Measuring contagion risk in international banking

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Abstract

We propose a distress measure for national banking systems that incorporates not only banks' CDS spreads, but also how they interact with the rest of the global financial system via multiple linkage types. The measure is based on a tensor decomposition method that extracts an adjacency matrix from a multi-layer network, measured using banks' foreign exposures obtained from the BIS international banking statistics. Based on this adjacency matrix, we develop a new network centrality measure that can be interpreted in terms of a banking system's credit risk or funding risk.

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1 Introduction

As a consequence of the very high degree of integration in the global financial system, financial stress that originates in one country can quickly spill across borders into other countries. The Great Financial Crisis of 2007-09 and the subsequent European Sovereign Debt Crisis both illustrated that in a very stark manner.

The rapid growth of the global financial system over the past several decades has increased the importance of properly measuring the risk of distress. This is true not only from a financial stability point of view, but also from a macroeconomic perspective, as financial crises tend to have significant and persistent negative effects on economic activity (see Acemoglu et al. (2012)). At the same time, the increased interconnectedness and complexity of the global banking system have made that task extremely challenging.

The existing literature has employed three main approaches to measuring systemic risk in the global financial system (Cerutti et al. (2012)). The first method uses information on (direct and indirect) aggregate country-level bilateral exposures (Peek and Rosengren (2000), Kaminsky and Reinhart (2003), McGuire and Tarashev (2008), Cetorelli and Goldberg (2012)). The second approach takes advantage of market data on Credit Default Swap (CDS) spreads, bond spreads and equity prices in order to quantify systemic risk (Huang et al. (2009), Acharya et al. (2010), Delatte et al. (2012)).[1] The third approach utilizes simulation techniques to assess the probability of various potential contagion paths (both within and across sectors), conditional on an initial set of exposures (Árvai et al. (2009)).

In principle, the market price-based approach should be complementary to the bilateral exposure-based approach (Cerutti et al. (2012)). Nevertheless, there are not many existing studies that have used the two approaches in a complementary fashion, mainly due to the practical difficulties associated with matching market-based data with data on bilateral exposures. Faced with this data constraint, researchers have usually resorted to the "maximum entropy" assumption, under which exposures are assigned as uniformly as possible across counterparties. Nevertheless, this assumption has multiple important shortcomings, especially in the global context (Upper (2011) and Drehmann and Tarashev (2011)).

In this paper, we propose a methodology that merges the market price-based approach and the exposure-based approach. Namely, the methodology that we utilize enhances the information contained in market prices (i.e. CDS spreads) with the information contained in bilateral exposures (i.e. banks' foreign claims, broken down by counterparty country and sector). We assume that the global matrix of banks' foreign claims can be used to map the links in the global financial network (see also Tintchev (2016), De Souza et al.

[1] A credit default swap is an agreement in which the seller commits to repay an obligation (e.g. a bond) underlying the contract at par in the event of a default; to produce this guarantee, a regular premium is paid by the buyer (the creditor of the reference loan), during a specified period. The premium is expressed in the form a "CDS spread", a measure which is a function of the expected loss for a unitary loan amount. For a given maturity, the expected loss is a function of the debtor's probability of default and loss given default. As a consequence, if an entity (e.g. a sovereign or a bank) is considered more likely to default by the market, a higher protection fee (CDS spread) is charged.

(2016)).

In theory, banks' CDS spreads should incorporate all available relevant information, including information about banks' (direct and indirect) exposures. In practice, however, this is not necessarily the case. It is true that the majority of banks' immediate (direct) exposures tend to be known to market participants (and therefore incorporated in banks' CDS spreads). For example, during the euro area crisis, it was well-known that French banks had large exposures to borrowers in the euro area periphery countries (Greece, Ireland, Italy, Portugal and Spain). Nevertheless, the available information about banks' indirect (higher-order) exposures was much less complete. For example, although US banks did not have large direct (first-order) exposures to the euro area periphery countries, they were indirectly exposed to them through their exposures to French banks. In turn, a number of other banking systems which were directly exposed neither to the euro area, nor to French banks, were indirectly exposed to them through their exposures to US banks. It is easy to see how market participants, who may have been able to quantify the direct exposures and even the first-order of indirect exposures reasonably well, may have had very incomplete information about the higher-order (indirect) exposures in the global network.

This motivates us to construct a measure that takes into account the higher-order exposures of banks. To achieve this goal, a good predictor of a banking crisis should be based not only on the past CDS spreads of a given banking system, which may vary in accordance with idiosyncratic factors, but also on how its fluctuations are affected by contagion from other systems, in a multidimensional way. When constructing our measure, we aim to achieve an improved prediction of CDS spreads and, ultimately, of crisis events.

As foreign exposures can be of different types, we represent them as a multi-layer financial network, which can describe multidimensional interlinkages between economic agents (see Poledna et al. (2015), Montagna and Kok (2016), Aldasoro and Alves (2016)). The modelling of multi-layer financial networks has been recently employed in the domain of interbank lending. The papers in the existing literature are mostly descriptive, rather than predictive, and aim to create an aggregate network representation by summing over single networks. This approach may fail to adequately capture important non-linearities generated by the multi-layer nature of the data (Gauvin and Panisson (2014)).

In this paper, we extend the above approach to international banking lending to multiple sectors (the banking sector, the non-banks private sector and the official sector), using the rich dimensionality of the BIS international banking statistics. More concretely, we consider a set of 23 countries. Financial relationships between them can be represented by a multi-layer network, which contains the foreign claims of each country vis-a-vis the other countries in three types of layers, which correspond to foreign claims on (i) the banking sector, (ii) the official sector and (ii) the non-bank private financial sector.

