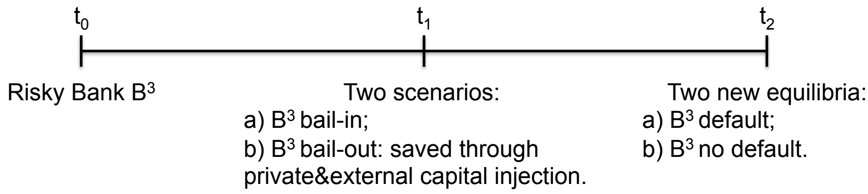
[Figure 6.8.1]

At any time t, all the three banks have an expected loss (EL^m), calculated as the product between their assets (A^m) and their default probability (PD^m): in addition, they are all (directly) correlated to each other through the partial correlation coefficients ρ_{mn}.

In order to understand whether bank 1 (B^1) and bank 2 (B^2) will benefit more from saving bank 3 (B^3) rather than from letting it default, we need to add the time component, so to derive the time evolution of the total default probabilities under the two hypothesis. We consider for simplicity, to have three times, t_0, t_1 and t_2, as shown in Figure 6.3.1.

According to Figure 6.3.1, we suppose to firstly observe the bank system at t_0: at this point, one bank (B^3) reveals to be risky, because of a high default probability. At the following time t_1, two events can occur: a) the bank B^3 defaults, in a Bail-in scenario;

Figure 6.3.1: Timeline



Notes: Simulated time evolution of three banks, under two scenarios. At time t_1 two events can occur: a) B^3 defaults, or b) B^3 is saved by B^1 and B^2. At time t_2 the bank system will reach a new equilibrium, without or with B^3, respectively if event a) or b) has verified.

b) the bank B^3 is "saved" by the other two banks in the system, B^1 and B^2, through a capital injection, in a Bail-out scenario. Finally, at time t_2, the bank system will reach a new equilibrium: a) without B^3 in case it has defaulted; b) with B^3 in case it has been saved by the other banks. In the following Sections we will analyse the two scenarios and compare them in terms of TPD, for the two "safe" banks in the system.

6.3.1 Bail-in scenario

Let us assume, without loss of generality, that each bank keeps the same amount of assets over time: in other words, $A^m_{t_0} = A^m_{t_1} = A^m_{t_2}$. Moreover, we assume that the two safe banks, B^1 and B^2, maintain the same default probability through time: $PD^{1,2}_{t_0} = PD^{1,2}_{t_1} = PD^{1,2}_{t_2}$. Finally, the risky bank B^3 is characterised by its default probability at time t_0, $PD^3_{t_0}$, while in the following time $PD^3_{t_1} = 1$, as it defaults and, then, disappears.

Marginal and, consequently, partial correlation coefficients can be derived from the correlation matrix between the expected losses: in particular, we suppose that the shock B^1 and B^2 receive at time t_1, due to the default of B^3, depends on the correlations between the two safe banks with the risky bank observed in the time just before t_1. For this reason, we will use the same correlation coefficients calculated at t_0 for propagating the default shock at t_1. After B^3 has defaulted, the bank system will be composed of only two banks, B^1 and B^2, which will reach a new equilibrium: the new correlation matrix will thus be a 2×2 rather than a 3×3 matrix, and from its inverse the partial correlation coefficients can be derived. A summary of the involved variables can be observed in Table 6.8.1.

[Table 6.8.1]

Consistently with equation (6.2.10), at each time t_j we can calculate the total default probability of each bank. By substituting the variables summarised in Table 6.8.1 in equation (6.2.10), the following results can be obtained:

$$t_0: \begin{cases} TPD^{1,a}_{t_0} = PD^1 + \rho_{12|S} \cdot PD^2 A^2/A^1 + \rho_{13|S} \cdot PD^3_{t_0} A^3/A^1 \\ TPD^{2,a}_{t_0} = PD^2 + \rho_{12|S} \cdot PD^1 A^1/A^2 + \rho_{23|S} \cdot PD^3_{t_0} A^3/A^2 \\ TPD^{3,a}_{t_0} = PD^3_{t_0} + \rho_{13|S} \cdot PD^1 A^1/A^3 + \rho_{23|S} \cdot PD^2 A^2/A^3 \end{cases} \tag{6.3.1}$$

$$t_1: \begin{cases} TPD^{1,a}_{t_1} = PD^1 + \rho_{12|S} \cdot PD^2 A^2/A^1 + \rho_{13|S} \cdot A^3/A^1 \\ TPD^{2,a}_{t_1} = PD^2 + \rho_{12|S} \cdot PD^1 A^1/A^2 + \rho_{23|S} \cdot A^3/A^2 \\ TPD^{3,a}_{t_1} = 1 \end{cases} \tag{6.3.2}$$

$$t_2: \begin{cases} TPD^{1,a}_{t_2} = PD^1 + \rho^a_{12|S,t_2} \cdot PD^2 A^2/A^1 \\ TPD^{2,a}_{t_2} = PD^2 + \rho^a_{12|S,t_2} \cdot PD^1 A^1/A^2 \end{cases} \tag{6.3.3}$$

Such total default probabilities can be aggregated over time, according to equation (6.2.11), in order to obtain one "overall" default probability, $TPD^{m,a}_T$ for each bank. These results will then be compared with the results obtained from the bail-out scenario ($TPD^{m,b}_T$).

6.3.2 Bail-out scenario

In case B^1 and B^2 decide to "save" B^3 through a capital injection, we assume, without loss of generality, a proportional assets allocation. More precisely, suppose B^3 needs an amount of assets X in order to be saved, with X being equal, for example, to the 8% of its total assets[3]: the other two banks in the system will lend, respectively, a fraction X^1 and X^2 of their assets, proportionally to their capital dimensions, as follows:

$$\begin{cases} X^1 = X \frac{A^1}{A^1+A^2}, \\ X^2 = X \frac{A^2}{A^1+A^2}. \end{cases} \tag{6.3.4}$$

Consistently with (6.3.4), at time t_1 and t_2 the total amount of assets of bank 1 and bank 2 is reduced by the amounts X^1 and X^2, while the assets of bank 3 are increased by an amount X. For what concerns default probabilities, we suppose that, as in the bail-in

[3]We have chosen this quantity consistently with the Total Loss Absorbing Capacity introduced by the Financial Stability Board for European Banks, and with the Minimum Required Eligible Liability criterion.

scenario, B^1 and B^2 maintain their PD over time: the difference lies in PD^3, since B^3 now does not default, and is thus characterised by a default probability $PD^3_{t_2} \neq 1$. As bank 3 has been helped with a recapitalisation, we can reasonably suppose that its default probability at time t_2 will be different than before. In particular, in the worst scenario, B^3 will have the same PD as before ($PD^3_{t_2} = PD^3_{t_1} = PD^3_{t_0}$) but, in principle, we can impose the constraint $PD^3_{t_2} \leq PD^3_{t_0}$.

Marginal and partial correlations can be derived as in the bail-in scenario, with the only difference being that now, at time t_2, the correlation matrix is a 3×3 matrix since B^3 is still part of the banking system. In this last case, we can safely assume that the correlation matrix remains the same as in t_0. The involved variable are summarised in Table 6.8.2.

[Table 6.8.2]

Consistently with equation (6.2.10), at each time t_j we can calculate the total default probability of each bank. By substituting the variables summarised in Table 6.8.1 in equation (6.2.10), the following results can be obtained:

$$t_0\colon \begin{cases} TPD^{1,b}_{t_0} = PD^1 + \rho_{12|S} \cdot PD^2 A^2/A^1 + \rho_{13|S} \cdot PD^3_{t_0} A^3/A^1 \\ TPD^{2,b}_{t_0} = PD^2 + \rho_{12|S} \cdot PD^1 A^1/A^2 + \rho_{23|S} \cdot PD^3_{t_0} A^3/A^2 \\ TPD^{3,b}_{t_0} = PD^3_{t_0} + \rho_{13|S} \cdot PD^1 A^1/A^3 + \rho_{23|S} \cdot PD^2 A^2/A^3 \end{cases} \tag{6.3.5}$$

$$t_1\colon \begin{cases} TPD^{1,b}_{t_1} = PD^1 + \rho_{12|S,t_0} \cdot PD^2 A^2/A^1 + \rho_{13|S} \cdot PD^3_{t_0} \frac{A^3+X}{A^1+A^2-X} \frac{A^1+A^2}{A^1} \\ TPD^{2,b}_{t_1} = PD^2 + \rho_{12|S} \cdot PD^1 A^1/A^2 + \rho_{23|S} \cdot PD^3_{t_0} \frac{A^3+X}{A^1+A^2-X} \frac{A^1+A^2}{A^2} \\ TPD^{3,b}_{t_1} = PD^3_{t_0} + \frac{A^1+A^2-X}{(A^3+X)(A^1+A^2)\left[\rho_{13|S} \cdot PD^1 A^1 + \rho_{23|S} \cdot PD^2 A^2\right]} \end{cases} \tag{6.3.6}$$

$$t_1\colon \begin{cases} TPD^{1,b}_{t_2} = PD^1 + \rho_{12|S,t_0} \cdot PD^2 A^2/A^1 + \rho_{13|S} \cdot PD^3_{t_0} \frac{A^3+X}{A^1+A^2-X} \frac{A^1+A^2}{A^1} \\ TPD^{2,b}_{t_2} = PD^2 + \rho_{12|S} \cdot PD^1 A^1/A^2 + \rho_{23|S} \cdot PD^3_{t_0} \frac{A^3+X}{A^1+A^2-X} \frac{A^1+A^2}{A^2} \\ TPD^{3,b}_{t_2} = PD^3_{t_2} + \frac{A^1+A^2-X}{(A^3+X)(A^1+A^2)\left[\rho_{13|S} \cdot PD^1 A^1 + \rho_{23|S} \cdot PD^2 A^2\right]} \\ PD^3_{t_2} \leq PD^3_{t_0} \end{cases} \tag{6.3.7}$$

As in the bail-in scenario, such total default probabilities can be aggregated over time, so $TPD^{m,b}_T$ can be obtained and compared to $TPD^{m,a}_T$.

6.3.3 Bail-in vs bail-out scenario

According to the previous equations, the final default probabilities for each bank m, conditional on their previous survival, can be summarised as follows:

$$\begin{cases} TPD_T^{m\neq 3,a} = 1-(1-TPD_{t_0}^{m,a})\cdot(1-TPD_{t_1}^{m,a})\cdot(1-TPD_{t_2}^{m,a}), \\ TPD_T^{3,a} = 1, \\ TPD_T^{m,b} = 1-(1-TPD_{t_0}^{m,b})\cdot(1-TPD_{t_1}^{m,b})\cdot(1-TPD_{t_2}^{m,b}). \end{cases} \tag{6.3.8}$$

Since banks 1 and 2 have to decide wether to help bank 3 or not, from a systemic risk perspective they are interested in analysing the difference between TPD_T^a and TPD_T^b. In particular, if $TPD_T^{m,a} - TPD_T^{m,b} > 0$, then saving bank 3 (bail-out scenario) decreases the total probability of default of bank m with respect to the bail-in scenario; on the contrary, if $TPD_T^{m,a} - TPD_T^{m,b} < 0$, than letting bank 3 default (bail-in scenario) decreases the total probability of default of bank m, with respect to the bail-out scenario.

In this section we compute such differences simulating alternative scenarios. We consider two large banks and a smaller one, with $A^1 = 40$, $A^2 = 20$ and $A^3 = 4$ (billion euros), with $X = A^3$ in case of bail-out scenario. Correlation coefficients are sampled from Gaussian distributions: $\rho_{mn} \sim \mathcal{N}(\mu_{\rho_{mn}}, \sigma^2_{\rho_{mn}})$, with $\mu_{\rho_{12}} = \mu_{\rho_{13}} = \mu_{\rho_{23}}$, and unit variances. Baseline default probabilities are also sampled from Gaussian distributions: $PD^m \sim \mathcal{N}(\mu_{PD^m}, \sigma^2_{PD^m})$, with $\mu_{PD^1}, \mu_{PD^2} = 0.01, 0.03, 0.05, 0.07$ and unit variances.

Last, the default probability of bank 3 at t_2 (in the bail-out scenario), is simulated according to different values of $PD^3_{t_2}$:

$$\begin{cases} PD^3_{t_j} \sim \mathcal{N}(\mu_{PD^3_{t_j}}, \sigma^2_{PD^3}), \\ \mu_{PD^3_{t_0,t_1}} = 0.10, \\ \mu_{PD^3_{t_2}} \sim \mathcal{U}([0, 0.10]), \end{cases} \tag{6.3.9}$$

with a unit variance. The resulting differences in TPD, as a function of the sampled PDs, are shown in Figure 6.8.2. As seen before, the higher the difference, the more convenient the bail-out is, with respect to the bail-in.

