

Depth-of-Field Enhancement in Filtered-Delay Multiply And Sum Beamformed Images Using Synthetic Aperture Focusing

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Abstract

1 The Synthetic Aperture Focusing (SAF) technique makes it possible to achieve a higher and more uniform
2 quality of ultrasound images throughout depth, as if both transmit and receive dynamic focusing were applied.

3 In this work we combine a particular implementation of SAF, called Synthetic Transmit Aperture (STA)
4 technique, in which a single element in turn transmits and all the array elements receive the ultrasound wave,
5 with the Filtered-Delay Multiply and Sum (F-DMAS) non-linear beamforming algorithm that we presented in
6 a previous paper. We show that using F-DMAS, which is based on a measure of backscattered signal spatial
7 correlation, B-mode images have a higher contrast resolution but suffer from a loss of brightness away from
8 the transmit focus, when a classical scan with receive-only dynamic focusing is performed. On the other hand,
9 when synthetic transmit focusing is achieved by implementing STA, such a loss is compensated for and a
10 higher depth of field is obtained, as signal coherence improves. A drawback of SAF/STA however is the
11 reduced signal-to-noise ratio, due to single-element transmission; in the paper we also analyze how this
12 influences F-DMAS images. Finally, a preliminary investigation on the use of the classical monostatic SAF
13 technique with F-DMAS beamforming is also carried out to evaluate its potential performances.

14
15 **Keywords:** beamforming; Filtered-Delay Multiply And Sum; Synthetic Aperture Focusing; ultrasound
16 medical imaging

18 1. INTRODUCTION

19 In signal processing, beamforming can be considered as a form of spatial filtering [1] aimed at reinforcing
20 the estimate of the signal received from the direction of interest, while rejecting as much as possible
21 interferences coming from off-axis directions.

22 To accomplish this task, in ultrasound medical imaging systems, the receive beamformer unit is in charge
23 of computing and applying a set of delays and weights to the echo signals received by the transducer elements
24 in the probe, in order to focus and steer the beam towards the desired direction, while optimizing its shape.

25 The standard beamforming technique implemented by commercial ultrasound scanners is the simple Delay
26 And Sum (DAS). However, the quality of ultrasound images remains still limited by the aperture size and
27 operating frequency of the system, which are directly related to the achievable lateral/axial resolution, depth
28 of field (DOF) and penetration depth. On the other hand, adaptive beamformers, such as the Capon/Minimum
29 Variance (MV) beamformer [2,3], have been developed to obtain higher resolution and contrast by controlling
30 the aperture apodization weights based on the spatial statistics of the received signals. Other ultrasound image
31 formation techniques have also been recently developed to improve the lateral resolution and gain a higher
32 contrast, as, for example, the Dual Apodization with Cross-Correlation (DAX) [4] or the Side Lobe Masking
33 [5] techniques. Besides, non-linear beamformers were proposed in the past, mainly for direction of arrival
34 estimation but also for beam formation [6,7].

35 In a previous work [8], we proposed and adapted a non-linear beamforming algorithm, called Delay
36 Multiply and Sum (DMAS), for application to ultrasound B-mode image formation. That beamformer had
37 been originally presented by Lim *et al.* in a paper on microwave image reconstruction for breast cancer
38 detection [9]. By introducing several further processing steps in the beamformation chain, both working on the
39 amplitude and frequency content of the echo signals, this improved DMAS algorithm, called Filtered-DMAS
40 (F-DMAS), was shown to achieve higher contrast resolution than DAS, both in simulation and *in vivo* tests,
41 and also when used jointly to other ultrasound imaging techniques [10-12]. The improved performance of the
42 F-DMAS beamformer arises from the computation of the aperture spatial auto-correlation, on which this
43 technique is based.

44 Actually, a slight decrease of brightness can be observed in F-DMAS B-mode images at low (i.e. near to
45 the probe surface) and high depths, as compared to DAS. A possible explanation of this phenomenon could be
46 that the spatial correlation of backscattered signals is higher in correspondence of the transmit focal depth. In
47 conventional B-mode imaging, in fact, a fixed focal depth is used during transmission and dynamic focusing
48 (DF) is usually implemented only in reception, otherwise the frame rate would be drastically reduced. An
49 acceptable trade-off between the required frame rate and an improved image quality could be achieved by
50 acquiring images with different transmit focal depths and splicing them together [13]; anyway, even in this
51 case the frame rate would be reduced.

52 In [14] we presented some preliminary results to validate the hypothesis that the intensity loss shown by F-
53 DMAS images away from the transmit focus could be due to the decrease of echo signal coherence when DF
54 is applied only in reception. As a matter of fact, backscattered signal coherence is expected to reach its
55 maximum at the transmit focus based on the Van Cittert-Zernike (VCZ) theorem [15].

56 A similar problem affects also Short-Lag Spatial Coherence (SLSC) imaging [16], which is another recently
57 proposed technique based on the spatial correlation of backscattered signals [17]. SLSC computation involves
58 the coupling, multiplying and summing of the short-lag echo signals. However, differently from B-mode image
59 formation techniques like DMAS, in this case such operations are used to generate images of the backscattered
60 signal spatial coherence, and not images of echo magnitudes, whose influence is removed by normalizing the
61 cross-correlation [17]. Therefore, being SLSC images a direct representation of backscattered signals
62 coherence, in [16] they were shown to suffer from a reduced DOF away from the transmit focal depth, based
63 on the VCZ theorem. The DOF was instead significantly improved by implementing Synthetic Aperture
64 Focusing (SAF) [18].

65 In a classical SAF implementation, the aperture is synthetically built by activating one single element at
66 each time to act as a transmitter and receiver. Instead, when a single element is used in turn to transmit the
67 ultrasound pulse but reception is performed by all the array elements, this technique is referred to as Synthetic
68 Transmit Aperture (STA) [19,20]. After the transmit-receive sequence has been repeated for all the elements
69 in the array, beamforming can be performed by synthetically focusing the acquired signals a posteriori in each
70 point of the image space, as if implementing both transmit and receive DF. Therefore, signals can be almost

71 correctly realigned by compensating for the two-way propagation delays at all depths, yielding to an image
72 with higher lateral resolution, improved DOF and more uniform quality, thanks to synthetic transmit focusing.
73 However, a drawback of these techniques is that they generally suffer from a poor SNR and low penetration
74 capability due to single-element transmission, and also from tissue motion artifacts, due to the higher number
75 of transmit events required to generate an image [18].

76 In this paper we hypothesize that there are mainly two factors that cause the decorrelation effect which
77 affects F-DMAS image intensity in standard B-mode scans with receive-only DF, i.e.: i) the broadening of the
78 transmit beam away from the transmit focus, and ii) noise (including both electronic noise and other
79 interferences related to the physics of the ultrasound beam). Thus, we aim to understand how F-DMAS images
80 are influenced by different focusing strategies. This would also provide further insights on the impact of
81 backscattered signal coherence on F-DMAS beamforming, widening the study presented in our previous work
82 [8]. We thus implemented F-DMAS with or without STA and synthetic transmit focusing, in order to analyze
83 decorrelation effects in F-DMAS beamformed images.