We employ a tensor decomposition method that extracts an adjacency matrix from this multi-layer network. In addition, we also develop a new network centrality measure that can be interpreted probabilistically in terms of the credit risk or the funding risk (depending on the side of the balance sheet that is being analyzed) of a given banking system, taking multiple type of foreign exposure into account. The multi-layer methodology that we propose summarizes the relevant information in a non-linear, rather than linear, way. As we demonstrate in the results section below, this non-linear approach has a superior predictive performance than alternative linear measures constructed using unadjusted CDS spreads.

The rest of this paper is organized as follows. Our methodology is described in the next section. Section 3 contains a description of the data. Section 4 describes the key empirical findings. Section 5 concludes, while offering several policy implications.

2 Methodology

The multi-layer network can be mapped into a tensor $\mathcal{X} \in \mathbb{R}^{I \times I \times K}$ where I represents countries and K the type of exposures. Each element of the tensor, x_{ij}^k represents the amount of funds claimed by country i from country j, in type k, normalized by the total amount of foreign claims. Doing so, x_{ij}^k represent the percentage share of a particular claim. The tensor is composed by three slices $\mathbf{X} \in \mathbb{R}^{23 \times 23}$, each of which represents the financial transactions between countries in each type of exposure.

2.1 Probabilistic Matrix Decomposition

Our aim is to extract the relevant features from a multi-layer network of financial relationships. To exemplify, we first show how this can be done in the simpler mono layer network setting.

We aim to summarise a network matrix, that contains a specific type k of financial claims between countries, with two scores, that represent, respectively, hub score and authority score as in Kleinberg (1999), associated to each banking system (a country, for brevity).

The economic intuition is that a country i that lends to many other countries with high authority score receives a high hub score, u_i. A country j that borrows from many other countries with a high hub score receives a high authority score, v_j.

We propose a model in which hub and authority scores are obtained from the transition probabilities of a Markov chain between countries, based on the claim matrix. This has the advantage of a better interpretation of the results, as probabilities are normalised by definition.

The network claim matrix $\mathbf{X}^k$ of a particular type k can be used to calculate conditional frequencies, as follows:

$$h^k_{i|j} = \frac{x^k_{ij}}{\sum_{i=1}^{I} x^k_{ij}} \quad i = 1, ..., I$$

$$a^k_{j|i} = \frac{x^k_{ij}}{\sum_{j=1}^{I} x^k_{ij}} \quad j = 1, ..., I,$$

Without loss of generality, when x^k_{ij} is equal to 0 for all $1 \leq i, j \leq I$ the values of $h^k_{i|j}$ ($a^k_{j|i}$) can be set to $1/I$.

The previous conditional frequencies can be used to estimate the transition probabilities of a Markov chain:

$$\Pr\left[X^k_\tau = i | Y^k_\tau = j\right]$$

$$\Pr\left[Y^k_\tau = j | X^k_\tau = i\right],$$

where X^k_τ and Y^k_τ are random variables referring to the visit at any particular country as a hub or as an authority at the Markov chain step τ. In other words, $h^k_{i|j}$ while estimates the probability that country i borrows from to country j, $a^k_{j|i}$ estimates the probability that country j lends to i.

The conditional frequencies $h^k_{i|j}$ and $a^k_{j|i}$ can also be used to estimate the stationary distribution of countries being hubs or authorities (borrowers or lenders). For a given τ step of the chain let:

$$\Pr\left[X^k_\tau = i\right] = \sum_{j=1}^{J} h^k_{i|j} \Pr\left[Y^k_\tau = j\right]$$

$$\Pr\left[Y^k_\tau = j\right] = \sum_{i=1}^{I} a^k_{j|i} \Pr\left[X^k_\tau = i\right]$$

The stationary probabilities of each country being a hub or an authority are then:

$$u^k_i = \lim_{\tau \to \infty} \Pr\left[X^k_\tau = i\right]$$

$$v^k_j = \lim_{\tau \to \infty} \Pr\left[Y_\tau = j\right]$$

According to the previous results, to practically compute u^k_i and v^k_j we need to solve the following system of equations:

$$u^k_i = \sum_{j=1}^{I} h^k_{i|j} v^k_j \quad i = 1, ..., I \tag{1}$$

$$v_j^k = \sum_{i=1}^{I} a_{j|i}^k u_i^k \quad j = 1, ..., I \tag{2}$$

The system can be solved iterating between equations 1 and 2 until $\left\|\mathbf{u}^{(\tau)} - \mathbf{u}^{(\tau-1)}\right\| + \left\|\mathbf{v}^{(\tau)} - \mathbf{v}^{(\tau-1)}\right\| < \epsilon$.

From an economic viewpoint, the hub score u_i^k of country i is the weighted sum of the authority scores v_j^k of the countries that i lends to. The weight associated with each borrower is the element of the transition probability matrix $h_{i|j}^k$ between i and the borrower j. The authority score v_j^k of a country j is the weighted sum of the hub scores u_i^k of the countries that borrow from j. The weight associated with each lender is the element of the transition probability matrix $a_{j|i}^k$.

We are now ready to present how the claim matrix can be decomposed. As authority and hub scores are stationary probability distributions, they can be employed to calculate the (joint) probability that an extra unit claim is placed from country i to country j. Let m_{ij}^k to represent the probability that, in sector k, the banking system of country i has an extra unit of claim on country j and let $\mathbf{M}^k$ be the stochastic matrix of all such probabilities, whose elements sum up to 1. Let $\mathbf{u}^k$ and $\mathbf{v}^k$ be, respectively, the hub and authority scores. From Markov chain theory it follows that:

$$\mathbf{M}^k = \mathbf{u}^k (\mathbf{v}^k)^T.$$

and, therefore, m_{ij}^k is the product between the hub score of i and the authority score of j.