[Figure 6.8.2]

The results plotted in Figure 6.8.2 can be summarised and interpreted according to both the different dimensions and the default probabilities of the two banks. First, in this special case of all positive correlations, it is always convenient for both banks to help bank 3 without letting it default. Secondly, by comparing the two graphs, it is clear that the smaller bank B^2 has a larger advantage of helping B^3 and that such advantage is positively dependent on the decreasing dimension of the "safe" banks. Thirdly, both graphs represent four lines according to four different values for $\mu_{PD^{1,2}}$: by comparing such four lines, the result is that the safer a bank is, the larger the advantage of helping the troubled bank B^3.

The previous simulation is based on the hypothesis of fixed and positive $\mu_{\rho_{mn}}$: however, this is an extremely simplified assumption, especially under the bail-in scenario. It is common knowledge, in fact, that when a bank is in default, or when the banking system faces a crisis period, correlations between them vary. In order to take this more realistic scenario into account, we now sample $\mu_{\rho_{mn}}$ as well, uniformly over the possible range:

$$\begin{cases} \rho_{mn} \sim \mathcal{N}(\mu_{\rho_{mn}}, \sigma^2_{\rho_{mn}}), \\ \mu_{\rho_{mn}} \sim \mathcal{U}([-1, 1]), \end{cases} \tag{6.3.10}$$

The resulting differences in TPD, as a function of the sampled correlations, and of the PD of the safe banks, keeping $\mu_{PD^3_{t_2}} = 0.10$, are shown in Figure 6.8.3.

[Figure 6.8.3]

Figure 6.8.3 represents the advantage/disadvantage of helping bank 3 as a function of the correlations between bank 1 and bank 3 (top-left), between bank 2 and bank 3 (top-right) and between the two safe banks 1 and 2 (bottom, left referred to bank 1, right referred to bank 2). By looking at the top two graphs it is first clear that the smaller the bank is, the stronger the dependence on correlations. Second, in case of positive correlations with B^3, the bail-out scenario is better, and the smaller or the safer a bank is, the larger the advantage. On the contrary, in case of negative correlations the bail-in scenario is preferred, and the advantage increases with the dimension of a bank.

The two bottom graphs show the relative convenience of the two scenarios in terms of the impact of the correlation between the two safe banks B^1 and B^2. It reveals that the impact is not particularly significant for large banks (such as bank 1), while it slightly changes the results referred to small banks: the weaker the correlation between bank 1 and bank 2 is, the bigger the advantage of helping bank 3.

By jointly reading Figures 6.8.2 and 6.8.3, the results show that, overall, B^1 and B^2 should prefer the bail-in rather than the bail-out scenario only in case of negative partial correlations. In addition, the convenience for the bail-out situation is a decreasing function of the default probabilities of the safe banks, a decreasing function of the dimension of the safe banks in system, and an increasing function of the correlation of safe banks with the troubled one.

6.4 Application to the Italian banking system

We now apply the methodology described in the previous Sections to the Italian banking system: since we wil use CDS spreads to derive market-implied idiosyncratic default probabilities, we will concentrate on the eight banks for which such data are available (source *Markit*): Banca Popolare di Milano (BPM), Banca Carige (CRG), Banco Popolare (BAPO), Intesa San Paolo (ISP), Mediobanca (MB), Monte dei Paschi di Siena (MPS), Unicredit (UCG), Unione Banche Italiane (UBI). Since the new bail-in regulation has been introduced in January 2016, we will focus on data from the 1st of January 2016 until the 30th of September 2016, with daily frequencies. The summary statistics for CDS spreads is reported in Table 6.8.3. Regarding Assets, we consider the book values referred to the 31st of December 2015 (source *Compustat*): they are reported in Table 6.8.4.

[Tables 6.8.3, 6.8.4]

Table 6.8.3 reveals that Monte dei Paschi di Siena is the most troubled bank in the system, having the highest CDS spreads and the highest volatility. On the contrary, the two biggest banks (Unicredit and Intesa San Paolo) appear to be the safest ones. The remaining banks, which are smaller in size, lie in an intermediate situation.

6.4.1 Contagion effects on individual banks

In order to compute partial correlations, we consider the time-series of expected losses (calculated as the product between default probabilities and assets, with the former implied by CDS spreads). The resulting partial correlation network is reported in Figure 6.8.4.

[Figure 6.8.4]

Figure 6.8.4 reveals that the two largest banks are not strongly connected to each other, as well as that they are not much connected to the Italian banking system. On the contrary, medium-size banks and the troubled MPS are strongly connected. Finally, the smallest bank in the sample (CRG) seems to be quite isolated with respect to the others.

We can now compute the difference in the total default probability of each bank in case one bank in the system is close to its default point. In order to understand how much the failure of a small rather than a big bank can impact other banks and the system, we will study two scenarios: (a) Banca Carige (the smallest bank in the sample) is in trouble; (b) Monte dei Paschi di Siena (a large bank, with a high idiosyncratic default probability) is in trouble. In each scenario we perform Monte Carlo simulations, structured as follows:

$$\begin{cases} PD_{t_j}^m \sim \mathcal{N}(\mu_{PD_{t_j}^m}, \sigma^2_{PD^m}), \\ \mu_{PD_{t_2}^{CRG}} \sim \mathcal{U}([0, 0.20]), \\ \rho_{mn} \sim \mathcal{N}(\mu_{\rho_{mn}}, \sigma^2_{\rho_{mn}}), \end{cases} \tag{6.4.1}$$

where the means $\mu_{PD_{t_j}^m}$ for $m \neq CRG$ are fixed and based on the average CDS spreads; similarly, $\mu_{PD_{t_0,t_1}^{CRG}}$ are fixed and based on CDS spreads for the first two periods; in the last time period t_2 we extract the mean of the PD distribution referred to Banca Carige from a uniform distribution, in order to allow the TPDs of the other banks to depend on the increase/decrease of the default probability of CRG in case of bail-out. The same structure is used to model the banking system in case Monte dei Paschi di Siena faces a bail-in or a bail-out scenario. The results are shown in Figure 6.8.5, and are referred to the bail-in/bail-out of Banca Carige (top) and Monte dei Paschi di Siena (bottom).

[Figure 6.8.5]

The top graph in Figure 6.8.5 shows the changes in TPDs due to a bail-in and a bail-out resolution for Banca Carige: more precisely, it shows the convenience of the bail-out as a function of the default probability of Carige at time t_2. The results are quite different for each bank, and have to be interpreted by jointly reading this graph with the network in Figure 6.8.4. More precisely, Banco Popolare and UBI seem to prefer a bail-out scenario almost independently from the $PD_{t_2}^{CRG}$: in other words, even if Carige does not start performing better after having been saved, BAPO and UBI will anyway prefer having it in the system rather than letting it default. This because both banks are little related to CRG and, therefore, are not affected by changes in its PD. On the other side, the largest banks (ISP, UCG, MPS) seem to be neutral with respect to the two scenarios, since their large dimensions make them independent from the default or the bail-out of a small bank from a systemic risk perspective. Finally, Mediobanca and Banca Popolare di Milano show completely different behaviours: they are both negatively affected by a bail-out, but their dependence on $TPD_{t_2}^{CRG}$ is opposite. Mediobanca increases its preference for a bail-in as the PD_{t_2} of Carige increases: since they are positively related, Figure 6.8.5 reveals that the persistence of Carige in the network has a more negative impact on MB with respect to the shock due to its failure. An opposite trend regards BPM, negatively correlated to CRG.

The bottom chart in Figure 6.8.5 shows the changes in TPDs due to a bail-in and a bail-out resolution for Banca Monte dei Paschi di Siena. These results are mixed, and strongly depend on the default probability of MPS at time t_2 after the bail-out. In case MPS maintains its PD (first vertical line), again BAPO and UBI prefer a bail-out intervention, ISP and UCG remain neutral while BPM and MB prefer a bail-in resolution. But the situation changes if MPS gets worse, decreasing its PD: in this case all banks (with the exception of MB) would prefer a bail-in resolution rather than a bail-out. This result can be explained by observing that all banks are positively and significantly related to MPS, meaning that the persistence of such large and troubled bank in the system worsens the PD of all the others, and this impact is bigger than the impact its default might have. Finally, the impact on MB is reversed because of its negative partial correlation with MPS.

6.4.2 Contagion effects on the banking system

The results presented so far analyse the impact of a bail-in rather than a bail-out on each banks' default probability: they thus consider the impact of systemic risk on the vulnerability of each financial institution. We now show the comparison between the two scenarios in terms of the increase/decrease of the expected losses for the entire banking system, as derived in Section 6.2.3. As previously described, we interpret the effects of the two scenarios on the banking system as a cascade effect: the transmission mechanism of expected losses (contagion) starts from one institution and propagates on all the others. Since this process strongly depends on the order of the propagating institutions, we perform random simulations aver all the possible permutations. For each scenario (bail-in or bail-out), time period (t_0, t_1, t_2) and troubled bank (Carige or MPS) we thus obtain a distribution.

Figure 6.8.6 shows the distribution of the expected losses of the entire banking system when Carige is close to its default point. The top graph refers to t_0, the middle one to t_1 and the bottom chart to t_2; each graph shows the comparison between the distributions obtained in case of bail-in (red line) or bail-out (green line).

[Figure 6.8.6]

The results shown in Figure 6.8.6 can be summarised as follows. (a) At time t_1 the shock due to the default of a bank, even if small as Banca Carige, strongly affects the banking system, deeply increasing the total expected losses with respect to the bail-out scenario; in addition, the standard deviation of the distribution becomes much higher, meaning a strong dependence on the cascade order. (b) At time t_2 the situation is reversed: the persistence of the troubled bank in the network increases the expected losses of the system in case of bail-out with respect to the bail-in scenario.

In order to understand which effect prevails over the other, we have aggregated the expected losses over time for both scenarios: the results are shown in Figure 6.8.7.

[Figure 6.8.7]

Figure 6.8.7 clearly shows that the bail-in resolution significantly increases the expected losses of the banking system with respect to the bail-out scenario. In other words, the shock produced by the failure of a bank, even if small, is much higher than the damage produced by the persistence of that troubled bank in the system.

In order to understand how much the improvement of the expected losses for the entire system in case of bail-in depends on the default probability of the troubled bank at time t_2, we have performed the same exercise with different values of $PD_{t_2}^{CRG}$. The results are shown in Figure 6.8.8.

[Figure 6.8.8]

Figure 6.8.8 reveals that a change in the default probability of Banca Carige at time t_2 in case of bail-out does not significantly affects the expected losses of the banking system.

We have then performed the same simulations in case Monte dei Paschi di Siena is under bail-in or bail-out resolution. Figure 6.8.9 shows the distributions obtained at each time-period in case of bail-in (red line) or bail-out (green line).

[Figure 6.8.9]

The results reported in Figure 6.8.9 are similar to those observed in the previous case, even if bigger in absolute values. At time t_1 the shock produced by the default of MPS strongly increases the expected losses of the banking system: such effect is greater than before, because of the much bigger size of MPS with respect to Banca Carige. At time t_2, on the contrary, the persistence of MPS in the network in case of bail-out increases the expected losses of the system.

The expected losses aggregated over time in case of bail-in (red line) or bail-out (green line) are reported in Figure 6.8.10.

[Figure 6.8.10]

Figure 6.8.10 clearly shows that, from a systemic risk viewpoint, the bail-out strongly reduces contagion effects with respect to a bail-in resolution; as previously underlined such preference is much bigger than before because of the bigger size of MPS, which makes the shock produced in case of bail-in at time t_1 strongly negative on the entire system. The comparison of this result with the graph in Figure 6.8.5 shows an interesting behaviour: even if each single bank in the system slightly benefits from a bail-out scenario, the entire system is strongly and positively affected by it. The difference between the individual and the system effects is due to the fact that MPS is strongly interconnected, as well as it has a large size: these two factors, combined together, make the cascate effect stronger than the contagion effects on each single financial institutions, thus amplifying the magnitude of bail-in or bail-out consequences.

Finally, we can analyse how much the expected losses of the banking system change according to different values of PD^{MPS} at time t_2 in case of bail-out. Such results are reported in Figure 6.8.11.

[Figure 6.8.11]

Differently from before, Figure 6.8.11 shows big differences according to an increase or a decrease of the PD of MPS at time t_2. As the troubled bank increases its default probability after the bail-out, the expected losses of the entire system increases, thus making at some point the bail-in resolution preferable to the bail-out.