84 Finally, we also investigate if F-DMAS beamforming can be used in a simpler monostatic SAF-based
85 system, making it possible to achieve adequate imaging performance. This technique, in fact, is generally
86 worse than STA in terms of contrast resolution, but could be more appealing for a possible hardware
87 implementation, as it involves only one single transducer element (and thus a simpler electronics with one
88 single channel) both to transmit and receive the ultrasound wave.

89 In the following pages, the F-DMAS algorithm as well as the SAF and STA techniques are first described
90 (Section 2). Henceforth, we will use the acronym SAF to refer to the classical monostatic implementation of
91 this technique. We then compare F-DMAS and DAS performance by reconstructing images, either with fixed
92 transmit focus and receive-only DF, so as to emulate a classical B-mode scan, or with STA and synthetic
93 transmit focusing (i.e. emulating both transmit and receive dynamic focusing), and we evaluate the results
94 achieved in simulations, phantom experiments and *in vivo* (Section 3). The performance of classical SAF
95 together with F-DMAS is analyzed in phantom experiments too. Finally, in Section 4 we discuss the results
96 and provide some conclusive remarks.

97

98 **2. MATERIALS AND METHODS**

99 **2.1. SAF and STA techniques**

100 The classical monostatic implementation of SAF consists in activating each time a single element of the
101 array to transmit an unfocused spherical wave and to receive the echo signal. If we consider N transducers and
102 we denote the active element with index i ($i=1\dots N$), then a set of raw radiofrequency (RF) signals V is collected
103 after all elements have been used one by one to transmit and receive:

104
$$V(t) = [v_1(t) \quad v_2(t) \quad \dots \quad v_N(t)]. \quad (1)$$

105 Each column v_i of matrix V represents the RF signal received by element i after it has transmitted.

106 In order to realign the received signals v_i , the focusing delays τ_{ii} are computed by considering the two-way
107 distance from element i to the focal point and vice-versa. For example, focusing delay τ_{ii} is computed as follows
108 (Fig. 1a):

109
$$\tau_{ii} = \tau_{i,TX} + \tau_{i,RX} = \frac{2}{c} \sqrt{(x_F - x_i)^2 + z_F^2}, \quad (2)$$

110 where $\tau_{i,TX}$ and $\tau_{i,RX}$ are the transmit and receive delay of element i , respectively, the coordinates of the active
111 element are $(x_i, z_i=0)$, and the focus is placed at (x_F, z_F) ; c is the sound speed in the medium. In this way, a new
112 set S of focused signals is obtained:

113
$$S(t) = [s_1(t) \quad s_2(t) \quad \dots \quad s_N(t)] \quad (3)$$

114 where $s_i(t) = v_i(t - \tau_{ii})$.

115 In order to implement STA, instead, we use the following procedure. Each single transducer element in the
116 active aperture is used in turn to transmit an un-focused spherical wave, and the backscattered echo signals are
117 received by all elements; this process is repeated for each transducer in the aperture. If index i refers to the
118 transmitting element and $j=1\dots N$ to the receiving elements, then a set of RF signals V_i is collected by the N
119 receivers for each i -th transmission:

120
$$V_i(t) = [v_{i1}(t) \quad v_{i2}(t) \quad \dots \quad v_{iN}(t)]. \quad (4)$$

121 where each column v_{ij} of the matrix represents the RF signal received by element j when element i transmits.
122 In order to realign these signals, delays τ_{ij} are computed by considering the two-way distance from the
123 transmitting element i to the focal point, and back to each receiving element j (Fig. 1b), as follows:

$$124 \quad \tau_{ij} = \tau_{i,TX} + \tau_{j,RX} = \frac{1}{c} \left[\sqrt{(x_F - x_i)^2 + z_F^2} + \sqrt{(x_F - x_j)^2 + z_F^2} \right], \quad (5)$$

125 where the coordinates of the transmitting element are $(x_i, z_i=0)$, those of the receiving element are $(x_j, z_j=0)$ and
126 $\tau_{i,TX}$ and $\tau_{j,RX}$ are the transmit and receive delay of elements i and j , respectively. The new set \mathbf{U}_i of focused
127 signals is given by:

$$128 \quad \mathbf{U}_i(t) = [u_{i1}(t) \quad u_{i2}(t) \quad \dots \quad u_{iN}(t)] \quad (6)$$

129 where $u_{ij}(t) = v_{ij}(t - \tau_{ij})$. The transmit-receive sequence is repeated for each i -th element in the aperture and the
130 N realigned signal sets \mathbf{U}_i are summed together:

$$131 \quad \mathbf{S}(t) = \sum_{i=1}^N \mathbf{U}_i(t) = [s_1(t) \quad s_2(t) \quad \dots \quad s_N(t)], \quad \text{with } s_j(t) = \sum_{i=1}^N u_{ij}(t) \quad (7)$$

132 so that one single signal $s_j(t)$ is obtained for each j -th receiving element in the aperture.

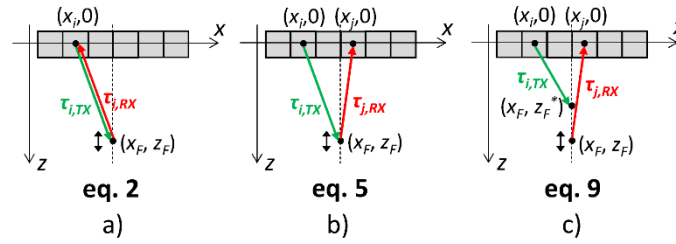
133 Both in the case of SAF and STA, the signals \mathbf{S} obtained respectively in (3) and (7) are subsequently used
134 for image formation by applying the considered beamforming algorithm: for F-DMAS, the processing steps
135 described in the next section are applied, while for DAS the signals are simply summed up:

$$136 \quad y_{DAS}(t) = \sum_{j=1}^N s_j(t). \quad (8)$$

137 Since with SAF and STA the synthetic focusing stage is totally (i.e. both for transmit and receive, eq. 3 and
138 6) implemented a-posteriori on the raw signal set, the advantage of these techniques is that they make it
139 possible to vary the focal point with depth, and therefore both transmit and receive DF may be implemented.
140 This allows us to considered also a third configuration in this work, in which STA is implemented by
141 synthesizing a focal point (x_F, z_F^*) with fixed depth (i.e. $z_F = z_F^*$) and varying only the receive focus (x_F, z_F) with
142 depth, so as to emulate a B-mode scan. In this case the delays are computed as follows (Fig. 1c):

$$143 \quad \tau_{ij} = \tau_{i,TX} + \tau_{j,RX} = \frac{1}{c} \left[\sqrt{(x_F - x_i)^2 + z_F^{*2}} + \sqrt{(x_F - x_j)^2 + z_F^2} \right]. \quad (9)$$

144 Of course SAF and STA have both pros and cons. For example, with SAF, each signal s_i is the result of one
145 single transmit-receive event and (with DAS) the beampattern has a narrower main lobe, high side and grating
146 lobes, as shown in [19]. With STA instead, each output signal s_j is obtained by combining the contributions of
147 all the aperture transmitting elements (7) and the beampattern has lower side lobes, no grating lobes, but a



148
149 Fig. 1. Schematic representation of delay calculation, corresponding to: a) equation (2), b) equation (5), c) equation (9).