Note that, when temporal data is available, such probability can be calculated at any given time point.

2.2 Probabilistic Tensor Decomposition

We now consider the more general multi-layer network, in which we aim to summarise a network tensor, that contains financial claims between countries, of different types, with three scores that represent, respectively, hub, authority scores associated to each country and a type score associated to each layer.

To achieve this aim, following Ng et al. (2011), we consider a Markov chain on the claim tensor, whose joint stationary distribution will be the product of hub, authority and type scores.

While authority and hub scores are the same as those previously defined, type scores are specific to the multi-layer framework. From an interpretational viewpoint, a financial relation type that connects countries with high hub score with countries with high authority score receives a high type score, w_k.

For a multi-layer network, the starting point of the construction of the decomposition is the computation of the (bivariate) conditional frequencies $\mathcal{H}$, $\mathcal{A}$ and $\mathcal{R}$ for hubs, authorities and types. They can be obtained by normalizing the entries of the tensor $\mathcal{X}$ as follows:

$$h_{i|jk} = \frac{x_{ijk}}{\sum_{i=1}^{I} x_{ijk}} \quad i = 1, ..., I$$

$$a_{j|ik} = \frac{x_{ijk}}{\sum_{j=1}^{I} x_{ijk}} \quad j = 1, ..., I$$

$$r_{k|ij} = \frac{x_{ijk}}{\sum_{k=1}^{J} x_{ijk}} \quad k = 1, ..., K$$

As done before, when x_{ijk} is equal to 0 for all $1 \leq i, j \leq I$ the values of $h_{i|jk}$ ($a_{j|ik}$) are set to $1/I$ ($1/K$ for $r_{k|ij}$).

The above quantities can be used to estimate the conditional probabilities:

$$\Pr\left[X_\tau = i | Y_\tau = j, Z_\tau = k\right]$$

$$\Pr\left[Y_\tau = j | X_\tau = i, Z_\tau = k\right]$$

$$\Pr\left[Z_\tau = k | Y_\tau = j, X_\tau = i\right]$$

where X_τ, Y_τ and Z_τ are random variables referring to the visit at any particular country as a hub or as an authority, and using any particular financial relation type, at the step τ of the Markov chain.

Note that $h_{i|jk}$ can now be interpreted as the probability of visiting the i-th country as a hub (or as an authority) given that the j-th (or i-th) country as an authority (or as a hub) is currently visited and that the k-th relation is used.

Similarly, $a_{j|ik}$ is the probability of visiting the j-th country as an authority given that the i-th country as a hub is currently visited and that the k-th relation is used.

In addition, $r_{k|ij}$ can be interpreted as the probability of using the k-th relation given that the j-th country as an authority is visited from the i-th country as a hub.

In analogy with the matrix decomposition setting, the conditional frequencies can also be used to derive the stationary marginal probabilities, first computing:

$$\Pr\left[X_\tau = i\right] = \sum_{j=1}^{J} \sum_{k=1}^{K} h_{i|jk} \Pr\left[Y_\tau = j, Z_\tau = k\right]$$

$$\Pr\left[Y_\tau = j\right] = \sum_{i=1}^{I} \sum_{j=1}^{K} a_{j|ik} \Pr\left[X_\tau = i, Z_\tau = k\right]$$

$$\Pr\left[Z_\tau = k\right] = \sum_{i=1}^{I} \sum_{j=1}^{I} r_{k|ij} \Pr\left[Y_\tau = j, X_\tau = i\right].$$

and, then, taking their limiting distributions:

$$u_i = \lim_{\tau \to \infty} \Pr\left[X_\tau = i\right]$$

$$v_j = \lim_{\tau \to \infty} \Pr\left[Y_\tau = j\right]$$

$$w_k = \lim_{\tau \to \infty} \Pr\left[Z_\tau = k\right],$$

which can be used as hubs, authority and type scores.

From a practical viewpoint, exploiting the result seen in the last subsection on the decomposition of stationary bivariate probabilities into the product of univariate ones, the scores can be derived solving the following system of equations:

$$u_i = \textstyle\sum_{j=1}^{I} \sum_{k=1}^{J} h_{i|jk} v_j w_k \quad i = 1, ..., I \tag{3}$$

$$v_j = \textstyle\sum_{i=1}^{I} \sum_{j=1}^{K} a_{j|ik} u_i w_k \quad j = 1, ..., I \tag{4}$$

$$w_k = \textstyle\sum_{i=1}^{I} \sum_{j=1}^{I} r_{k|ij} u_i v_j \quad k = 1, ..., K \tag{5}$$

From an economic viewpoint, the hub score u_i of country i is the weighted sum of the authority scores v_j of the borrowing countries that i points to. The weight associated with each borrower is the product of the element of the transition probability tensor $\mathcal{H}$ between i and the borrower j times the type score w_k of the layer in which the transaction is performed.

The authority score v_j of country j is the weighted sum of the hub scores u_i of the lending countries that point to j. The weight associated with each lender is the product of the element of the transition probability tensor $\mathcal{A}$ times the type score w_k of the layer in which the transaction is executed. Finally, the type score of layer k is the sum, over all pairs of countries (i, j) involved in a transaction in layer k, of the product between the hub score u_i of lender i with the authority score v_j of borrower j and with the element of the transition probability tensor $\mathcal{R}$ between i and j.

The above system can be solved in a recursive way, with an algorithm that takes as inputs the three tensors $\mathcal{H}$, $\mathcal{A}$ and $\mathcal{R}$, the two initial probability vectors $\mathbf{u}$, $\mathbf{v}$ and the tolerance parameter ϵ. The algorithm iterates through equations 3, 4 and 5 until $\left\|\mathbf{u}^{(\tau)} - \mathbf{u}^{(\tau-1)}\right\| + \left\|\mathbf{v}^{(\tau)} - \mathbf{v}^{(\tau-1)}\right\| + \left\|\mathbf{w}^{(\tau)} - \mathbf{w}^{(\tau-1)}\right\| < \epsilon$.