6.5 Case study: Atlante

In order to test the methodology proposed in the previous Sections to a real case, we now concentrate on the Italian equity fund called Atlante. This case study analysis is particularly interesting since it represents the first critical situation that occurred in a banking system after the new bail-in regulations introduced in Europe in January 2016.

In March 2016, in fact, a bank called Banca Popolare di Vicenza was found to be almost in default: in order to avoid its bail-in, almost all the other Italian banks in the system decided to organise themselves in an equity fund, called Atlante. Each bank put an amount of its capital into Atlante, with the aim of re-capitalising it.

Since not all Italian banks are listed, as well as for not all of them CDS spreads are available, we used different data from before. More precisely, instead of using assets as A in (6.2.1) we considered market capitalisations Cap as the time-varying data series (source, *Borsa Italiana*). For deriving expected losses, we obtained default probabilities in different ways, as follows. For Carige (CRG), Monte dei Paschi di Siena (MPS), Banco Popolare di Milano (BPM), Banco Popolare (BAPO), Istituto San Paolo (ISP), Mediobanca (MB), Unione Banche Italiane (UBI) and Unicredit (UCG) we employed the PDs obtained from daily CDS spreads, averaging them over the period January-September 2016. For the banks Banca Popolare dell'Emilia Romagna (BPER), Banca Popolare di Sondrio (POPSO), Credito Emiliano (CREDEM), Credito Valtellinese (CREVAL) and Banca Mediolanum (MDL), we used the default probabilities calculated by the Risk Management Institute of the National University of Singapore, calibrated with those implied by the Fitch ratings. For the troubled Banca Popolare di Vicenza, which is not listed and it does not have either CDS data or a PD from the Singapore Institute, we used different approximations: in particular, since its Fitch rating is equal to that of MPS, its average default probability has been set equal to PD^{MPS}; regarding capital, we used the book value reported in December 2015.

The banks considered in the sample, together with their default probabilities and capital values, are listed in Table 6.8.5.

[Table 6.8.5]

6.5.1 Contagion effects on individual banks

We can now calculate the expected losses of each bank by multiplying daily capitalisations with PD values, as in (6.2.1): we then derive partial correlations and, consistently, the partial correlation network. The results are depicted in Figure 6.8.12.

[Figure 6.8.12]

The network proposed in Figure 6.8.12 shows both positive (green lines) and negative (red lines) partial correlations: the ticker the line, the stronger the connection. It is interesting to observe that positive correlations prevail: in particular, they are strong between "troubled" banks such as MPS and CRG. Moreover, Banca Popolare di Vicenza seems to be quite isolated with respect to the others: this may be due to the fact that it is a small regional bank, with business evolution really different from that of the others. Note that, by construction, the correlation between POPVIC and CRG is equal to that between MPS and CRG, whereas that between MPS and POPVIC has been set equal to that between MPS and CRG.

We now derive the TPDs of each bank according to the bail-in and the bail-out scenarios. As a capital value $Cap^m_{t_0}$, we consider the average capitalisation values reported in Table 6.8.5. In case of bail-out (scenario b) at time t_1, such amounts are decreased by a fraction X^m, which indicates the amount of capital each bank has decided to put into Atlante. According to the previous Section, such amounts X^m have been allocated proportionally to the size of each bank, as follows:

$$\begin{cases} Cap^m_{t_1} = Cap^m_{t_0}\left(1 - \frac{X}{\sum\limits_m Cap^m_{t_0}}\right), & \text{if } m \neq POPVIC \\ Cap^m_{t_1} = Cap^m_{t_0} + X, & \text{if } m = POPVIC \end{cases} \tag{6.5.1}$$

with $X = 1500$ Mlns euros.

For robustness purposes we performed Monte Carlo simulations as in the previous Section: more precisely, correlations have been extracted from a normal distribution, $\rho_{mn|S} \sim \mathcal{N}(\mu_{\rho_{mn|S}}, \sigma^2_{\rho_{mn|S}})$, with $\mu_{\rho_{mn|S}}$ and $\sigma^2_{\rho_{mn|S}}$ calculated using the available expected losses time-series; similarly, default probabilities are $PD^m \sim N(\mu_{PD^m}, \sigma^2_{PD^m})$, with μ_{PD^m} as in Table 6.8.5. In order to examine how much contagion effects depend on the default probability of the troubled bank POPVIC at time t_2 after the bail-out (scenario b), we assume the following variation range:

$$\begin{cases} PD^{POPVIC}_{t_j} \sim N(\mu_{PD^{POPVIC}_{t_j}}, \sigma^2_{PD^{POPVIC}}), \\ \mu_{PD^{POPVIC}_{t_2}} \sim \mathcal{U}([0.03, 0.2]). \end{cases} \tag{6.5.2}$$

Consistently with the previous Section, we now calculate the differences between the default probabilities in the bail-in and the bail-out scenario ($TPD^m_{bail-in} - TPD^m_{bail-out}$) aggregated over time. The results are shown in Figure 6.8.13.

[Figure 6.8.13]

Figure 6.8.13 clearly shows that some banks would benefit, even if little, from helping and saving POPVIC through the Atlante equity fund, while some others would not. In particular, Monte dei Paschi di Siena, Banca Popolare di Sondrio, Credito Valtellinese and Unione Banche Italiane would little benefit from a bail-our scenario: they are quite heterogenous banks, being the first one a big bank but strongly and positively connected to POPVIC, while the others are small financial institutions, not related to the troubled bank POPVIC. As in the previous Section, the biggest banks Intesa San Paolo and Unicredit, together with Banca Mediolanum, seem to be neutral to the choice of a bail-in rather than a bail-out. Finally, two banks show a preference for the bail-in resolution: they are Credito Emiliano and Banca Carige. CREDEM is not partially related to POPVIC and, more generally, is not strongly connected to the other banks in the system. CRG, on the contrary, is strongly and positively related to POPVIC: this means that the persistence of POPVIC in the network after a bail-out choice increases the default probability of Banca Carige much more than what a bail-in resolution would do; in addition, such increase is even stronger as the default probability of POPVIC at time t_2 increases.

Figure 6.8.13 also reveals an interesting result: none of the lines crosses the *x-axis*. Only two banks show a decreasing shape, and they are Monte dei Paschi di Siena and Banca Carige, since they are the only two strongly and positively connected to POPVIC. However, they present different behaviours: the slope of MPS, in fact, is much lower than that of CRG, and this is due to their different dimensions. Monte dei Paschi is much bigger than Carige and, consequently, its dependence on the default probability of POPVIC at time t_2 is weaker. Banca Carige, on the other side, shows a preference for the bail-in even if the default probability of POPVIC decreases after the bail-out, because it suffers contagion deriving from its strong connection to POPVIC, and such strong spillover effects are due to its small size.

6.5.2 Contagion effects on the banking system

Figure 6.8.13 shows the preference for a bail-in rather than a bail-out scenario from the banks' perspective. We now show the results referred to the system perspective: more precisely, we simulate cascade effects and, consistently, the increase or decrease in the total expected losses of the system in each scenario.

Figure 6.8.14 shows the distribution of the expected losses of the entire banking system when Banca Popolare di Vicenza is subject to the bail-in (red line) rather than to the bail-out (green line) choice. The top graph refers to t_0, the middle one to t_1 and the bottom chart to t_2.

[Figure 6.8.14]

Figure 6.8.14 shows results quite similar to the ones presented in the previous Section. At time t_1 the bail-out reduces the expected losses of the entire system since the shock produced by the default of a bank increases contagion effects; at time t_2 the situation is reversed, because the persistence of a troubled bank in the system increases the default probabilities of all the others. Finally, it is interesting to observe that such differences between the two scenarios are quite small, together with the total expected losses of the system: both these effects are due to the small dimension of Banca Popolare di Vicenza with respect to the other Italian banks.

The expected losses aggregated over time in case of bail-in (red line) or bail-out (green line) are reported in Figure 6.8.15.

[Figure 6.8.15]

The aggregation of expected losses over time reveals that the long-run effects of a bail-out should be preferred with respect to the ones deriving from a bail-in resolution, even if the difference between the two scenarios is lower than in the previous examples. This fact is again due to the small size of Banca Popolare di Vicenza, whose default or recapitalisation does not strongly affect the Italian banking system (differently from what could happen in case of default of MPS).

Finally, we can analyse how much the expected losses of the banking system change according to different values of PD^{POPVIC} at time t_2 in case of bail-out. Such results are reported in Figure 6.8.16, in this case aggregated over time.

[Figure 6.8.16]

Even when considering results aggregated over time as in Figure 6.8.16, the advantage of the bail-out results to be an increasing function of the survival probability of Banca Popolare di Vicenza at time t_2 in case of bail-out. This result is a further confirm of the simulation exercises provided in the previous Section.

6.5.3 Simulations vs real data

In the previous Sections we have used equation (6.5.1) in order to homogeneously and proportionally distribute the capital amount X POPVIC needs in order to not default. Indeed, we know the real amounts X^m of capital that each bank has transferred to the

Atlante fund, by the 29th of April, 2016. They are shown in Table 6.8.6 (source, *Milano Finanza*).

[Table 6.8.6]

Table 6.8.6 reveals that only two banks have not taken part of the Atlante fund: Credito Emiliano and Mediobanca. The choice Italian banks have taken is consistent with our results (Figure 6.8.13), especially regarding CREDEM. Furthermore, Credito Valtellinese and UBI Banca are the banks that transferred the biggest fractions of their capital amount to Atlante, and this choice is again consistent with our simulated results. Carige has decided to invest in the Atlante funds even if our model suggests they would not benefit from it: this choice, however, can be explained not in terms of systemic risk, but by other strategic factors (such as the possible acquisition of their non performing loans).

6.6 Conclusions

We have compared the consequences of a bail-in and a bail-out scenario from a systemic risk perspective. To understand which choice should be the best one, we have derived the contagion effects on financial institutions by using two different perspectives: the single banks' perspective and the system perspective. The former aims at understanding how much the default probability of each bank changes according to the two scenarios; the latter examines how much the expected losses of the entire banking system are affected by the default or the bail-out of a troubled bank.

To understand the relationships between the involved variables (default probabilities, sizes, correlation networks), we have firstly performed simulations on a stylised system composed by three banks. We have then applied the above described methodology to the Italian banking system: as troubled bank, we have considered both a small (Banca Carige) and a large (Monte dei Paschi di Siena) bank. We have finally compared the results of our simulation exercises to what occurred in the actual Atlante bail-out.

Our findings can be summarised as follows. First, in the stylised setting of three banks, the simulation results reveal that the smaller or the safer a bank is, the larger the advantage of choosing a bail-out scenario. The advantage increases with the correlation with the troubled bank; it decreases with the correlation between the safe banks and it decreases with the default probability of the troubled bank. Second, the application to the Italian banking system reveals that, in the long-run, the bail-out should always be preferred to the bail-in resolution from a system's perspective. Such preference, moreover, strongly increases as the size of the troubled bank increases. Regarding the single bank perspective, the correlation pattern is the main driver for the choice of a bail-in or a bail-out.

6.7 References

Acharya, V.V., Pedersen, L.H., Philippon, T., Richardson, M., 2010. Measuring Systemic Risk. Technical Report. New York University.

Acharya, V.V., Engle, R., Richardson, M., 2012. Capital Shortfall: A New Approach to Ranking and Regulating Systemic Risks. American Economic Review: Papers and Proceedings, 102(3), 59-64.

Adrian, T., Brunnermeier, M.K., 2011. CoVar. NBER Working Paper 17454. National Bureau of Economic Research.

Ahelegbey, D.F., Billio, M., Casarin, R., 2015. Bayesian Graphical Models for Structural Vector Autoregressive Processes. Journal of Applied Econometrics.

Barigozzi, M., Brownlees, C., 2013. Nets: Network Estimation for Time Series. Technical Report.

Battiston, S., Delli Gatti, D., Gallegati, M., Greenwald, B., Stiglitz, J.E., 2012. Liasons dangereuses: Increasing connectivity risk sharing, and systemic risk. Journal of Economic Dynamics and Control, 36(8), 1121-1141.

Betz, F., Oprica, S., Peltonen, T.A., Sarlin, P., 2014. Predicting Distress in European Banks. Journal of Banking and Finance, 45(C), 225-241.

Billio, M., Getmansky, M., Lo, A.W., Pelizzon, L., 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of Financial Economics, 104, 535-559.

Brownlees, C., Engle, R., 2012. Volatility, Correlation and Tails for Systemic Risk Measurement. Technical Report. New York University.

Das, S.R., 2015. Matrix Metrics: Network-Based Systemic Risk Scoring. Technical Report. Santa Clara University.

Diebold, F.X., Yilmaz, K., 2014. On the network topology of variance de- compositions: Measuring the connectedness of financial firms. Journal of Econo- metrics, 182, 119-134.