150 wider main lobe [19]. Anyway, SAF could be very attractive for its simpler hardware implementation,
151 especially if used in conjunction with a beamforming algorithm as F-DMAS, which is expected to improve its
152 performances in terms of contrast resolution.

153 2.2. Filtered-Delay Multiply And Sum beamforming

154 After the signal focusing/delaying phase, the following procedure is applied to implement F-DMAS [8].
155 First, the signed squared root is applied to the realigned RF signals s_j , which are then combinatorially coupled,
156 multiplied and summed. In the practice, it is as if a new set of "equivalent RF signals" is computed, whose
157 amplitude has been rescaled just before entering the multiplication stage:

$$158 \hat{s}_j(t) = \text{sign}(s_j(t)) \cdot \sqrt{|s_j(t)|}. \quad (10)$$

159 Hence, the DMAS beamformed signal is given by:

$$160 y_{DMAS}(t) = \sum_{n=1}^{N-1} \sum_{m=n+1}^N \hat{s}_n(t) \hat{s}_m(t). \quad (11)$$

161 In [8] we explain that this operation is equivalent to the aperture spatial auto-correlation, except for the fact
162 that the new set of RF signals (10) is employed in the calculation, the auto-products ($n=m$) are not considered
163 and each signal couple is used only once (i.e., $s_n s_m$ and not $s_m s_n$). As the multiplication stage also changes the
164 frequency content of the output signal y_{DMAS} , a final band-pass (BP) filtering stage is implemented to generate

165 the beamformed output y_{F-DMAS} . The filter is designed to pass only the second harmonics almost unaltered while
166 attenuating the lower and higher spectral components, especially the baseband one.

167 The spatial autocorrelation of a N -element aperture with uniform weights (i.e. a *rect* function) is a triangle
168 made of $2N-1$ coefficients. This means that the contribution of short-lag coefficients, obtained by cross-
169 correlating signals received by close elements in the array, is higher than that of long-lag ones. We shall also
170 consider that, the higher the lag between the elements, the less the RF signals will be correlated; therefore, a
171 higher contribution is provided by more strongly correlated signal couples. The described operations make it
172 possible to achieve several advantages compared to DAS:

- 173 1) images have a higher lateral resolution, since it is as if a wider aperture is employed (the auto-correlation
174 has $2N-1$ coefficients) and the central frequency is doubled;
- 175 2) side lobes are lowered thanks to the cross-correlation (i.e. couple and multiply) stage, which turns out
176 into a higher contrast resolution;
- 177 3) for the same reason, un-correlated noise is better rejected.

178 In order to generate the desired B-mode image, the focusing and beamforming (i.e. F-DMAS or DAS)
179 processing steps are repeated for each scan line, and the image lines are then demodulated by applying the
180 Hilbert transform. Each image is normalized to its maximum value, logarithmically compressed, interpolated
181 and finally displayed.

182 ***2.3. Influence of focusing strategies on the coherence of received signals***

183 To understand the expected decrease of echo signal correlation in B-mode images away from the transmit
184 focus when DF is applied only in reception, and consequently the loss of amplitude in the F-DMAS image, the
185 VCZ theorem should be recalled.

186 The VCZ theorem provides a theoretical basis for the calculation of the spatial coherence of backscattered
187 signals. In [15,21] it was applied to ultrasound, and it was demonstrated that the spatial correlation function of
188 the pressure field produced by an incoherent source (such as diffuse scatterers) is proportional to the Fourier
189 transform of the source intensity distribution. For discrete time signals, assuming that the source is incoherent

190 and quasi-monochromatic, and that no phase aberrations are present, the sample spatial correlation R of the
191 ultrasound field evaluated at two points (e.g., at two array elements E_1 and E_2) can be approximated as [21,22]:

$$192 \quad R \approx e^{-j2\pi f_0 \tau} \sum_{m=1}^M I_m e^{-j2\pi x_m \frac{(x_1 - x_2)}{\lambda_0 z}}, \quad (12)$$

193 where M discrete point sources are considered; x_m , x_1 and x_2 are the lateral spatial coordinates respectively of
194 the m -th source, of element E_1 or element E_2 , f_0 is the center frequency, $\lambda_0 = c/f_0$, I_m is the ultrasound field
195 intensity at the m -th point, and z is the distance between the source and the aperture (z is assumed to be much
196 greater than the aperture dimension).

197 In (12), the summation term is the discrete Fourier transform of the source intensity distribution. The phase
198 term outside the summation instead corresponds to the time delay to be applied for focusing at point m ;
199 consequently, this term cancels if signals are properly phased shifted (i.e. realigned) for focusing during
200 reception. When $\tau=0$, the aperture autocorrelation function is the Fourier transform of the field intensity;
201 therefore, for a uniformly weighted/apodized aperture, the field in the focal plane is a sinc^2 and the spatial
202 correlation is a triangle function. This means that, the wider the beam is (e.g. away from the focus), the
203 narrower the autocorrelation function becomes [22].

204 For this reason, spatial coherence is maximized only at the focus, where the beam is narrower than at other
205 depths. This is the case of conventional B-mode imaging, where the receive beamformer correctly compensates
206 for the two-way propagation delays only at the transmit focus, as DF is applied only in reception. On the other
207 hand, SAF/STA makes it possible to implement synthetic transmit focusing, and the backscattered signals can
208 be correctly realigned at each time instant; thus, the aperture autocorrelation is expected to improve at all
209 depths [16].

210 **2.4. Simulation and experimental setup**

211 The results obtained by applying DAS and F-DMAS beamforming to reconstruct images, either with
212 receive-only DF or with STA and synthetic transmit focusing, were compared in simulated and experimental
213 conditions. Finally, an analysis on the performance of SAF with F-DMAS was carried out in experimental
214 phantom scans.

215 A 192-element linear array (element width=215 μm , height=5 mm, kerf=30 μm), with a center operating
216 frequency of 5 MHz, was modelled in Matlab for simulations using Field II [23,24]. The elevation focus was
217 fixed at 23.5 mm. A 64-element active aperture was considered and 129 lines were scanned (scan line spacing
218 = 1 pitch). In the case of receive-only DF, the fixed focus in transmission was set at $z=24$ mm. The F-number
219 (F#) in all cases varied with depth, as no dynamic apodization was applied in reception and the full aperture
220 was always used.

221 Received signals were first BP pre-filtered by simply windowing their spectrum; when F-DMAS was used,
222 a further BP filter was applied to the beamformed lines. The parameters of the filters were empirically
223 determined by observing the signal spectra and they are given in Table 1. In the simulations, a 2-cycle,
224 Gaussian-windowed sinusoidal burst at 5 MHz (fractional bandwidth $\approx 70\%$) was employed as excitation
225 signal, with a sampling frequency of 50 MHz.