Ng et al. (2011) demonstrated the existence and the uniqueness of a limiting probability distribution for the above algorithm, which can be interpreted as the stationary distribution of the Markov chain on the multi-layer network tensor.

As before, we can now focus on tensor decomposition. Let m_{ij} be the probability that the lending banking system i has an extra unit of claim on country j, in any of the type sectors, and let $\mathbf{M}$ be the stochastic matrix of all such probabilities.

From Markov chain theory it follows that:

$$\mathbf{M} = \mathbf{u}(\mathbf{v})^T,$$

and, therefore, as in the previous subsection, the probability m_{ij} can be obtained as the product between the hub score of i and the authority score of j. However, differently from the single-layer case, $\mathbf{M}$ is not sector specific but, rather, calculated integrating information from all relationship types.

Figure 1 below helps to further understand the meaning of hub, authority and type scores, the key elements of the multi-layer network tensor decomposition.

FIGURE 1 ABOUT HERE

2.3 Contagion measures

Many summary measures of network models have been proposed in the literature (see e.g. Kramer et al. (2009)). Among them we refer to the Alpha-centrality (see e.g. Bonacich and Lloyd (2001), Ide et al. (2013)), which can be converted into a probability measure. The alpha-centrality measures the total number of paths from a node, exponentially attenuated by their length. It can be interpreted as a cumulative probability of an infection spreading through the network.

More formally, let $\mathbf{c}$ be a vector of positive quantities, let $\mathbf{A}$ be an adjacency matrix, and let β be a non negative parameter. The alpha-centrality vector $\mathbf{S}$ is the steady state solution of the recursive equation:

$$\mathbf{S}_{\mu}= \mathbf{C}+ \beta\, \mathbf{A}\, \mathbf{S}_{\mu-1},$$

with μ indicating the number of iterations. An equivalent representation can be given in cumulative terms, as follows:

$$\mathbf{S}_{\mu}= \left(\mathbf{I}+\beta\mathbf{A}+\beta^2\mathbf{A}^2 + ... + \beta^{\mu}\mathbf{A}^{\mu}\right) \mathbf{C},$$

where the (i, j) element of the powered matrix $\mathbf{A}^{\mu}$ represents the number of paths that, from node i, can reach node j in μ steps.

Note that, to achieve convergence, β^k must be equal to:

$$\beta = \frac{\alpha}{\lambda_1(\mathbf{A})}$$

where λ_1 the largest eigenvalue of $\mathbf{M}$ and α can be interpreted as an "average" contagion probability $(0 \leq \alpha \leq 1.)$.

To derive the Alpha-centrality measure in our context, we need some preliminary assumptions.

The adjacency matrix $\mathbf{A}$ can be set equal to $\mathbf{M}$, the tensor decomposition that contains the probabilities that an extra claim is placed from i to j. In this way, the adjacency matrix describes "exposure" to contagion.

Concerning the rate of contagion α, we can assume that:

$$\alpha^o = w_o, \alpha^b = w_b, \alpha^p = w_p$$

which implies that, the higher the importance of a type of financial relationship, the higher the probability that it can be used to channel funds between countries, the higher a contagion could come from that sector. Note that, consequently:

$$\beta^k = \frac{\alpha^k}{\lambda_1(\mathbf{M})}$$

Further assumptions concerns the channels of contagion.

As in Tintchev (2016) we assume that contagion can come both from the lending side (credit risk) and from the borrowing side (funding risk). In our context, c_{ik} is the CDS spread of country i in type k, and $\mathbf{c}^k$ is the vector of all spreads of type k. We then take $\mathbf{c} = \frac{1}{k} * \sum_{k=1}^{K} \mathbf{c}^k$.

In addition, the transmission of contagion to a banking sector has the possibility to reverberate into the system through higher order connections (as discussed in Battiston et al. (2012) and Hale et al. (2016)). Contagion transmission to the official and the non bank private sectors, instead, does not have this feature, and stops at the first order iteration.

Following the stated assumption, we obtain a multivariate spread, respectively on the borrowing (SB) and on the lending side (SL), as:

$$\mathbf{SB} = \mathbf{c} + \mathbf{c}^b \sum_{\mu=1}^{\Delta} \beta^{b,\mu} \mathbf{M}^{\mu} + \mathbf{c}^b \beta^o \mathbf{M} + \mathbf{c}^b \beta^p \mathbf{M}$$

$$\mathbf{SL}= \mathbf{c} + \mathbf{c}^b \sum_{\mu=1}^{\Delta} \beta^{b,\mu} \left(\mathbf{M}^T\right)^{\mu} + \mathbf{c}^o \beta^o \mathbf{M}^T + \mathbf{c}^p \beta^p \mathbf{M}^T,$$

where b, o, p indicate the bank, official and non bank sectors and where Δ indicates the network diameter (the longest of the shortest path between any two nodes).

From an interpretational viewpoint, both solutions, **SB** and **SL**, can be interpreted as network-modified, or multivariate, CDS spreads. They are based on a diffusive non conservative process that distributes through the network as an infection, or, in other words, as a random walk which replicates at each step on the network.