Duffie, D., Lando, D., 2001. Term Structures of Credit Spreads with Incomplete Accounting Information. Econometrica, 69(3), 633-664.

Duffie, D., Saita, L., Wang, K., 2007. Multi-Period Corporate Default Prediction with Stochastic Covariates. Journal of Financial Economics, 88(3), 635-665.

Dumitrescu, E., Banulescu, D.G., 2014. Which are the SIFIs? A Component Expected Shortfall (CES) Approach to Systemic Risk. Journal of Banking and Finance, 50, 575-588.

Duprey, T., Klaus, B., Peltonen, T.A., 2015. Dating systemic financial stress episodes in the EU countries. Technical Report. ECB.

Giudici, P., Parisi, L., 2016. CoRisk: measuring systemic risk through default probability contagion. Technical Report. University of Pavia.

Giudici, P., Spelta, A., 2016. Graphical Network Models for International Financial Flows. Journal of Business and Economic Statistics, 34(1), 128-138.

Halaj, G., Hüser, A.C., Kok, C., Perales, C., Van der Kraaij, A., 2016. Systemic implications of the European bail-in tool: a multi-layered network analysis. Financial Stability Review, May 2016, 120-129.

Hautsch, N., Schaumburg, J., Schienle, M., 2015. Financial Network Systemic Risk Contributions. Review of Finance, 19(2), 685-738.

Koopman, S.J., Lucas, A., Schwaab, B., 2012. Dynamic factor models with macro, frailty, and industry effects for U.S. default counts: the credit crisis of 2008. Journal of Business and Economic Statistics, 30(4), 521-532.

Lando, D., Nielsen, M.S., 2010. Correlation in corporate defaults: contagion or conditional independence. Journal of Financial Intermediation, 19(3), 355-372.

Longstaff, F.A., 2010. The subprime credit crisis and contagion in financial markets. Journal of Financial Economics, 97(3), 436-450.

Lorenz, J., Battiston, S., Schweitzer, F., 2009. Systemic risk in a unifying framework for cascading processes on networks. The European Physical Journal B - Condensed Matter and Complex Systems, 71(4), 441-460.

Shleifer, A., Vishny, R.W., 2010. Unstable banking. Journal of Financial Economics, 97(3), 306-318.

SRM Regulation, 2014. Regulation No. 806/2014 of the European Parliament and of the Council. Official Journal of the European Union.

6.8 Appendix F: Tables and Figures

Table 6.8.1: Variables time-evolution, bail-in

		t_0	t_1	t_2
Assets	B^1	A^1	A^1	A^1
	B^2	A^2	A^2	A^2
	B^3	A^3	A^3	-
PD	B^1	PD^1	PD^1	PD^1
	B^2	PD^2	PD^2	PD^2
	B^3	$PD^3_{t_0}$	$PD^3_{t_1} = 1$	-
Marg. Corr.	B^1 B^2 B^3	C_{t_0} (3×3)	C_{t_0} (3×3)	C_{t_2} (2×2)
Part. Corr.	B^1 B^2 B^3	$[(C_{t_0})^{-1}]^{mn} = \rho_{mn\|S}$	$[(C_{t_0})^{-1}]^{mn} = \rho_{mn\|S}$	$[(C_{t_2})^{-1}]^{mn} = \rho^a_{mn\|S,t_2}$

Notes: Time evolution of the variables that determine the total default probabilities of the three banks in the system, under the hypothesis that bank 3 defaults at time t_1 (scenario a, bail-in).

Table 6.8.2: Variables time-evolution, bail-out

		t_0	t_1	t_2
Assets	B^1	A^1	$A^1(1-\frac{X}{A^1+A^2})$	$A^1(1-\frac{X}{A^1+A^2})$
	B^2	A^2	$A^2(1-\frac{X}{A^1+A^2})$	$A^2(1-\frac{X}{A^1+A^2})$
	B^3	A^3	A^3+X	A^3+X
PD	B^1	PD^1	PD^1	PD^1
	B^2	PD^2	PD^2	PD^2
	B^3	$PD^3_{t_0}$	$PD^3_{t_0}$	$PD^3_{t_2} \leq PD^3_{t_0}$
Marg. Corr.	B^1 B^2 B^3	C_{t_0} (3×3)	C_{t_0} (3×3)	C_{t_0} (3×3)
Part. Corr.	B^1 B^2 B^3	$[(C_{t_0})^{-1}]^{mn}=\rho_{mn\|S}$	$[(C_{t_0})^{-1}]^{mn}=\rho_{mn\|S}$	$[(C_{t_0})^{-1}]^{mn}=\rho_{mn\|S}$

Notes: Time evolution of the variables used for estimating the total default probabilities of the three banks in the system, under the assumption that bank 3 is saved by the other two through a capital-lending operation at time t_1 (scenario b, bail-out).

Table 6.8.3: CDS spreads

Bank	μ (%)	Max (%)	Min (%)	σ $(\cdot 10^{-2})$
CRG	1.811	1.958	1.689	0.090
MPS	7.321	8.836	3.714	1.429
BPM	3.318	4.043	2.168	0.456
BAPO	3.771	4.871	2.608	0.484
MB	2.250	3.081	1.601	0.351
UCG	1.430	1.584	1.292	0.097
UBI	2.915	3.417	2.067	0.354
ISP	1.693	2.395	1.168	0.291

Notes: CDS spreads referred to eight Italian banks(CRG=Banca Carige; MPS=Monte dei Paschi di Siena; BPM=Banca Popolare di Milano; BAPO=Banco Popolare; MB=Mediobanca; UCG=Unicredit; UBI=Unione Banche Italiane; ISP=Intesa San Paolo). The most troubled bank is MPS, with the highest mean value and volatility. The biggest banks (UCG and ISP) have the lowest and less volatile values.

Table 6.8.4: Assets

Bank	Assets (10^9 €)
CRG	30.30
MPS	169.01
BPM	50.20
BAPO	120.51
MB	71.55
UCG	860.43
UBI	117.20
ISP	676.50

Notes: Total Assets referred to eight Italian banks (expressed in billion euros). The biggest banks are Unicredit (UCG) and Intesa San Paolo (ISP), while the smallest one is Banca Carige (CRG).

Table 6.8.5: Capitalisations

Ticker	μ_{PD}(%)	Cap (10^6 €)
MPS	7.23	1905.90
BPER	2.22	2433.00
BPM	3.45	2859.10
POPSO	2.68	1504.30
BAPO	3.89	2249.10
CRG	4.23	613.60
CREDEM	2.23	2084.10
CVAL	3.82	759.60
ISP	1.87	41200.30
MB	2.42	5818.40
UBI	3.06	3343.70
UCG	2.41	19838.90
MDL	1.70	5246.40
POPVIC	7.23	145.16

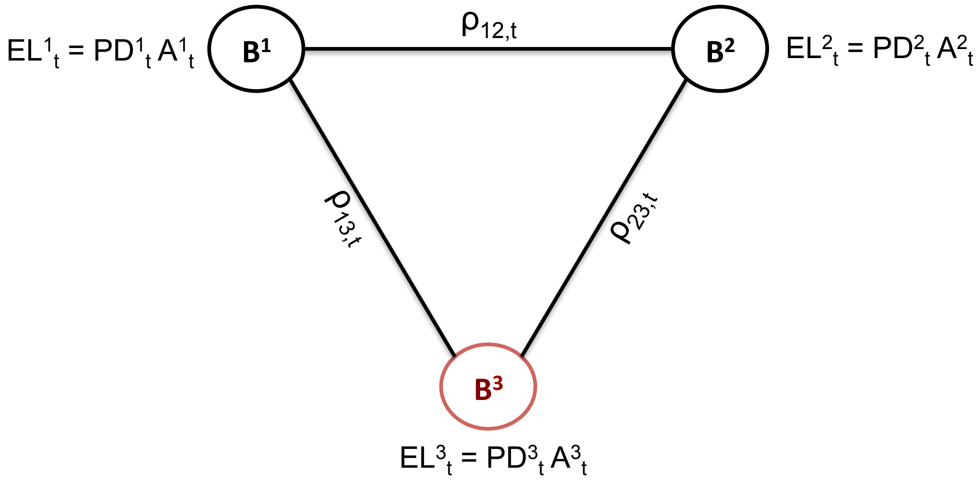
Notes: 14 Italian banks in terms of default probabilities (expressed in percentage points) and market capitalisation (expressed in million euros). Default probabilities have been calculated as follows: (a) implied by CDS spreads and averaged over the period January-September 2016 (MPS, BPM, BAPO, CRG, ISP, MB, UBI, UCG); (b) reported by the Credit Institute of the National University of Singapore, calibrated with Fitch ratings (BPER, POPSO, CREDEM, CVAL, MDL, POPVIC).

Table 6.8.6: Atlante, capital injections

Ticker	**Cap** (10^6 €)	X^m (10^6 €)	$\Delta Cap(\%)$	$C^m_{t_1}$ (10^6 €)
MPS	1906	-50	-2.62%	1855.9
BPER	2433	-100	-4.11%	2333
BPM	2859	-100	-3.50%	2759.1
POPSO	1504	-50	-3.32%	1454.3
BAPO	2249	-50	-2.22%	2199.1
CRG	614	-20	-3.26%	593.6
CREDEM	2084	0	0.00%	2084.1
CVAL	760	-60	-7.90%	699.6
ISP	41200	-1000	-2.43%	40200.3
MB	5818	0	0.00	5818.4
UBI	3344	-200	-5.98%	3143.7
UCG	19839	-1000	-5.04%	18838.9
MDL	5246	-50	-0.95%	5196.4
POPVIC	145	+1500		1510.11

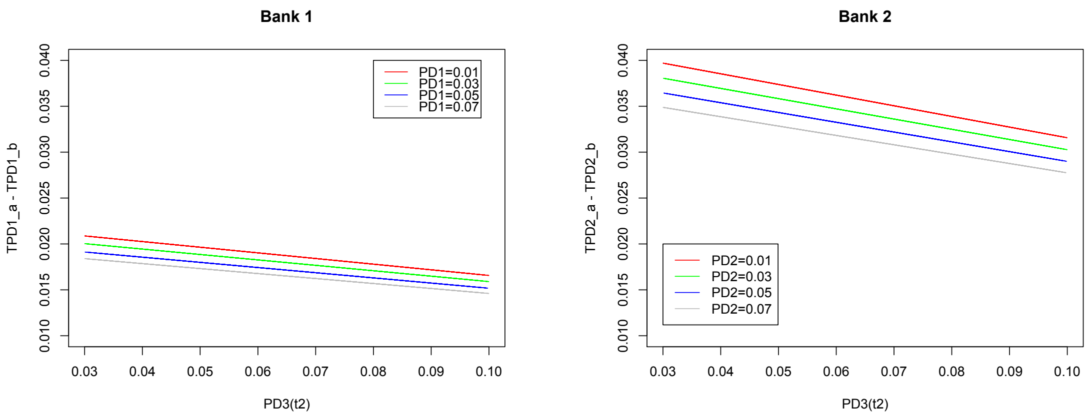
Notes: 14 Italian banks in terms of market capitalisation (expressed in million euros), the amount of capital they transferred to the Atlante fund (expressed in million euros), their capital change (expressed in percentage points) and their final capital amount (expressed in million euros). CREDEM and Mediobanca decided not to participate to the Atlante fund; on the contrary, two medium-size banks such as CREVAL and UBI experienced the biggest percentage change in their capital amounts.