226 First, a numerical phantom scan was simulated. The phantom was homogeneous; it was made of 150,000
227 scattering points with a Gaussian-distributed reflectivity, randomly spread in a $12 \times 1 \times 40 \text{ mm}^3$ volume, centered
228 at $(x, y, z)=(0, 0, 30)$ mm.

229 Experimental acquisitions were performed by connecting a linear array probe (model LA-532, Esaote
230 s.p.a., Florence, Italy) to the ULA-OP system [25] and using the same scan parameters set in simulations.

231

232

TABLE 1. Parameters of the BP-filters.

	Filter	Window	Frequency boundaries
Simulations	RF signals pre-filter	Tukey ($\alpha=0.5$)	1.5–9 MHz
	F-DMAS final filter	Tukey ($\alpha=0.5$)	3–15 MHz
Experiments	RF signals pre-filter	Tukey ($\alpha=0.3$)	1.4–7 MHz
	F-DMAS final filter	Tukey ($\alpha=0.5$)	3.3–13 MHz

233 Actually, a custom routine was developed to implement the SAF/STA transmit-receive sequence with
234 ULA-OP, by activating each time only one element to transmit and receiving with all the array. The voltage

235 excitation signal was a 5 MHz, 2-cycle, Hanning-windowed sinusoidal burst, with a 64 V_{pp} amplitude. The
236 sampling frequency was 50 MHz.

237 A CIRS phantom model 040GSE (CIRS Inc., Norfolk, VA) was scanned to obtain an image of anechoic
238 cysts at different depths and also of a homogeneous region, for comparison with simulations; raw pre-
239 beamforming data were then processed in Matlab for offline image reconstruction. In the case of the
240 homogeneous-region scan, a further set of images was generated by artificially worsening the quality of RF
241 signals used for reconstruction, so as to investigate the performance of F-DMAS in noisier conditions. To do
242 so, synthetic white noise (Gaussian distribution, SNR=40 dB) was added to the raw RF signals in Matlab. A
243 time/depth-varying gain of 2.5 dB/cm was applied to the noise signal, since this was the same gain set for Time
244 Gain Compensation (TGC) in the ULA-OP system during acquisitions.

245 Finally, *in vivo* images of the human carotid artery were acquired; here the transmit focal depth (for the
246 receive-only DF case) was set to $z=15$ mm.

247 By synthesizing the focusing delays as in eq. 9 or eq. 2/5, it was thus possible to use the same set of RF-
248 signals (and thus to perform a single acquisition) to reconstruct images both with receive-only DF or with
249 SAF/STA with synthetic transmit focusing (i.e. transmit/receive dynamic focusing), respectively.

250 For cyst phantom images, three parameters were measured for performance evaluation, i.e. the contrast
251 ratio (CR), contrast-to-noise ratio (CNR) and SNR parameters; they were computed as follows [17]:

$$252 \quad CR = 20 \log_{10} \left(\frac{\mu_{cyst}}{\mu_{bck}} \right) \quad (13)$$

$$253 \quad CNR = \frac{|\mu_{bck} - \mu_{cyst}|}{\sqrt{\sigma_{bck}^2 + \sigma_{cyst}^2}} \quad (14)$$

$$254 \quad SNR = \frac{\mu_{bck}}{\sigma_{bck}} \quad (15)$$

255 where μ_{cyst} and μ_{bck} are the mean image values measured on the envelope-detected signals (before log-
256 compression) respectively inside the cyst and in the surrounding background, and σ_{cyst} , σ_{bck} are the
257 corresponding standard deviations.

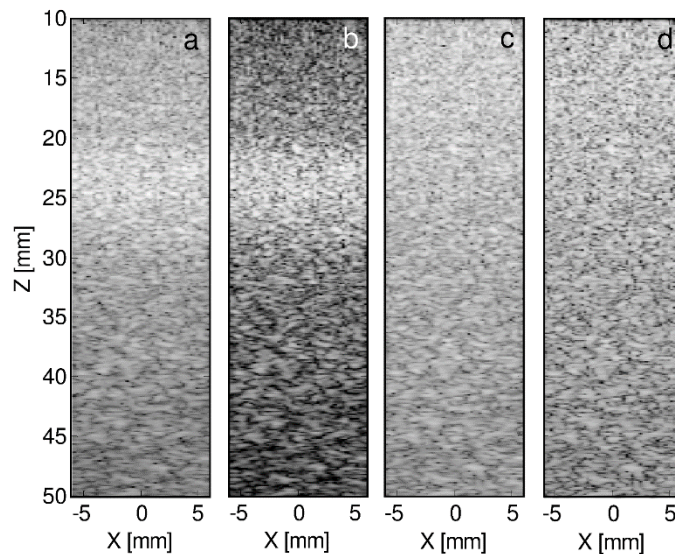
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259 3. RESULTS

260 3.1. Simulated homogeneous phantom images

261 Fig. 2 represents the images of the phantom obtained with DAS and F-DMAS by implementing receive-
262 only DF or STA with synthetic transmit focusing.

263 As expected, in the first case (Fig. 2a-b) the image intensity lowers away from the transmit focal depth (i.e.
264 $z=24$ mm). This intensity loss is visibly more pronounced in the F-DMAS image (Fig. 2b). On the other hand,
265 with synthetic transmit focusing, not only the peak intensities of both the DAS and F-DMAS images are more
266 uniform at all depths (Fig. 2c-d), but they are also very similar all along the z axis. A higher contrast resolution
267 can be always observed in the case of F-DMAS (Fig. 2b, d) as the speckle pattern looks better defined.



268 Fig. 2. Simulated images of the numerical homogeneous phantom, acquired by implementing receive-only DF (a, b) or
269 STA with synthetic transmit focusing (c, d), and DAS (a, c) or F-DMAS (b, d) beamforming. Images are displayed over
270 a 60 dB dynamic range.
271

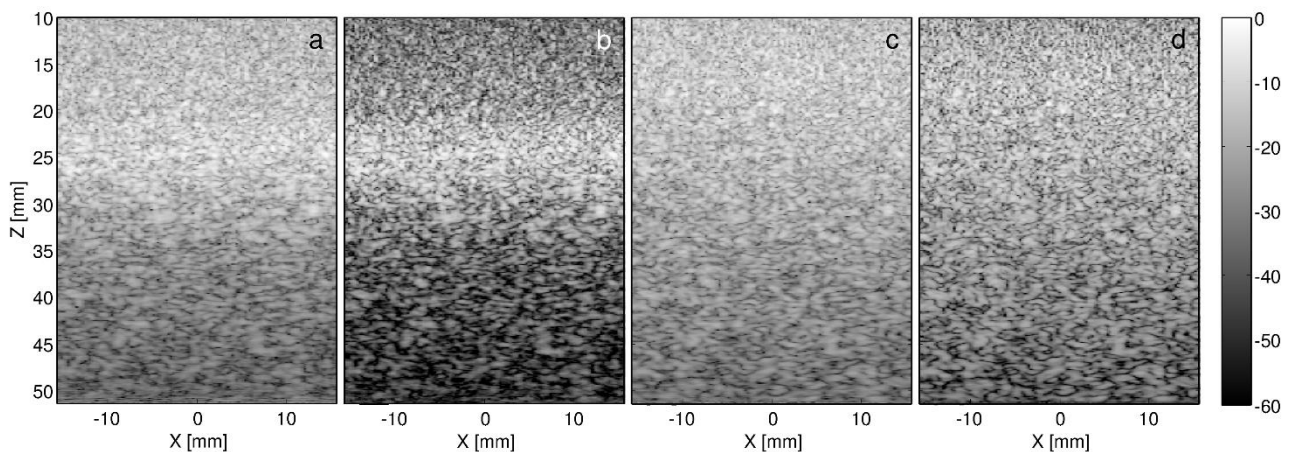
272 3.2. Experimental homogeneous phantom images