It is important to note that the notion of *contagion* differs from that of *interdependence*. The seminal work of Forbes and Rigobon (2002) highlights the difference between the two concepts by examining cross-country stock market co-movements. In particular, they empirically identify cases of interdependencies between economic quantities without contagion. For instance, during the 1997 East Asian crisis, the 1994 Mexican peso collapse, and the 1987 U.S. stock market crash, there was no contagion, but only interdependence. In our case, while co-movements in national banking systems' exposures capture the notion of interdependence, co-movements in CDS spreads capture the notion of contagion. Indeed, our measure of centrality assesses to what extent interdependence between banking sectors exposure drives contagion by letting CDS spreads interact in a financial network environment. Thus, besides being able to identify systemically important national banking systems, our approach can quantify the potential capital losses arising from contagion effects. In addition, it allows for an intuitive visualization of both interdependence and contagion effects.

Interpreting network spreads as modified CDS spreads, we can also compute the expected losses of each country. Let E be the total amount of foreign claims of the system. The expected credit loss for a country i, on the lending side, can be obtained as:

$$EL_i^L = E \sum_j \mathbf{M}_{ij}^T SL_i. \tag{6}$$

The expected funding loss for a country i, on the borrowing side, can be obtained as:

$$EL_i^B = E \sum_j \mathbf{M}_{ij} SB_i \tag{7}$$

The above expected losses can be calculated in any given time point, and provide a very relevant indicator measure of the overall risk of each banking system, both in terms of funding risk and credit risk. The technique can also be easily adapted to a stress test scenario, making alternative assumptions on the distribution of banking claims between countries.

We finally remark that, if we work separately on each sector using the probabilistic matrix decomposition approach, we can separately consider each sector's exposure matrix. We can then calculate multivariate spreads in a similar fashion, however with three different measures, on the borrowing and the lending side. In formulae:

$$
\begin{aligned}
\mathbf{SB}^o &= \mathbf{C^o} + \mathbf{C}^o\beta^o\mathbf{M^o} & \mathbf{SL}^o &= \mathbf{C^o} + \mathbf{C}^o\beta^o\mathbf{M}^{o,T} \\
\mathbf{SB}^b &= \mathbf{C^b} + \mathbf{C}^b{\textstyle\sum_{\mu=1}^{\Delta}}\beta^{b,\mu}\mathbf{M}^{b,\mu} & \mathbf{SL}^b &= \mathbf{C}^b + \mathbf{C}^b{\textstyle\sum_{\mu=1}^{\Delta}}\beta^{b,\mu}\left(\mathbf{M}^{b,T}\right)^{\mu} \\
\mathbf{SB}^p &= \mathbf{C^p} + \mathbf{C}^p\beta^p\mathbf{M^p} & \mathbf{SL}^p &= \mathbf{C}^p + \mathbf{C}^p\beta^p\mathbf{M}^{p,T}
\end{aligned}
$$

The resulting disaggregated expected loss can be found using equations (6-7) using each sector's total exposure.

2.4 Predictive analysis

Any proposed systemic risk measure should be validated in a predictive performance setting. This is the aim of this subsection.

Specifically, we investigate whether the inclusion of contagion helps in predicting the next period CDS spreads. We compare the predictions obtained using only the information on the past CDS spreads values with those obtained using both past CDS spreads and past multi-layer values, merged in the multivariate CDS spread derived in the previous subsection.

In more detail, when we rely only on past CDS values we use the average of the last n_1 values to forecast the CDS spread in next period as:

$$\widehat{\mathbf{CDS}} = -\frac{\sum_{x=t-n_1}^{t}\mathbf{CDS}_x}{n_1}.$$

When we also employ multi-layer network information, we embed the stochastic matrices $\mathbf{M}_t$, obtained in different time periods, in a three dimensional tensor $\mathcal{M} \in \mathbb{R}^{I\times I\times T}$ where I represents countries, T the time periods and accordingly, the generic element m_{ijt} represents the probability of having a connection from the borrowing country i to the lending country j at time t.

We then follow Spelta (2017) in predicting the next period slice of $\mathcal{M}$ which, together with the average of the last n_1 values of the CDS spreads, are used to forecast the CDS. More formally, we employ a tensor decomposition technique called CP decomposition (see e.g. Harshman (1970) and Carroll and Chang (1970)).

The decomposition aims at writing the tensor $\mathcal{M}$ as the outer product of two spatial vectors $\mathbf{a}$ and $\mathbf{b}$, that contain the importance of each country in terms of borrowing and lending probabilities, and one temporal vector $\mathbf{g}$, containing the temporal profile of each slice $\mathbf{M}_t$.

The vectors **a**, **b** and **g** can be found solving the following minimization problem:

$$\min_{\mathbf{a}>\mathbf{0},\mathbf{b}>\mathbf{0},\mathbf{g}>\mathbf{0}} \|\mathcal{M} - \mathbf{a} \circ \mathbf{b} \circ \mathbf{g}\|^2$$

This is equivalent to minimizing the difference between each of the modes $\mathbf{M}_{(i)}$ and their respective approximation in terms of factors:

$$\begin{aligned} &\min_{\mathbf{a}>\mathbf{0}} \left\|\mathbf{M}_{(1)} - \mathbf{a}\left(\mathbf{b} \odot \mathbf{g}\right)\right\|_F^2 \\ &\min_{\mathbf{b}>\mathbf{0}} \left\|\mathbf{M}_{(2)} - \mathbf{b}\left(\mathbf{a} \odot \mathbf{g}\right)\right\|_F^2 \\ &\min_{\mathbf{g}>\mathbf{0}} \left\|\mathbf{M}_{(3)} - \mathbf{g}\left(\mathbf{a} \odot \mathbf{b}\right)\right\|_F^2 \end{aligned} \tag{8}$$

Since financial flows are always non negative, a non-negative tensor factorization method can be employed to solve (8). Specifically, the Block Coordinate Descent Method for Regularized Multiconvex Optimization (Xu and Yin (2013)) and the Matlab Tensor Toolbox (Bader and Kolda (2004)) can be used to solve (8). We let the reader to refer to the Appendix for a more complete description of tensor calculus.