Figure 6.8.1: Correlation structure, stylised banking system



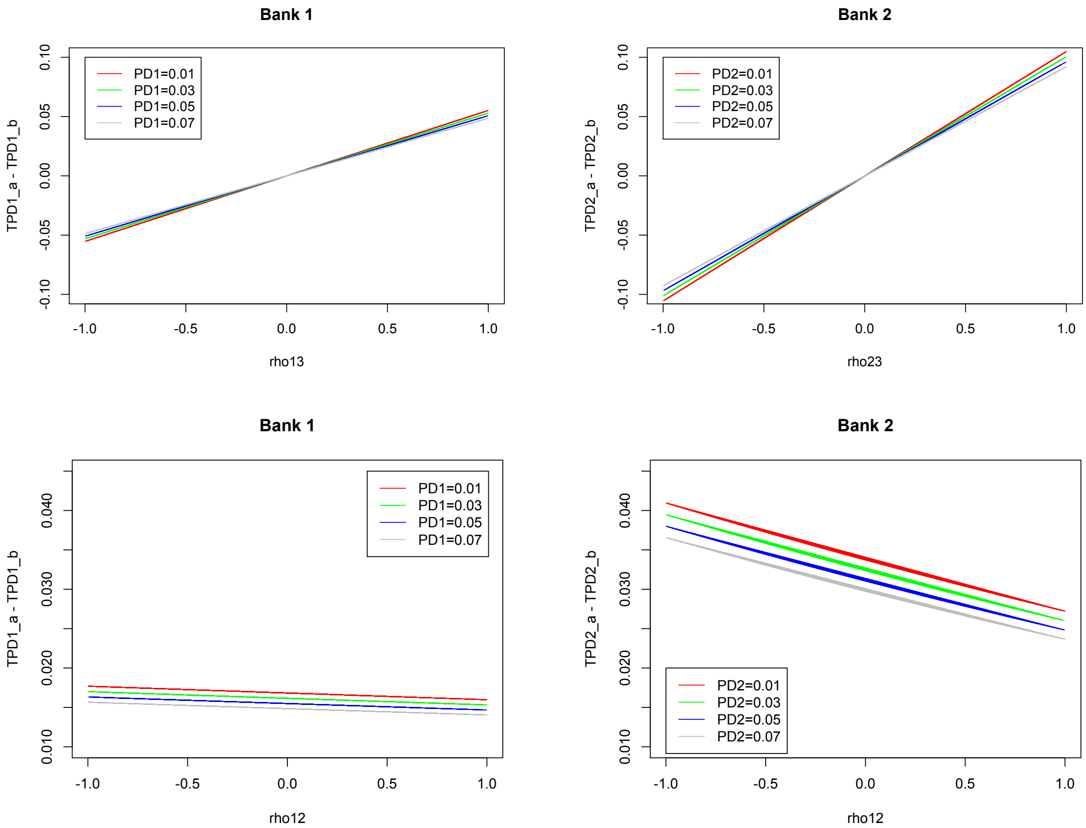
Notes: Simulated correlation structure between two "safe" banks, B^1 and B^2, and a "troubled" bank B^3, at a certain time t. All banks are associated with their expected losses, and links between each other are based on the partial correlation coefficients ρ_{mn}.

Figure 6.8.2: Changes in TPDs as functions of $PD^3_{t_2}$, stylised banking system



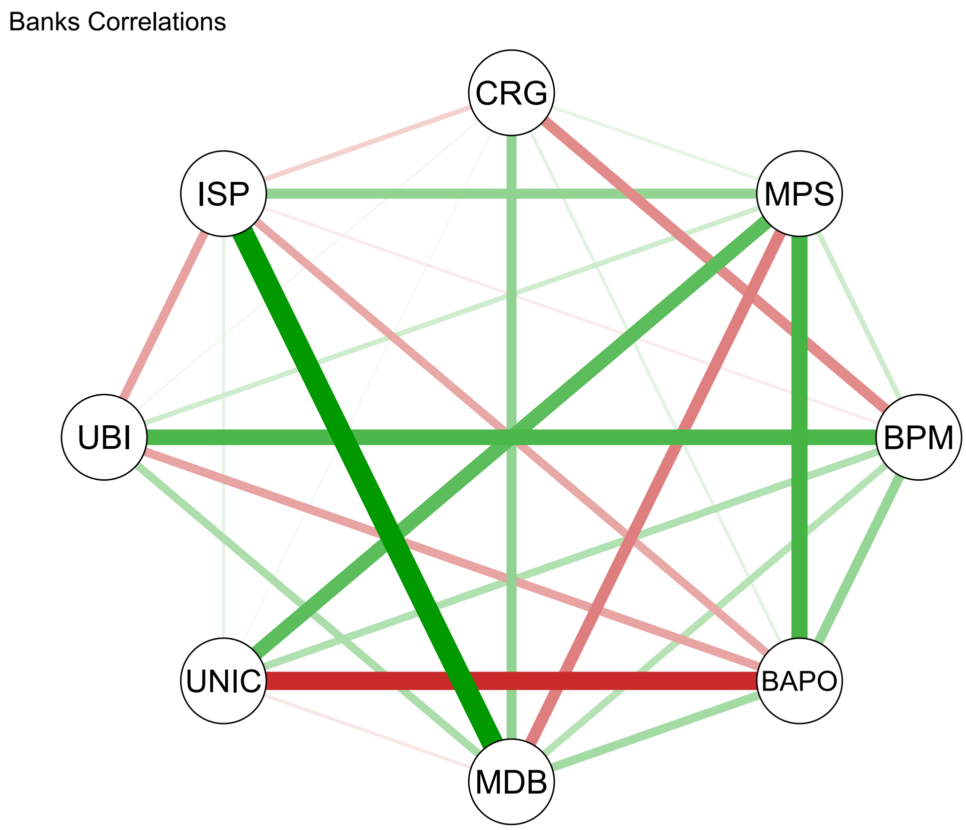
Notes: Monte Carlo simulated differences between the total default probabilities in case of bail-in and bail-out for bank 1 (left) and bank 2(right), plotted as functions of $\mu_{PD^3_{t_2}}$. The safer and the smaller the bank, the bigger the advantage of a bail-out; this advantage, however, decreases as the $PD^3_{t_2}$ increases in case of bail-out.

Figure 6.8.3: Changes in TPDs as functions of partial correlations, stylised banking system



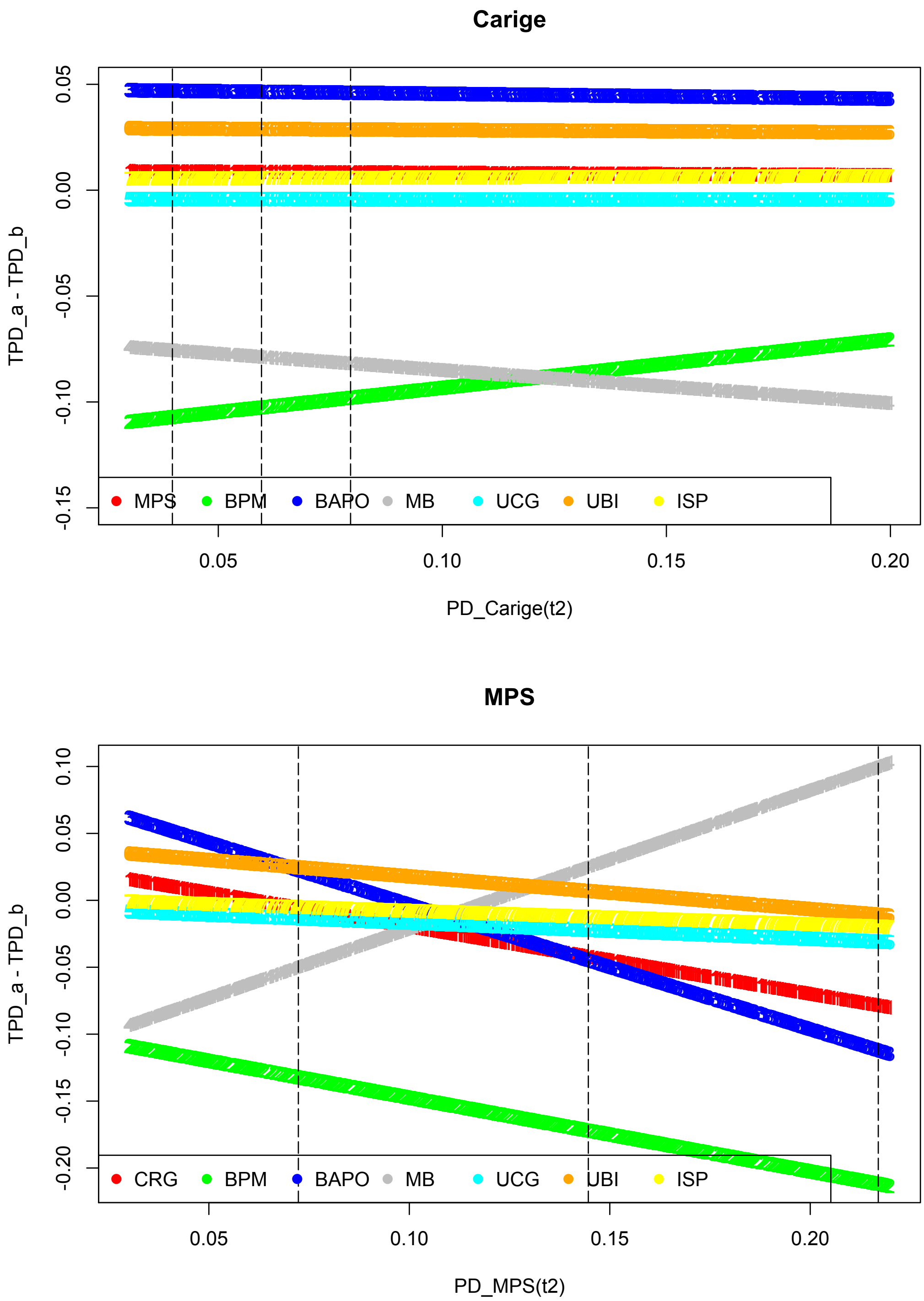
Notes: Monte Carlo simulated differences between the total default probabilities in case of bail-in and bail-out for bank 1 (left) and bank 2(right), plotted as functions of $\mu_{\rho_{13}}$ *(top-left),* $\mu_{\rho_{23}}$ *(top-right) and* $\mu_{\rho_{12}}$ *(bottom). The stronger the correlation with the troubled bank, the bigger the advantage of the bail-out scenario; for the small bank (bank 2), such advantage is a decreasing function of the partial correlation with the other safe bank.*

Figure 6.8.4: Partial correlation network, Italian banking system



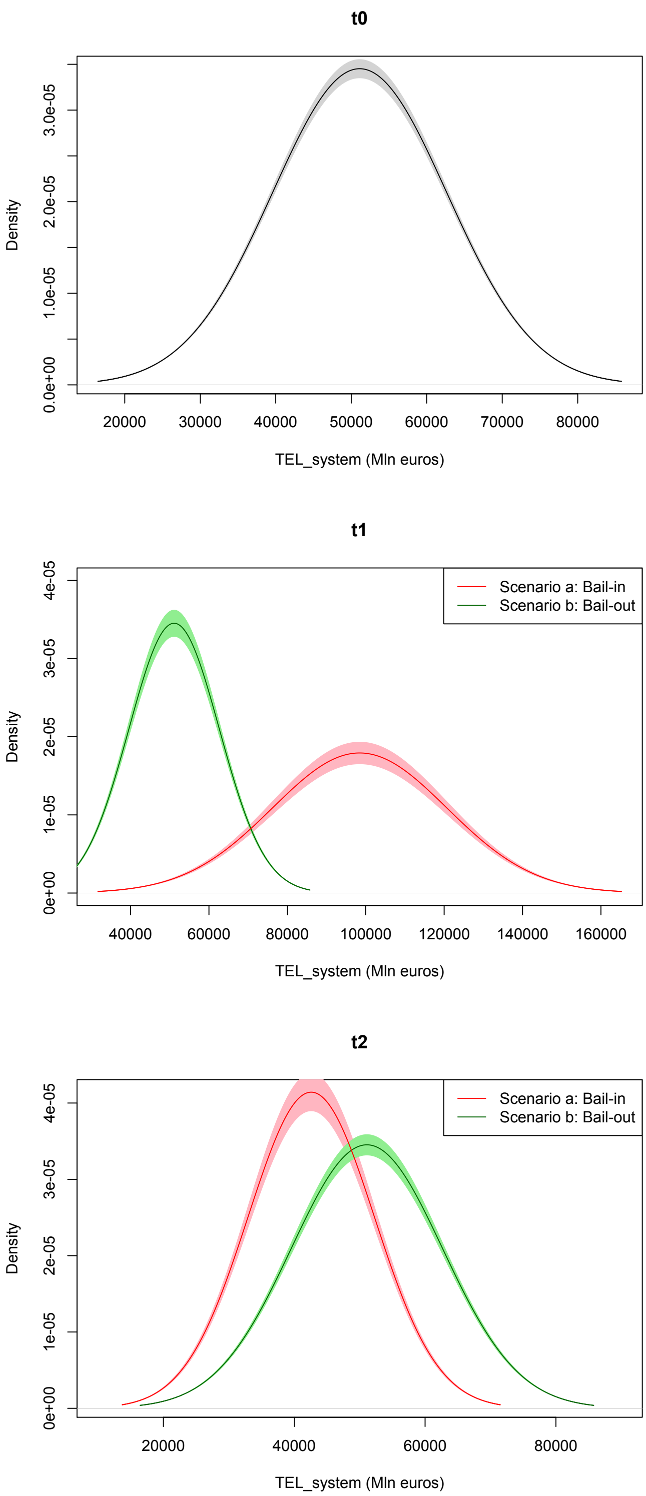
Notes: Partial correlation network between eight Italian banks, based on partial correlations between expected losses. The smallest bank in the system (CRG) is the less connected.