273 The same test was carried out also on experimental data, by acquiring images of a homogeneous area of the
274 CIRS phantom. The phantom images in the four considered conditions (i.e. receive-only DF or STA with
275 synthetic transmit focusing, and with DAS or F-DMAS) are represented in Fig. 3.

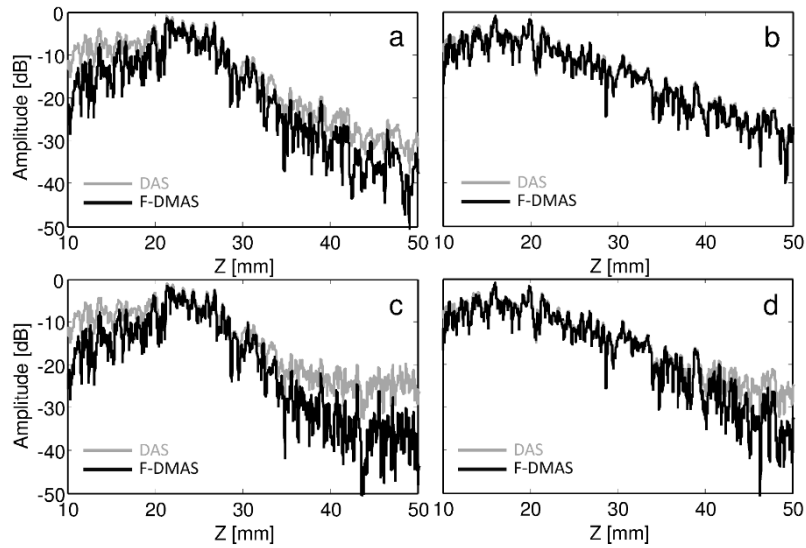
276 In both the focusing modalities, F-DMAS performs better than DAS in terms of image contrast resolution
277 (Fig. 3b, d). Moreover, in the synthetic transmit focusing case (Fig. 3c, d) the speckle peak intensities of the
278 two images become similar at all depths.

279 When F-DMAS is used in conjunction with STA and synthetic transmit focusing in the noisier case, the
280 amplitude loss shown by the image with receive-only DF away from the transmit focal depth is compensated
281 for, but only partially. Actually, at higher depths the results are quite different from the original images, and a
282 lower intensity is still shown by synthetic-transmit-focusing images with F-DMAS.

283 The image maximum amplitudes at each depth may be thus analyzed to effectively highlight the different
284 behavior of F-DMAS in the two focusing modalities, and also in the original and noisier conditions. The
285 speckle peak values (in the $x = [-1; 1]$ mm range) were detected in each DAS/F-DMAS image of the
286 homogeneous phantom, and plotted in Fig. 4. Overall, Fig. 4 confirms that with receive-only DF the difference
287 between DAS and F-DMAS maximum image amplitudes is low only near to the focal depth, while before and
288 after the transmit focus it reaches values higher than 10-15 dB (Fig. 4a, c). On the other hand, the plots in Fig.
289 4b show that this difference is very low at all depths for original images with synthetic transmit focusing in
290 Fig. 3c-d. This is not true in the case of images with synthetic transmit focusing and additive noise. In Fig. 4d



291
292 Fig. 3. Homogeneous phantom images, experimentally acquired by implementing receive-only DF (a, b) or STA with
293 synthetic transmit focusing (c, d), with DAS (a, c) or F-DMAS (b, d) beamforming. Images are displayed over a 60 dB
294 dynamic range.



295
 296 Fig. 4. Maximum amplitudes of the DAS and F-DMAS images of the CIRS uniform phantom in the $x=[-1; 1]$ mm
 297 range, with receive-only DF (a, c) or STA and synthetic transmit focusing (b, d), and with DAS (gray lines) or F-DMAS
 298 (black line). Panels (c, d) refer to images generated from RF signals with synthetic additive white noise.

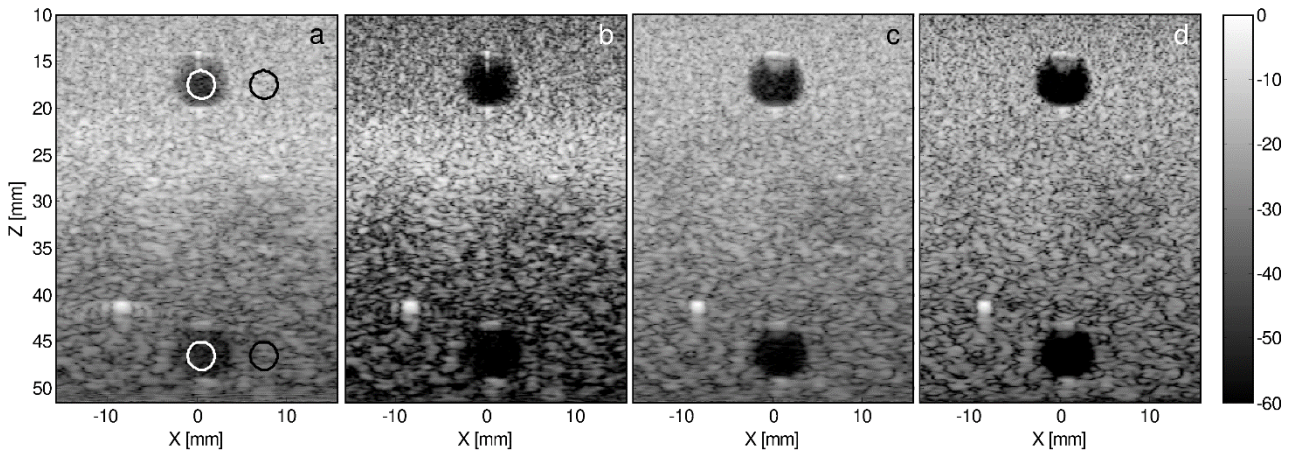
299 in fact the difference between DAS and F-DMAS also increases at depths close to 40-50 mm, reaching again
 300 values higher than 10 dB, due to the lower amplitude of the F-DMAS image, caused by noise. Also close to
 301 the probe (i.e. $z < 15$ mm), the difference between the DAS and F-DMAS images slightly increases in Fig. 4b,d,
 302 due to the fixed receive apodization scheme, which implies the use of a low-F# focused aperture, causing an
 303 increased decorrelation.