The next step consists in generating the adjacency matrix of the predicted network. A simple average, applied to the last n_1 observations of the temporal profile vector **g**, extracts a scalar $\hat{g}$ representing the forecast of the next period value of such vector. The adjacency matrix containing the predicted probability of a link between all country pairs can then be obtained as a linear combination of the two spatial vectors **a** and **b** and of the forecast $\hat{g}$ of the temporal profile vector. In matrix terms: $\hat{\mathbf{M}} = \hat{g} * \mathbf{a} \circ \mathbf{b}$ or, elementwise $\hat{m}_{ij} = \hat{g} a_i b_j$.

The predicted spreads are finally found as:

$$\widehat{\mathbf{SB}} = \hat{\mathbf{C}} + \hat{\mathbf{C}}^b \beta \hat{\mathbf{M}} + \hat{\mathbf{C}}^b \textstyle\sum_{\mu=1}^{\Delta} \beta^\mu \hat{\mathbf{M}}^\mu + \hat{\mathbf{C}}^p \beta \hat{\mathbf{M}},$$

for the borrowing side, and:

$$\widehat{\mathbf{SL}} = \hat{\mathbf{C}} + \hat{\mathbf{C}}^o \beta \hat{\mathbf{M}}^T + \hat{\mathbf{C}}^b \textstyle\sum_{\mu=1}^{\Delta} \beta^\mu \left(\hat{\mathbf{M}}^T\right)^\mu + \hat{\mathbf{C}}^p \beta \hat{\mathbf{M}}^T,$$

for the lending side. Note that, for simplicity, we set $\beta = \frac{1}{3}$ and $\hat{\mathbf{C}}^k = \frac{\sum_{x=t-n_1}^{t} \mathbf{C}_x^k}{n_1}$ where $k = \{o, b, p\}$.

3 Data

We obtain data on banks' foreign claims using the BIS Consolidated Banking Statistics (CBS). The BIS CBS measure banks' country risk exposures. They capture the worldwide consolidated claims of internationally active banks headquartered in BIS reporting countries. The consolidated statistics include the claims of banks' foreign affiliates, but exclude intragroup positions, in accordance with the consolidation approach followed by banking supervisors.

There are two main BIS CBS datasets. The CBS on an immediate counterparty basis (CBS/IC) provide information on banks' direct counterparties without accounting for any associated credit risk transfers. By contrast, the CBS on an ultimate risk basis (CBS/UR) account for credit risk transfers (BIS (2015a)).

In our empirical exercise, we use the CBS/UR in order to construct a quarterly panel of 23 bank nationalities (home countries) and 23 borrowing countries for the period Q1 2005 to Q4 2015[2]. These 23 nationalities include the major internationally active banks, and account for more than 95% of the aggregate stock of global foreign claims at end-Q4 2014 (see BIS (2015b)). The main variables we focus on are bilateral foreign claims, broken down by reporting national banking systems and borrowing countries. Foreign claims (FC) are defined as the sum of (i) cross-border claims (XBC) and (ii) local claims (LC), booked by banks' affiliates located in the borrowing country (i.e. FC = XBC + LC).

Furthermore, we define three distinct layers of foreign exposures by taking advantage of the fact that the CBS/UR contain breakdowns not only by lending banking systems and borrowing countries, but also by borrowing sectors. More concretely, foreign claims in the CBS/UR dataset can be divided into claims on the official sector, claims on other (non-related) banks and claims on the non-bank private sector. This counterparty sector breakdown allows us to examine the financial relationships between lending banks and borrowers from different countries and sectors in the context of the multi-layer network framework outlined in Section 2.2. More specifically, each layer in the network we examine is defined by the foreign exposures of a given national banking system to one of the three counterparty sectors (listed above) in a given borrowing country and corresponds to a weighted adjacency matrix. All networks have 23 nodes, corresponding to the full set of reporting national banking systems.

We construct three sets of CDS spreads series – one for each of the three counterparty sectors in the BIS CBS/UR foreign claims dataset. First, for the public sector of each borrowing country, we use the respective five-year sovereign on-the-run CDS spread, obtained from Markit. Second, for the banking sector of each

[2]The 23 national banking systems for which we obtain data from the BIS CBS are: 'AU - Australia' 'AT - Austria' 'BE - Belgium' 'CA - Canada' 'CL - Chile' 'TW - Chinese Taipei' 'FR - France' 'FI - Finland' 'DE - Germany' 'GR - Greece' 'IN - India' 'IE - Ireland' 'IT - Italy' 'JP - Japan' 'NL - Netherlands' 'PT - Portugal' 'SG - Singapore' 'ES - Spain' 'SE - Sweden' 'CH - Switzerland' 'TR - Turkey' 'GB - United Kingdom' 'US - United States'.

borrowing country, we construct an equally-weighted average of senior five-year CDS spreads (obtained from Markit) for a sample of domestic financial institutions (following a methodology similar to the one used in Avdjiev et al. (2012)). Finally, for the non bank private sector of each borrowing country, we use the option-adjusted spread (OAS) between the yield on a US dollar-denominated non bank financial bond index for the respective country and the corresponding yield on US-Treasuries of a similar maturity.[3]

FIGURE 2 ABOUT HERE

Some summary network statistics are reported in Figure 2. The Network density (fraction of existing links over all possible links) is reported in the left panel. It is quite high (relative to its counterparts in usual financial networks) and increasing over time. Across the different borrowing sectors, the sovereign sector is the most sparse one, whereas the bank and the non bank private sector layers have approximately the same number of links. The percentage of reciprocated links (displayed in the right panel) exhibits similar patterns both, over time and across sectors.