Figure 6.8.5: Changes in TPDs as functions of $PD_{t_2}^{CRG}$ and $PD_{t_2}^{MPS}$



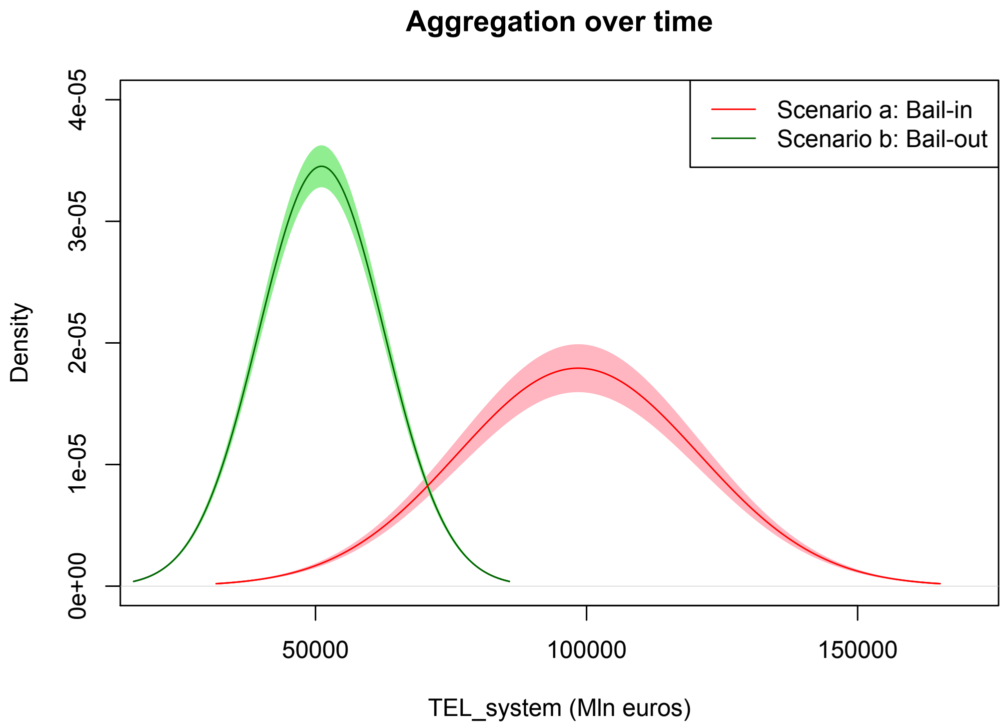
Notes: Monte Carlo simulated differences between the total default probabilities in case of bail-in and bail-out, if Banca Carige (top) or Monte dei Paschi di Siena (bottom) is close to its default point. In both graphs the biggest banks appear to be neutral with respect to the two alternative scenarios. In addition, the advantage of the bail-out (bail-in) is an increasing (decreasing) function of the $PD_{t_2}^{CRG,MPS}$ *in case of negative correlations; an opposite relationship regards positive correlations.*

Figure 6.8.6: Total expected losses of the system when CRG is in trouble



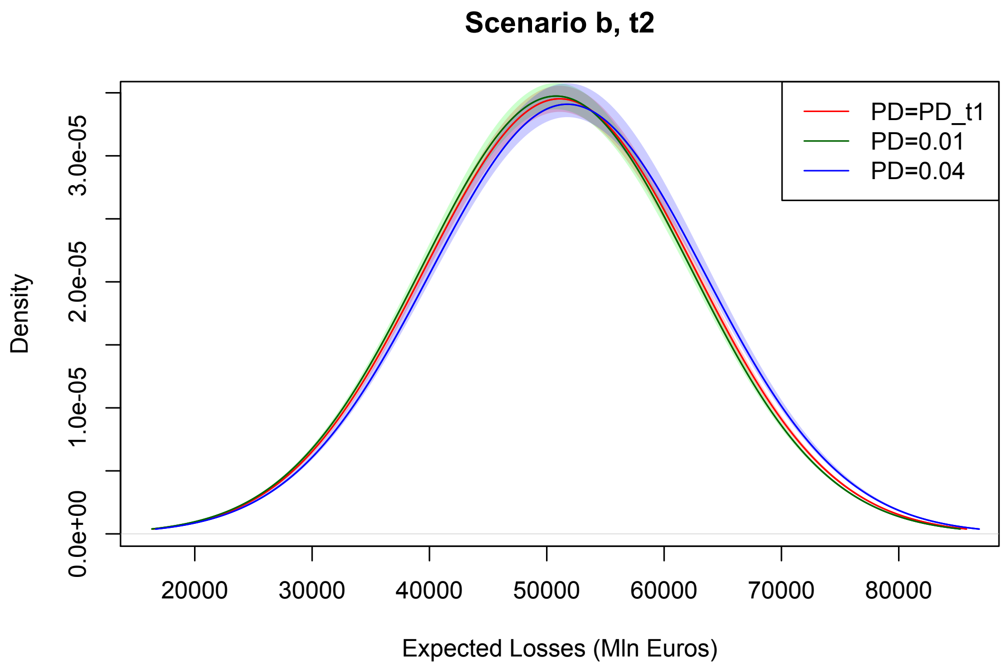
Notes: Monte Carlo simulated distributions of the expected losses of the entire banking system in case Banca Carige is close to its default point, calculated at time t_0 (top), t_1 (middle) and t_2 (bottom). Red lines represent the bail-in scenario, green lines stand for the bail-out scenario. Expected losses are lower for the bail-out scenario at time t_1, since the shock produced by the default of a bank increases contagion effects. On the contrary, at time t_2 the bail-in resolution should be preferred since the persistence of the troubled bank in the network in case of bail-out increases the expected losses of the system.

Figure 6.8.7: Total expected losses of the system when CRG is in trouble aggregated over time



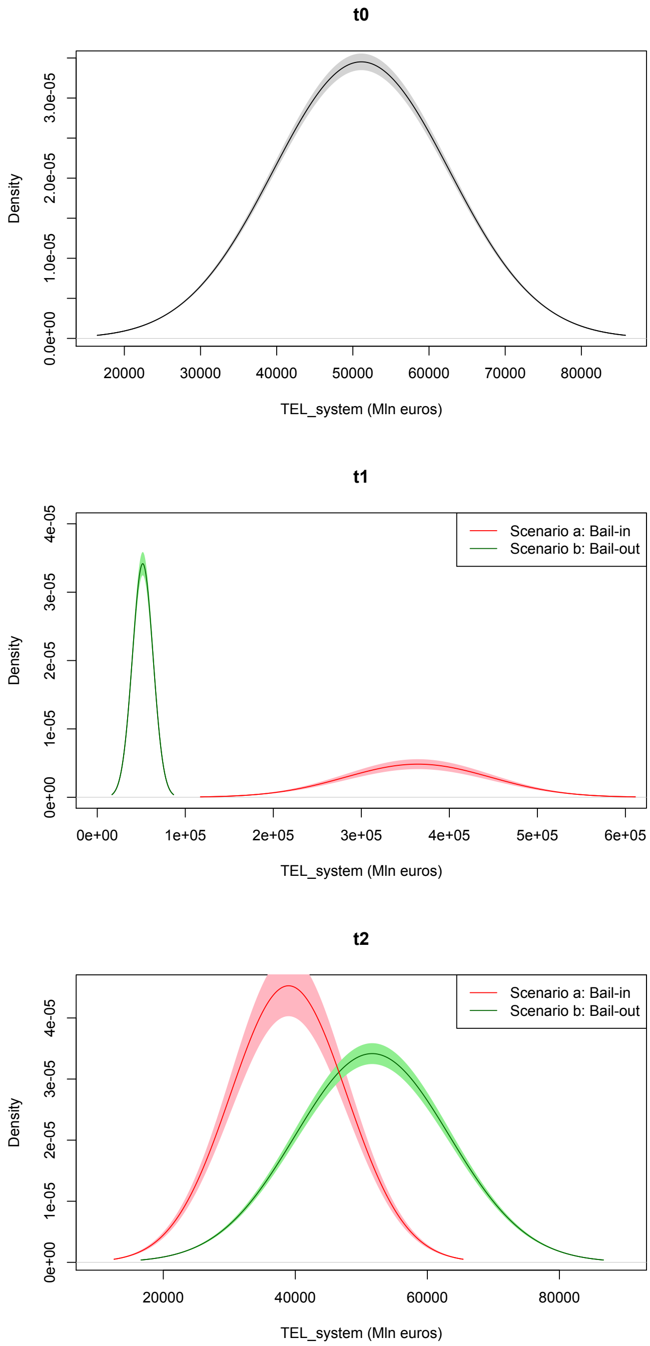
Notes: Monte Carlo simulated distributions of the expected losses of the entire banking system in case Banca Carige is close to its default point, aggregated over time. Red lines represent the bail-in scenario, green lines stand for the bail-out scenario. Overall, the bail-in resolution seems to increase the expected losses of the entire banking system.

Figure 6.8.8: Total expected losses of the system when CRG is in trouble as a function of $PD_{t_2}^{CRG}$



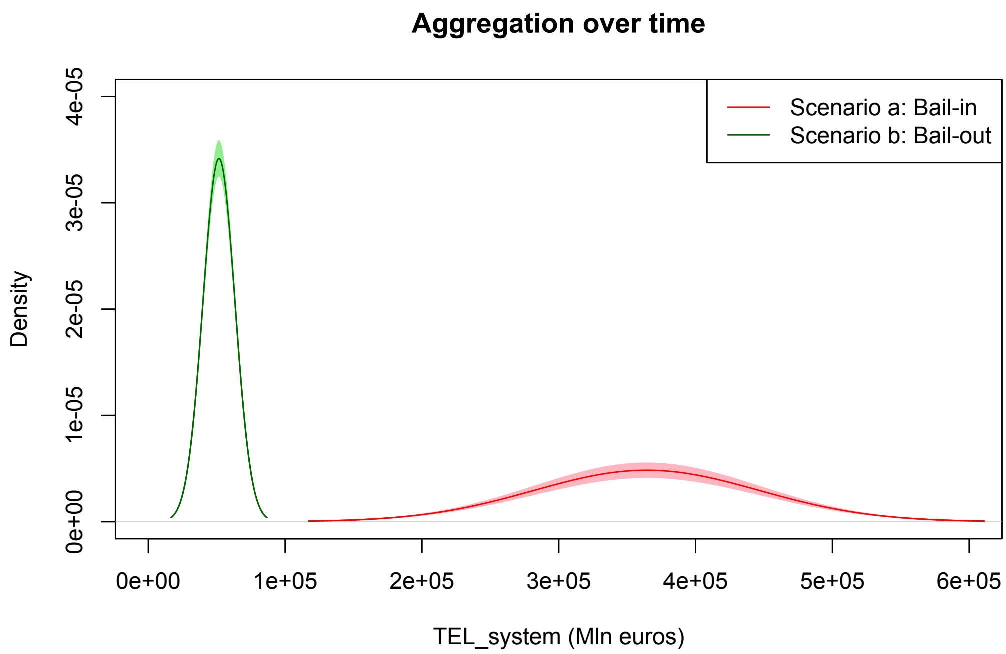
Notes: Monte Carlo simulated distributions of the expected losses of the entire banking system in case Banca Carige is close to its default point, calculated at time t_2 as functions of the default probability of Carige after the bail-out choice. The advantage of the bail-out scenario is a decreasing function of $PD_{t_2}^{CRG}$.