304 3.3. Experimental cyst phantom images

305 Acquisitions of phantom images with cysts at increasing depths were also performed in order to analyze
 306 the CR, CNR and SNR. The experimental phantom images are shown in Fig. 5; the circular regions (3 mm
 307 diameter) marked on Fig. 5a enclose the image areas used to compute the CR, CNR and SNR. In this
 308 experiment, the RF signals relative to 12 consecutive frames were averaged, before doing image
 309 reconstruction, to improve the SNR at higher depths. This number was empirically chosen in order to improve
 310 the quality of images at higher depths as much as needed, by suppressing electronic noise. Albeit such
 311 procedure would be unfeasible in a real-time application, due to the too high signal acquisition time and
 312 memory resources required, frame averaging allows us to analyze the DOF improvement with STA in more

313 ideal conditions, i.e. in the presence of acoustical noise only (e.g. clutter, aberrations, etc.) and with an as low
 314 as possible electronic noise.

315



316

317 Fig. 5. Cyst phantom images, experimentally acquired by implementing receive-only DF (a, b) or STA with synthetic
 318 transmit focusing (c, d), with DAS (a, c) or F-DMAS (b, d) beamforming and frame averaging. Images are displayed
 319 over a 60 dB dynamic range.

320

321 Again, the STA images with F-DMAS and synthetic transmit focusing (Fig. 5d) show a compensation of
 322 the speckle amplitude at lower and higher depths, with respect to the receive-only DF case (Fig. 5b). The
 323 contrast resolution of the F-DMAS images is always higher than with DAS; besides, with synthetic transmit
 324 focusing this higher quality is achieved more uniformly at all depths.

325 As reported in Table 2, the CR with F-DMAS is in any case higher (in absolute value) than with DAS.
 326 When STA with synthetic transmit focusing is implemented, the CR increases with respect to the receive-only
 327 DF case, especially for the F-DMAS image. Furthermore, the difference between the CR values measured at
 328 $z=17.5$ mm and $z=46.5$ mm in the F-DMAS image decreases from 17.6 dB with receive-only DF to 7.4 dB
 329 with synthetic transmit focusing, becoming similar to the value obtained with DAS (i.e. 7.7 dB).

330

331 **TABLE 2. CR, CNR and SNR values for the experimental cyst phantom images shown in Fig. 5 and 7**

Beamformer	z [mm]	Receive-only DF (frame averaging) (Fig. 5)			STA with synthetic transmit focusing (frame averaging) (Fig. 5)			SAF with synthetic transmit focusing (frame averaging) (Fig. 7)			SAF with synthetic transmit focusing (single frame) (Fig. 7)		
		CR [dB]	CNR	SNR	CR [dB]	CNR	SNR	CR [dB]	CNR	SNR	CR [dB]	CNR	SNR
DAS	17.5	-28.9	1.84	1.91	-34	1.81	1.85	-7.9	1.03	1.85	-8	1.03	1.83
F-DMAS	17.5	-41.2	1.06	1.07	-47.1	1.39	1.39	-13.6	0.75	0.97	-13.6	0.75	0.96
DAS	46.5	-14.2	1.72	2.17	-26.3	2.04	2.15	-8.6	1.14	1.98	-4.8	0.71	1.87
F-DMAS	46.5	-23.6	1.14	1.22	-39.7	1.84	1.86	-14.6	0.85	1.06	-9.4	0.63	0.98

332

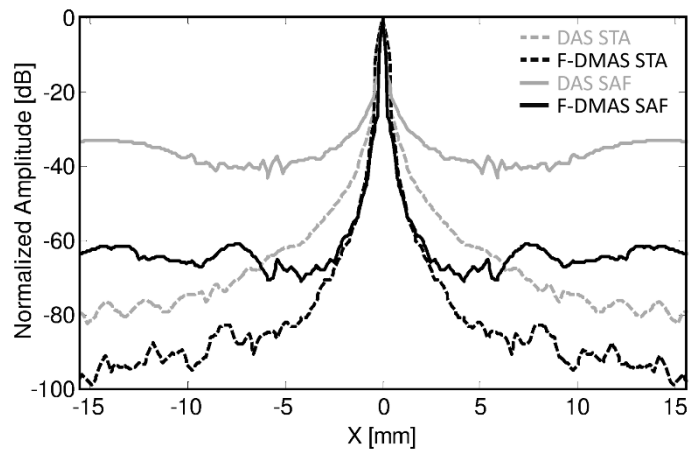
333 The CNR and SNR values for F-DMAS are instead always lower than for DAS. Since F-DMAS is designed
334 to suppress incoherent contributions in the backscattered signals received by the aperture, the beam has lower
335 side-lobes and a narrower main-lobe, which makes the Point Spread Function (PSF) sharper and the contrast
336 resolution improve. This is why the speckle pattern looks "finer" and many more dark regions appear in the
337 image instead of grey areas (compared to DAS), which turns out into a decrease of the CNR [3,8,26].

338 3.4. Evaluation of monostatic SAF performance with F-DMAS

339 In order to analyze also the performance of F-DMAS combined with the simpler monostatic SAF technique,
340 the simulated beampatterns obtained with STA and SAF (applying synthetic transmit focusing in both cases)
341 were compared. Fig. 6 shows that, as expected, the SAF beampattern has a narrower main lobe but higher side
342 lobes compared to STA. However, thanks to F-DMAS, the contrast-resolution performance of SAF can be
343 significantly improved, achieving a beampattern with a side-lobe level under -60 dB and an even narrower
344 main lobe.

345 For what concerns phantom scans, Fig. 7 shows both the single-frame images and those reconstructed by
346 averaging the RF signals of 12 frames (for comparison with Fig. 5). In fact, in the case of classical monostatic
347 SAF, which is much more critical from a SNR point of view compared to STA, it could be interesting to take
348 into account the effect of electronic noise. Besides, as we are investigating SAF for its simpler hardware
349 implementation, the single-frame images should be considered, as these are the ones that would be obtained in
350 a real application.