4 Empirical findings

4.1 Tensor decomposition

As discussed above, the proposed methodology distinguishes between Systemically Important Lender countries (hubs) and Systemically Important Borrower countries (authorities).

FIGURE 3 ABOUT HERE

Figure 3 presents the hub and the authority scores for each countries' banking sector obtained from the proposed tensor decomposition approach together with the type score associated to each network layer. The figure reveals that the United States and the United Kingdom are clearly the main Systemically Important Borrower (left panel). Meanwhile, the Systemically Important Lenders distribution (central panel) is much more evenly spread across bank nationalities, with Germany, Japan and France joining the United States and the United Kingdom as the main Systemically Important Lenders. Thus, the latter two countries have a dual role - they are systemically important both as hubs and as authorities.

The right panel of Figure 3 also reveals that the non bank private sector layer contributes more than the other two sectors to the systemic importance of countries. The relative importance of the other two sectors

[3]We obtain the spreads the non bank private sector from two sources. Spreads for Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom and the United States are obtained using the Barclays Global Aggregate Corporate Index. Spreads for Chile, Chinese Taipei, India, Singapore and Turkey are obtained using JPMorgan's Corporate Emerging Markets Bond Index (CEMBI).

has been reversed after the global financial crisis: whereas the bank sector was more systemically important than the official sector before the crisis, the opposite became true during the post-crisis period.

4.2 Contagion measures

We now derive the network modified CDS spreads separately for each country. Figure 4 plots, in the upper panels, the network based PD on the borrowing side.

FIGURE 4 ABOUT HERE

For the sake of clarity, we present the results only for two groups of countries: the largest five national banking systems (the United States, the United Kingdom, Japan, Germany, France) in the upper right panel and the euro area periphery countries (Greece, Portugal, Italy, Spain, Ireland) in the upper left panel.

Figure 4 suggests that, for most countries, there are non-negligible contagion effects, as approximated by the difference between the two measures (reported in the bottom panels). For example, Greece, Portugal, Italy and, to a lesser extent, Spain and Ireland have considerable differences. On the other hand, countries such as the United States, the United Kingdom, Japan and France have a low univariate PD (reported in the central right panel), but, when contagion is taken into account (bottom right panel), their overall PD reaches levels similar to those of countries such as Italy and Portugal.

The evolution of the contagion measure is consistent with the observed timing of crisis events. For example, the contagion measure for Greece spikes up in 2010 and remains at elevated levels until the second half of 2012 (after Draghi's "whatever it takes speech" in July 2012). It spikes again in late 2014/early 2015, on the eve of the January 2015 parliamentary elections and June 2015 referendum in Greece. The CDSs for Italy and Portugal exhibit similar, slightly delayed, dynamics, in line with the timing of the occurrence of their respective fiscal shocks.

The prominent role that Greece played in the euro area crisis is well-known and extensively documented. The BIS CBS allow us to quantify the foreign exposures of various national banking systems to residents of Greece and track their evolution over time. As Table 1 clearly illustrates, foreign bank's exposures to Greece, which were very high on the eve of the crisis ($310 billion as of end-Q2 2008), declined very rapidly and steadily in the next few years (reaching $22 billion as of end-Q4 2015). The table further illustrates that all major global banking systems (US, UK, Japan, France, and Germany), as well as all euro area periphery banking systems (Italy, Spain, Ireland and Portugal) drastically reduced their exposures to Greece.[4]

TABLE 1 ABOUT HERE

[4]We would like to thank an anonymous referee for the suggestion to include a table that tracks the evolution of foreign banks' claims on Greece.

The multivariate CDS of Ireland and Spain are also fairly high. Nevertheless their dynamics are somewhat different from those of the CDS of Greece, Italy and Portugal. This is most likely related to the initial contagion trigger along the bank-sovereign nexus for the two groups of countries. In the case of the former group, the original shock hit the banking system and was later transmitted to the sovereigns (which were initially in good shape); conversely, for the second group, the initial shock came from the sovereign and was later transmitted to the banking system (whose balance sheet was relatively healthy prior to that).

Comparing the time dynamics of low- and high-spread countries, it is notable that the multivariate CDS for the former peak at the time of the global financial crisis, whereas the multivariate CDS of the latter spike during the Euro area sovereign debt crisis. This is in line with the conventional wisdom that the largest banking systems in the world were more severely impacted by the global financial crisis, whereas the banking systems of the Euro area periphery were hit harder during the European sovereign crisis.

FIGURE 5 ABOUT HERE

Figure 5 reports the results on the lending side. It reveals that the overall dynamics of the multivariate CDS on the lending side for the top five banking systems are roughly similar to those on the borrowing side. Once again, there are two peaks in the contagion measure for this set of countries: one around the global financial crisis and one around the European debt crisis, with the former being higher.

Nevertheless, there are important differences among individual countries. For example, the United States has the highest contagion effect on the borrowing side. By contrast, on the lending side, the leading role is taken by United Kingdom, France and Germany, followed by the United States and Japan. The above differences between the borrowing and the lending side of the top five systems can be explained by their different roles. The United States is by far the largest borrowing country in the world. Meanwhile, even though it is also a relatively large lender, its share of global lending is not as large as those of United Kingdom, France and Germany. When it comes to the euro area periphery countries, the dynamic on the lending side is quite similar to that of the borrowing side. One notable exception is Spain, for which the contagion measure tends to be higher on the lending side. This is likely due to the fact that the share of Spain as a lender is higher than its share as a borrower.