Figure 6.8.9: Total expected losses of the system when MPS is in trouble



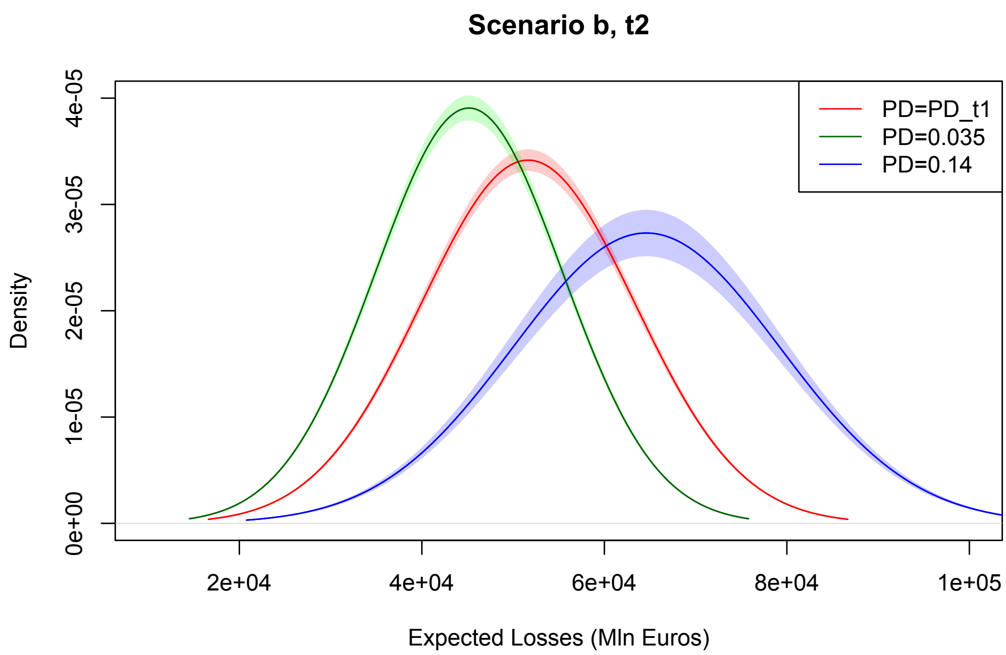
Notes: Monte Carlo simulated distributions of the expected losses of the entire banking system in case Monte dei Paschi di Siena is close to its default point, calculated at time t_0 (top), t_1 (middle) and t_2 (bottom). Red lines represent the bail-in scenario, green lines stand for the bail-out scenario. Expected losses are much lower for the bail-out scenario at time t_1, since the shock produced by the default of a big bank strongly increases contagion effects. On the contrary, at time t_2 the bail-in resolution should be preferred since the persistence of the troubled bank in the network in case of bail-out increases the expected losses of the system.

Figure 6.8.10: Total expected losses of the system when MPS is in trouble aggregated over time



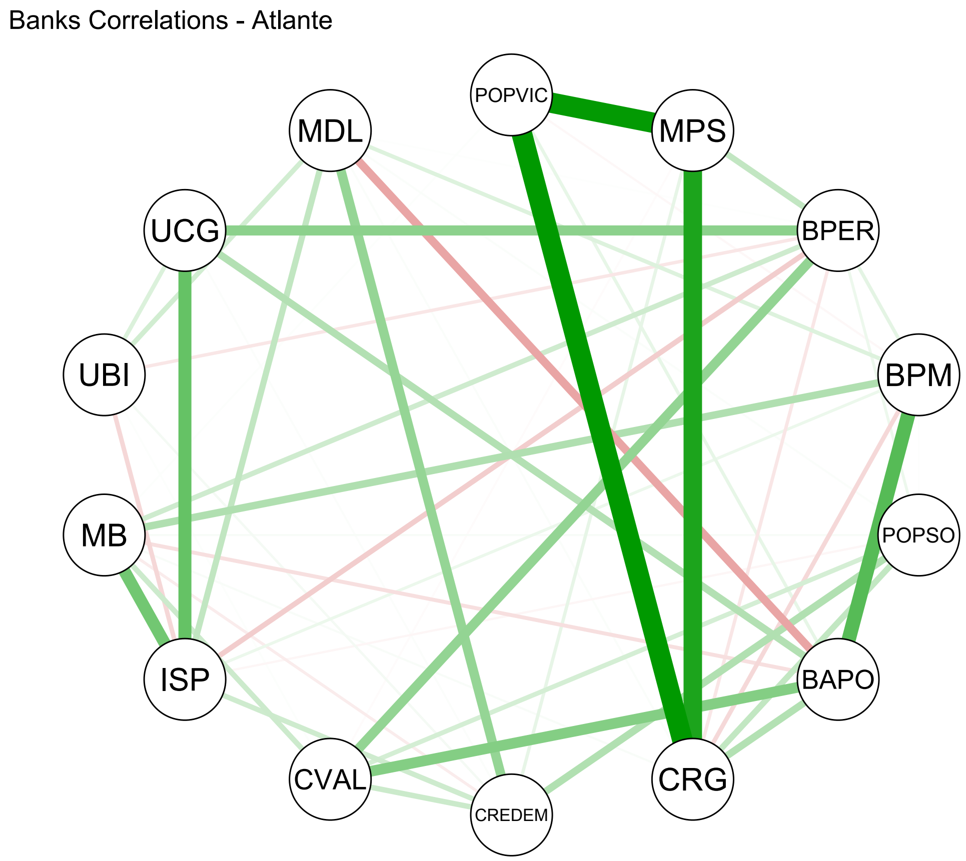
Notes: Monte Carlo simulated distributions of the expected losses of the entire banking system in case Monte dei Paschi di Siena is close to its default point, aggregated over time. Red lines represent the bail-in scenario, green lines stand for the bail-out scenario. Overall, the bail-in resolution seems to strongly increase the expected losses of the entire banking system.

Figure 6.8.11: Total expected losses of the system when MPS is in trouble as a function of $PD_{t_2}^{MPS}$



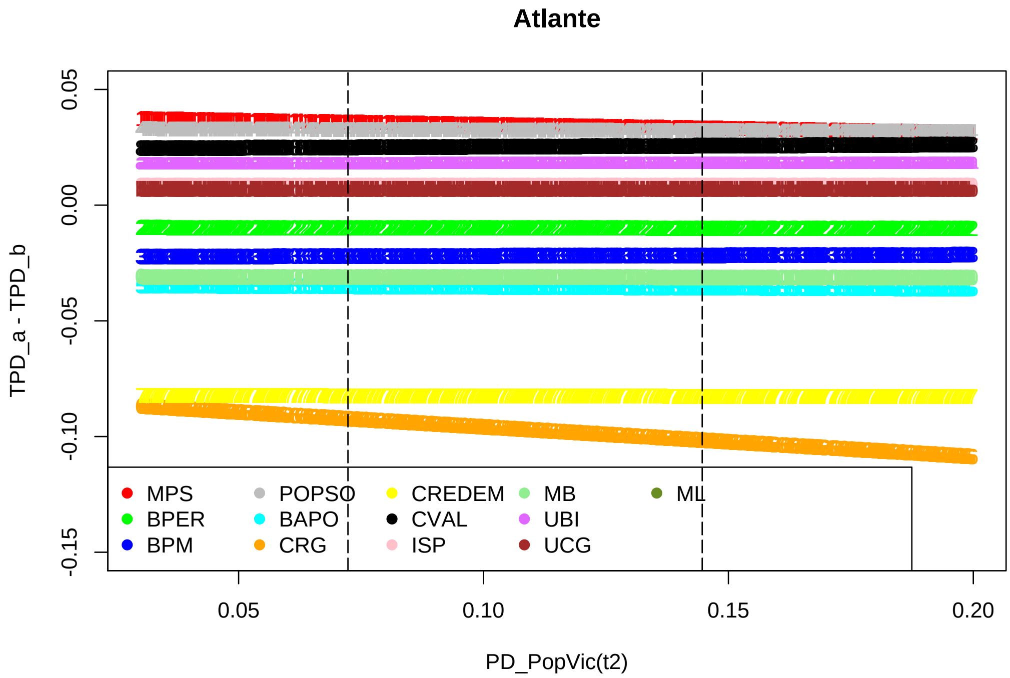
Notes: Monte Carlo simulated distributions of the expected losses of the entire banking system in case Monte dei Paschi di Siena is close to its default point, calculated at time t_2 as functions of the default probability of MPS after the bail-out choice. The advantage of the bail-out scenario is a decreasing function of $PD_{t_2}^{MPS}$, and such dependence seems to be stronger than in the Carige example.

Figure 6.8.12: Partial correlation network, Atlante



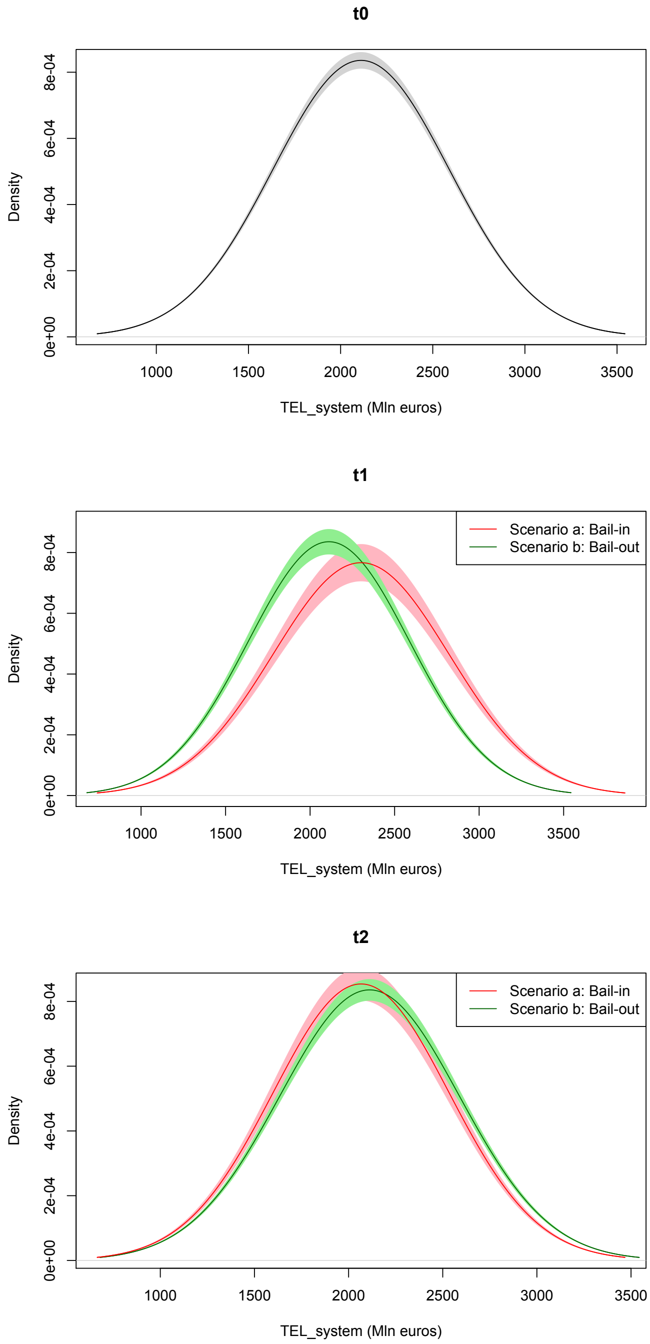
Notes: Partial correlation network between 14 Italian banks, based on partial correlations between expected losses. Expected losses have been calculated as the product between time-varying market capital values and default probabilities averaged over the period January-September 2016. The smallest bank in the system (POPVIC) is the less connected.

Figure 6.8.13: Changes in TPDs as functions of $PD_{t_2}^{POPVIC}$



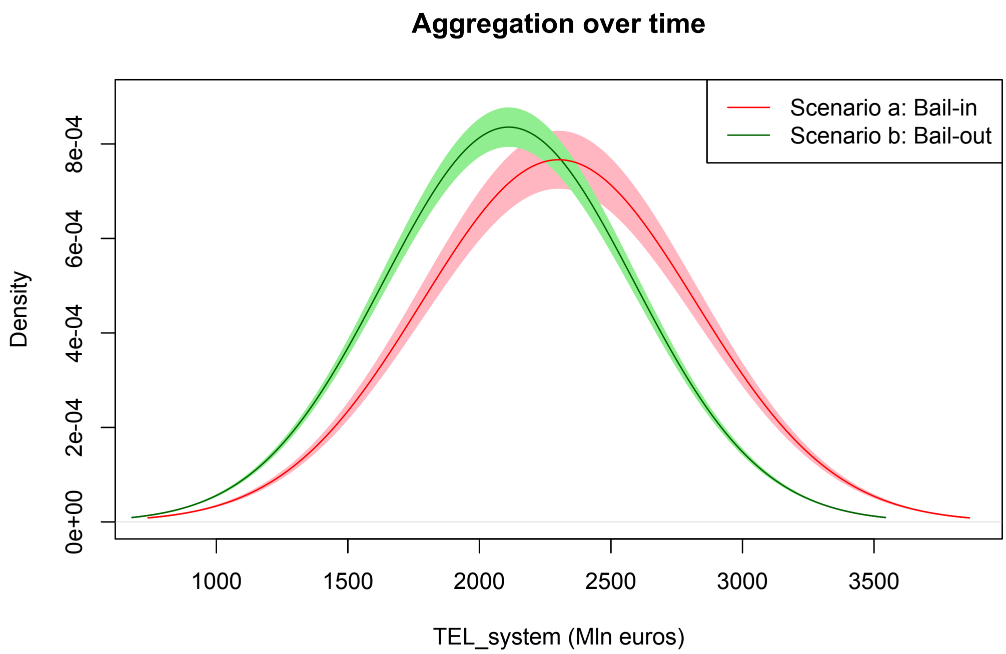
Notes: Monte Carlo simulated differences between the total default probabilities in case of bail-in and bail-out, consistently with the real situation of Banca Popolare di Vicenza close to its default point. The biggest banks appear to be neutral with respect to the two alternative scenarios. Only two banks appear to prefer the bail-in resolution: Banca Carige and CREDEM. In addition, for Carige the preference for the bail-in resolution is an increasing function of $PD_{t_2}^{POPVIC}$*, because of their high and positive partial correlation coefficients.*