351

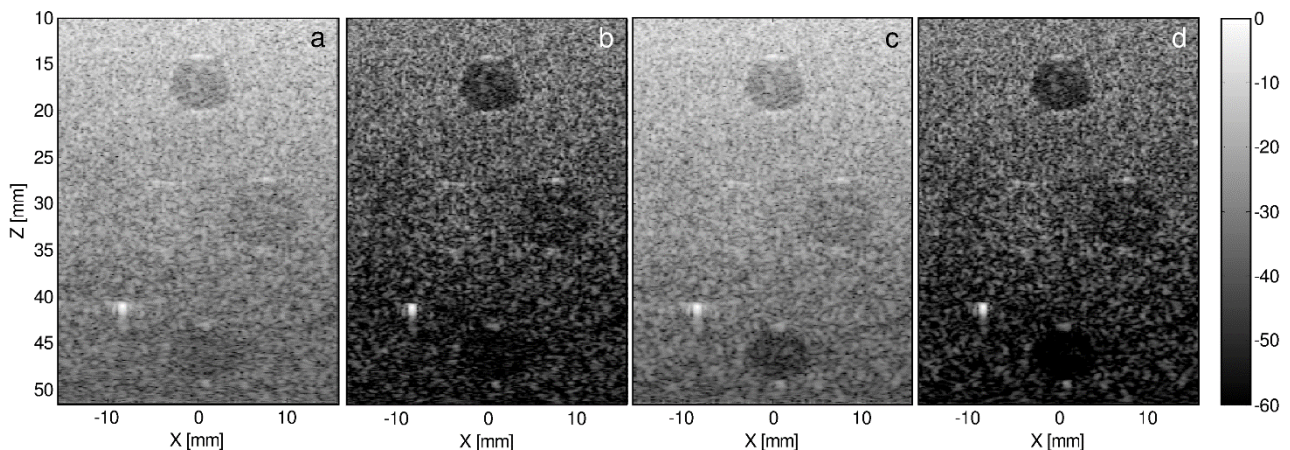


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Fig. 6. Simulated beampatterns at 24 mm depth, obtained by implementing STA (dashed lines) or SAF (solid lines) with DAS (gray lines) or F-DMAS (black lines) beamforming.



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Fig. 7. Cyst phantom images, experimentally acquired by implementing a SAF scan with synthetic transmit focusing, DAS (a, c) or F-DMAS (b, d) beamforming, and without (a, b) or with (c, d) frame averaging. Images are displayed over a 60 dB dynamic range.

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When SAF is applied together with F-DMAS, the quality of the cyst phantom image in Fig. 7 improves in terms of CR (Table 2). The cysts look darker with F-DMAS and the boundaries of the more superficial cyst are better defined. The speckle pattern is finer compared to the STA case; this effect is to be expected, as the SAF beampattern has a narrower main lobe, especially when F-DMAS is employed.

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Overall, the F-DMAS images look much darker, since the side-lobe contribution, whose presence is significant in the DAS images, is lowered. For a similar reason, the black cyst at higher depths is hardly

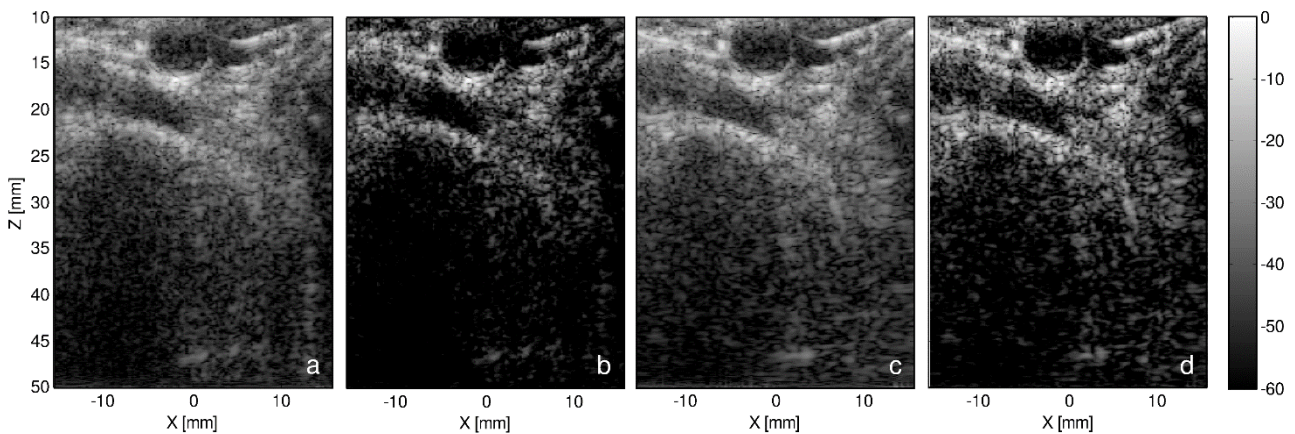
366 detectable in the F-DMAS images; in this region, in fact, the received signal SNR is low, and therefore many
 367 dark areas appear in the background speckle where the clutter is suppressed by F-DMAS.

368 Frame averaging improves image quality, most of all in the deeper regions (Table 2). Anyway, even in the
 369 single-frame case (Fig. 7a-b), the pixels inside the deeper cyst are darker with F-DMAS and a higher CR is
 370 achieved compared to DAS.

371 It is also worth pointing out that with SAF, thanks to the improved detail resolution (i.e. narrower main
 372 lobe), the borders of cysts are better highlighted, which improves the visibility of a further cyst (at 30-35 mm
 373 depth) in Fig. 7, which instead was hardly noticeable in Fig. 5.

374 3.5. *In vivo images of the carotid artery*

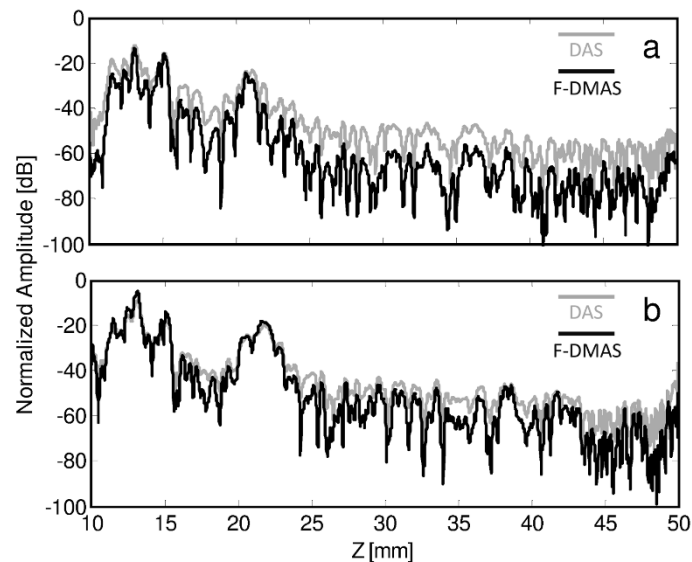
375 In order to further validate the obtained results, Fig. 8 shows the *in vivo* single-frame images of the human
 376 carotid artery, obtained by implementing receive only DF or STA with synthetic transmit focusing, and with
 377 DAS or F-DMAS.



378
 379 Fig. 8. *In vivo* images (single-frame) of the carotid artery, obtained by implementing receive-only DF (a, b) or STA
 380 with synthetic transmit focusing (c, d), and by applying DAS (a, c) or F-DMAS (b, d) beamforming. Images are
 381 displayed over a 60 dB dynamic range.

382 In Fig. 8b, it can be seen that the F-DMAS image with receive-only DF shows a reduced intensity away
 383 from the transmit focus (i.e. $z=15$ mm), especially in the higher depth regions. This is also confirmed by the
 384 plots represented in Fig. 9a (actually in Fig. 9 we preferred to plot the image axial sections at $x=-10$ mm and
 385 not the difference between the peak amplitudes, as in Fig. 4, since in this case the tissues are not homogeneous).

386 Such an effect is however compensated for by synthetic transmit focusing (Fig. 8d and 9b), making the
387 anatomical structures more visible at all depths. Compared to DAS, F-DMAS images show in every case a
388 better contrast resolution and clutter suppression.



389 Fig. 9. Plot of the axial sections (at $x=-10$ mm) of the carotid artery images in Fig. 8: a) receive-only DF, or b) STA
390 with synthetic transmit focusing. Gray and black lines refer respectively to DAS and F-DMAS images.
391

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393 4. DISCUSSION AND CONCLUSION

394 The initial hypothesis made in this work was that the reduced pixel intensity shown by B-mode F-DMAS
395 images away from the transmit focus, compared to DAS (Fig. 2a-b), could be related to a decorrelation effect,
396 caused by the broadening of the transmit beam when DF is implemented only in reception. To confirm this
397 assumption, a SAF technique is required, which makes it possible to beamform the received signals as if they
398 are dynamically focused both in transmission and reception; in a classical B-mode scan, in fact, this would not
399 be practically feasible without drastically limiting the frame rate. We thus compare the results obtained by
400 applying F-DMAS to data acquired in different scenarios by implementing STA, both with receive-only DF
401 and synthetic transmit focusing (which allows us to compare these two different focusing modes on the same
402 set of RF signals, acquired by performing one single scan of exactly the same region of interest), and also
403 monostatic SAF with synthetic transmit focusing.

404 When employing STA to simulate the scan of a numerical homogeneous phantom in a noise-free scenario,
405 results confirm that the amplitude loss of F-DMAS images can be compensated by the synthetic transmit
406 focusing of received signals at all depths (Fig. 2d), which makes it possible to improve the image DOF.

407 We should however point out that in this work DAS and F-DMAS images are compared in terms of speckle
408 maximum values, and not of average image intensities, which would be lower for F-DMAS. This is due to the
409 fact that F-DMAS improves the image contrast resolution, which turns out into a better defined speckle pattern,
410 with many more dark areas. Therefore, to compare the intensities of DAS and F-DMAS images, speckle
411 "peaks" should be used, and not "valleys", which usually reach a lower intensity value with F-DMAS.

412 We also show that, even in the case of a real ultrasound scan, the results confirm our initial assumption
413 (Fig. 3), but not always in a straightforward way. Actually, in fact, when synthetic transmit focusing is applied,
414 the amplitude of experimental F-DMAS images can still decrease at higher depths compared to DAS (Fig. 4d).

415 This is due to the fact that decorrelations among the backscattered signals – which turn into an intensity
416 drop in F-DMAS images – can be due not only to a misalignment of the received signals during the
417 beamformation process, but also generally to noise. The addition of synthetic white noise to the acquired
418 images allowed us to further enhance this effect. By comparing the original images to the noisier ones, we
419 have in fact shown that possible residual amplitude losses in F-DMAS images with STA and synthetic transmit
420 focusing in the deeper regions should be related to the improved noise-rejection performance of the F-DMAS
421 beamformer, and not to any residual signal de-focusing. Indeed, it is the higher sensitivity to noise of the spatial
422 cross-correlation function that allows F-DMAS to suppress unwanted clutter better than DAS does. Instead,
423 when the SNR improves and electronic noise lowers, the amplitude loss at higher depths is again compensated
424 for (Fig. 4b), demonstrating also the robustness of F-DMAS to other decorrelation sources.

425 Measurements on the experimental cyst phantom images show that with F-DMAS the CR is always higher
426 than with DAS, but only when synthetic transmit focusing is implemented the contrast values become more
427 similar at all depths (Fig. 5 and Table 2). This demonstrates that the combined use of STA with synthetic
428 transmit focusing and F-DMAS makes it possible to achieve images with a higher contrast throughout the scan
429 depth.

430 The SNR and CNR values of F-DMAS are anyway lower than those of DAS, due to the more defined
431 speckle texture, which makes speckle variance increase, as previously pointed out. Probably this effect - which
432 was observed also in other works e.g. on Capon/Minimum Variance beamforming [3,26] or Side Lobe Masking
433 [5] - might be compensated for by applying time averaging [27] or spatial compounding [3] techniques, so that
434 a higher CNR/SNR could be obtained, improving anechoic targets detectability. We foresee to further explore
435 this topic in future works.

436 *In vivo* images of the carotid artery also confirm the previous results, showing that with STA and synthetic
437 transmit focusing a more uniform higher quality can be achieved with F-DMAS at all depths.

438 All in all, however, in a real hardware setup, monostatic SAF could be more easily implemented than STA,
439 as in this case only a single-channel architecture is required, which makes it attractive for its potential low cost
440 and power consumption. Unfortunately, the problem with SAF is that not only the SNR is reduced but also the
441 contrast resolution is low. With this technique in fact, a rectangle-shaped, under-sampled (the pitch is doubled)
442 equivalent aperture is achieved, as shown in [19], which turns into a beampattern with a narrow main lobe, but
443 high side and grating lobes.

444 Since F-DMAS allows to significantly improve the contrast resolution of B-mode images [8], it is worth
445 investigating how F-DMAS beamforming behaves when used in conjunction to SAF. In Fig. 6 we demonstrate
446 that F-DMAS improves the performance of SAF by lowering the side and grating lobes, still preserving a
447 narrow main lobe; thus, images show a higher definition and contrast (Fig. 7). Actually, however, the strong
448 presence of noise at higher depths compromises the image quality, and F-DMAS images look darker due to
449 the decreased signal coherence. Nevertheless, if we look at single-frame images, where both electronic and
450 acoustical noise affect the image, F-DMAS has a positive impact on system performance and it improves the
451 CR even of the deeper cyst. *In vivo* images with SAF were instead not shown, as in this case the image quality
452 was poor due to the very low SNR. Anyway we shall consider that, in an ad-hoc developed single-channel
453 system, with a transmit/receive-path electronics optimized for such application, probably better performance
454 could be achieved.

455 On the other hand, F-DMAS would imply higher computational times compared to standard DAS, as it
456 involves more complex operations like the square root and multiplications [8, 28]. Nevertheless, such

457 operations can be efficiently simplified using approximate solutions and architecture-optimized instructions,
458 as pointed out in [12]. A real-time implementation of F-DMAS is foreseen in future work, and thus also an
459 actual accurate estimation of the required computational effort.

460 Overall, we have shown that F-DMAS beamforming can potentially make SAF an appealing technique,
461 since adequate contrast and resolution performances could be achieved with a very simple hardware
462 architecture. This represents a first step towards a more in-depth study on the use of F-DMAS in conjunction
463 to alternative SAF schemes, in which few elements may be used by following "smart" activation patterns,
464 which could make it possible to keep the hardware architecture complexity low, optimizing the beam shape at
465 the same time.

466

467

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