Figure 6.8.14: Total expected losses of the system when POPVIC is in trouble



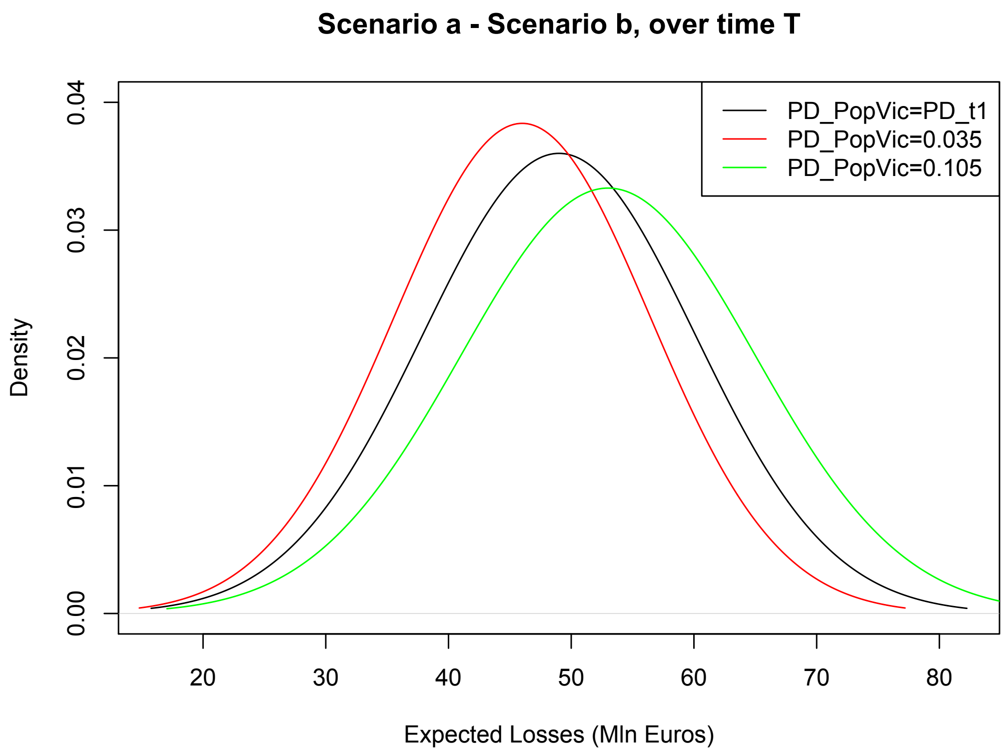
Notes: Monte Carlo simulated distributions of the expected losses of the entire banking system in the real case of Banca Popolare di Vicenza really close to its default point, calculated at time t_0 (top), t_1 (middle) and t_2 (bottom). Red lines represent the bail-in scenario, green lines stand for the bail-out scenario. Expected losses are lower for the bail-out scenario at time t_1, while at time t_2 the bail-in resolution should be preferred since the persistence of the troubled bank in the network in case of bail-out increases the expected losses of the system.

Figure 6.8.15: Total expected losses of the system when POPVIC is in trouble aggregated over time



Notes: Monte Carlo simulated distributions of the expected losses of the entire banking system in the real case of Banca Popolare di Vicenza really close to its default point, aggregated over time. Red lines represent the bail-in scenario, green lines stand for the bail-out scenario. Overall, the bail-in resolution seems to increase the expected losses of the entire banking system.

Figure 6.8.16: Total expected losses of the system when POPVIC is in trouble as a function of $PD_{t_2}^{MPS}$



Notes: Monte Carlo simulated distributions of the expected losses of the entire banking system in case Banca Popolare di Vicenza is close to its default point, aggregated over the whole time period as functions of the default probability of POPVIC after the bail-out choice. The advantage of the bail-out scenario is a decreasing function of $PD_{t_2}^{POPVIC}$.

Chapter 7

Conclusions

In this thesis we have analysed two aspects of prudential supervision: monetary policy transmission on one side, and systemic risk on the other. The first paper (*Predicting bank interests when monetary rates are close to zero*) concentrates on the modelling of the transmission mechanism of monetary rates on bank administered rates on overnight deposits, using data aggregated at the Italian level. In addition, after having tested all models, it provides an application to interest rate risk, using both the income and the economic value perspective. The results show that our proposed model better explain the transmission mechanism with respect to the traditionally used error correction model. In addition, we observe a regime switching in the explanation of bank administered interest rates in the recent time period: while until 2008 bank rates were significantly influenced by monetary rates, such relationship does not hold any more, being bank rates explained only by their autoregressive component. This radical change in the transmission mechanism, however, does not seem to influence interest rate risk.

The second paper (*Dynamic hierarchical models for monetary transmission*) evolves the methodology proposed in Chapter 2, from many viewpoints. First of all, we now introduce the cross-sectional dimension together with the time dimension: more precisely, we consider a set of Eurozone countries, and for each of them we model the transmission mechanism to three different kinds of interest rates. Consistently, we suppose a correlation structure between the time evolutions of different kinds of loans within the same country: in order to take this issue into account we introduce seemingly unrelated equations. Secondly, the first paper has shown us that monetary rates are not able to explain bank administered interest rates in the recent time period. Moreover, interest rates on different kinds of loans and in different countries are extremely heterogenous not only through time, but also according to the cross-sectional dimension. In order to implement all these kinds of variability we introduce dynamic equations (time-dependent parameters) and a hierarchical modelling: the first step of the process analysis how monetary rates have been able to explain the three kinds of bank rates in different countries, and how such relationship has changed through time. The second step of the process analysis the non-explained part of the first process by means of macroeconomic, country-specific fundamentals. This methodology allows to understand how the transmission mechanism on bank rates has changed through time, what are the differences across countries and kinds of loans, and how the heterogeneity in their behaviours can be explained by other factors different from monetary rates. Our results show that the effect of monetary rates on bank rates strongly depends on the lending types: large corporate loans are the most affected, whereas small-medium corporate loans and household loans depend less on monetary rates and more on country-specific macroeconomic factors, such as interest rates on deposits and GDP variations. This dependency can be explained, respectively, as the consequence of the bank need for returns, mostly determined by interest spreads, and as the impact of corporate and family risk. The dependence of bank rates on monetary rates considerably varies also across countries. In core countries, such as Germany and the Netherlands, bank rates depend almost exclusively on monetary rates and, therefore, the transmission of monetary policy is expected to be effective. In peripheral countries, instead, all lending rates depend on bank risk, corporate risk and, more recently, sovereign risk, as reflected by deposit rates, GDP variations and sovereign bond rates. Hence, in these countries the transmission of the monetary policy appears to be more problematic.

The following three papers concentrate on systemic risk. From a methodological viewpoint they all take advantage of partial correlation networks, but they focus on different objectives and research questions. The first paper in this stream of research (*Sovereign Risk in the Euro Area: a multivariate stochastic process approach*) aims at jointly modelling financial and real systemic effects of sovereign risk by means of correlated stochastic processes, with an application to Eurozone countries. To achieve this objective, we derive two different formulas for the PD calculation: one based only on financial data (interest rates on government bonds), and the other based on the leverage ratio Debt/GDP. All macro variables are modelled by means of multivariate stochastic processes, in order to take into account endogeneity issues, correlation structures and predictive performance assessments. The results show that, when using interest rate spreads, the Eurozone appears to be divided into northern (core) and southern (peripheral) economies, strongly interrelated within each cluster but with not significant or negative cross-cluster correlations. Such pattern, however, slightly changes when considering real economic variables such as the Debt/GDP ratio. Similar results also emerge from the analysis of default probabilities: in particular, the inclusion of the GDP growth rate exchanges the roles of France and Italy, with the former getting closer to peripheral countries, and the latter approaching an intermediate situation between peripheral economies and Germany. Furthermore, Ireland shows a radical regime switching through time: the drop of interest rates on government bonds combined with a strong increase in its GDP growth rate in the last two years, in fact, determined its extreme positioning change between clusters.

The second paper on systemic risk (*CoRisk: modelling systemic risk through default probability contagion*) focuses on the derivation of contagion effects due to interrelations between Eurozone countries and economic sectors. More precisely, we model each country as composed by three economic sectors: sovereigns, corporates and banks. Each sector is modelled as a linear combination of stochastic processes, one component being the idiosyncratic one, and the other representing the common, systematic factor. Since we use interest rates for both components, the resulting measure is an interest rate spread. Through partial correlation networks, we can thus identify the contagion channels between different economic sectors in different countries, as well as the interest rate spread measure allows the derivation of the market implied default probability of each node in the network. The combination of these two factors allows the determination of the diffusion mechanism of default probabilities through the network, thus providing a total default probability: such TPDs are composed by two parts: one typical of that single node, the other determined by the propagation of the PDs of the other nodes in the graph, that we call *CoRisk* (Contagion Risk). The methodology derived in this paper is also able to distinguish between vulnerability (how much a single node is affected by its neighbours) and systemic importance (how much a single node can affect its neighbours through its default probability propagation). Finally, the default probabilities derived at the sector level can be aggregated at the country level. The results can be summarised according to three dimensions: (a) the economic sector dimension, (b) the country dimension and (c) the time dimension at the aggregate country level. Concerning (a), the corporate sector is strongly influenced by the systematic component, and this implies that it responds to monetary policy changes more than sovereigns and banks. On the other hand, the sovereign and bank sectors deeply suffered, respectively, the sovereign and the financial crisis. In both sectors, the sovereign crisis has generated two distinct clusters, that have created loop effects further alienating troubled and strong economies. In a situation in which core economies benefit from positive contagion while peripheral countries suffer negative contagion, risk propagation seems to not act as a mean for balancing inequalities across countries; on the contrary, it weakens the weakest and strengthens the strongest countries. Concerning (b), core countries mostly behave as importers, rather than exporters of system risk. As a consequence, core economies are mostly affected by contagion risk and are rather vulnerable than systemic important; peripheral countries, instead, strongly suffer high sector-specific default probabilities and high contagion deriving from cluster effects, so they are both vulnerable and systemic important. Concerning (c) the sovereign crisis has had a larger impact on systemic risk with respect to the financial crisis. Peripheral countries, with high public debts, had little fiscal space to improve balance sheets and bailing out banks after 2008: as a consequence, the financial crisis triggered their imbalances to emerge in the subsequent sovereign crisis. The time sequence of these two events has determined an irreversible phase change, leading to a new non-stable equilibrium, where instability derives from peripheral-countries trajectories diverging from core ones.

The last paper (*Bail-in: a systemic risk perspective*) again deals with systemic risk, but is more related to macroprudential supervision issues. It exploits the methodology introduced in the previous paper (derivation of the contagion component) but it substantially evolves it and applies it into a different context. First of all, we now deals with banks at the micro level rather than on aggregated data. Secondly, in order to take different dimensions of banks into account, we propagate expected losses rather than default probabilities. Third, the research question is now the following: from a systemic risk perspective, banks should bailed-in or -out? The answer we provide depends on two perspectives: the banks perspective and the system perspective. Let us suppose there is a troubled bank in the system: we can identify two scenarios. The bank fails (bail-in); the bank is saved by the others banks in the system through a capital injection operation (private bail-out). The banks perspective analysis how convenient is the bail-out or the bail-in according to the decrease of its own systemic risk deriving from contagion effects; the system perspective analysis how convenient is the bail-out or the bail-in according to the decrease of the total expected losses of the banking system. Both perspectives can be analysed on the short-run or on long-term basis. We apply the methodology to the Italian banking system: as troubled bank we consider both a small (Banca Carige) and a large (Monte dei Paschi di Siena) bank. Our findings can be summarised as follows. First, the simulation results reveal that the smaller or the safer a bank is, the larger the advantage of choosing a bail-out scenario. The advantage increases with the correlation with the troubled bank; it decreases with the correlation between the safe banks and it decreases with the default probability of the troubled bank. Second, the application to the Italian banking system reveals that, in the long-run, the bail-out should always be preferred to the bail-in resolution from a system's perspective. Such preference, moreover, strongly increases as the size of the troubled bank increases. Regarding the single bank perspective, the correlation pattern is the main driver for the choice of a bail-in or a bail-out.