For the sake of a summary comparison, Tables 2-4 present some descriptive statistics for each country in the dataset. In particular, Table 2 refers to the original CDS values while Tables 3 and 4 refer to the multivariate cases (the borrowing and the lending cases respectively). In both tables we report the mean of the observed (univariate) market CDS spreads, and the mean of the estimated (multivariate) spreads, on both the borrowing and the lending sides, along with its maximum, standard deviation, skewness and kurtosis.

TABLE 2 ABOUT HERE

TABLE 3 ABOUT HERE

TABLE 4 ABOUT HERE

From the tables note that contagion increases the univariate spreads of the smaller peripheral countries (Greece,Portugal,Ireland) about three times, similarly on both the borrowing and the lending sides. Larger peripherals, that is Italy and Spain, increase instead by four times, indicating a larger vulnerability to contagion. Core countries have much smaller univariate CDS spreads, whose mean does not exceed 10 basis points. However, contagion sensibly increases the spreads, and especially for the most exposed countries. Note, in particular, the US, whose borrowing modified spread increases at 80 basis points versus 51 on the lending side, indicating a much higher liquidity, rather than credit risk. Conversely, France and, to a lesser extent, Japan and Germany, are instead more influenced on the lending side, due to their relevant credit risk exposure.

FIGURE 6 ABOUT HERE

Figure 6 reports the expected losses on the borrowing side. It is dominated by the United States, the largest borrower. The impact of the very large outstanding amount for the United States dominates the impact of its relatively low CDS spread. It is worth pointing out that, in terms of total expected losses, the peak associated with the sovereign crisis of 2011 is larger than the peak associated with the Global Financial Crisis of 2008. This is especially true for the top five banking systems, which account for the overwhelming majority of the expected losses during crisis period. At the same time, the ordering of peripheral European countries is only roughly correlated with their relative sizes as borrowing destinations. This is due to the relatively more powerful offsetting impact of the market CDS effect, which is particularly strong for Greece and for Ireland in late 2010/early 2011. For that group of countries, the two (Global Financial Crisis and euro area crisis) peaks are comparable.

FIGURE 7 ABOUT HERE

Figure 7 reports the expected losses results, on the lending side. Their overall distribution is more evenly spread among countries than the respective distribution on the borrowing side. A more detailed analysis, at the country level, reveals that the EL dynamics vary quite a bit across the euro area periphery countries, depending on the geographical distribution of their lending exposures. Finally, it is also worth pointing out that the peaks on the borrowing tend to be higher than those on the lending side.

FIGURE 8 ABOUT HERE

Figure 8 reports the expected losses, cumulated over all the considered time, on both the borrowing and the lending side. This allows to identify the countries that, throughout the considered period, are associated with the highest expected losses. The largest losses are expected by the United States, distantly followed by the United Kingdom, with the rest of the countries far behind. By contrast, on the lending side, losses are much more evenly distributed. The greatest expected losses are observed for the two largest European countries France and Germany, followed by Japan, the United Kingdom and the United States.

The results we discussed up to this point were all based on claims on all sectors. We now focus on a sector-wise comparison, in terms of cumulated losses over time.

FIGURE 9 ABOUT HERE

Figure 9 compares the overall expected losses in the official sector. Note that, in the borrowing side, the United States expected losses are much larger than those of the United Kingdom. It is notable that those of Germany are larger than those of France and those of Italy are larger than those of Spain. On the lending side we have the opposite relationships, for each one of the above pairs. This in line with the relative importance of those countries as lenders and borrowers, respectively.

FIGURE 10 ABOUT HERE

Figure 10 compares the expected losses, cumulated over time, in the banking sector. It reveals the dominant role that the United States and United Kingdom play on the borrowing side. Conversely, on the lending side, we find a prominent role for Spain, France, Germany and Japan. In addition, the Netherlands and Switzerland also appear relevant. These results are consistent with the relative international presence of the respective banking systems.

FIGURE 11 ABOUT HERE

Figure 11 compares the cumulative expected losses in the non bank private sector. Once again, the United States and the United Kingdom are the most relevant on the borrowing side. When it comes to lending, the United Kingdom is considerably larger than the United States and France is greater than Germany. Japan also appears quite relevant on the lending side.

Finally, we examine whether non-linear, higher order effects are present.

FIGURE 12 ABOUT HERE

Figure 12 compares the expected losses obtained with the tensor decomposition methods presented in Section 2.2, applied to all three sectors simultaneously, with the sum of the expected losses, calculated separately for each sector using the matrix decomposition of Section 2.1. The difference between the two, if positive, indicates the presence of non-linear, higher order contagion effects that make the overall expected losses higher than the sum of the sector-wise losses. Figure 12 reveals that this is indeed the case during the main crisis periods not only for many individual countries (upper two panels), but also at the global level (lower two panels).

4.3 Predictive analysis

To evaluate the predictive power of our proposed method, in Figure 13 we present the results from an out-of-sample predictive analysis. Specifically, the aim is to predict, at time $t-1$, the one-step ahead value of the CDS spread (at time t).

FIGURE 13 ABOUT HERE

To achieve this goal we compare: i) a model based only on the average of the last n_1 observed past spread values; ii) a model based on the same observed past spreads, modified by network contagion, as described in Section 2.4. For both models, we compare the predictive Root Mean Squared Error (RMSE), for different n_1 lengths: $N = 1, \ldots, 8$ quarters. We also compare their binarised versions, in which we set a performance indicator equal to 1 when the RMSE of i) is higher than that of ii) and zero otherwise.

As illustrated by Figure 13, our proposed method is clearly superior on the eve of and during crisis periods (as indicated by its considerably lower RMSE), in line with the results obtained when examining the non-linear effects in the preceding sub-section. Moreover, as suggested by the above figure, there appears to be a structural break in Q1-2013. Indeed, between that quarter and Q1-2014 our measure performs slightly worse than the alternative model (based on the average of past CDS spread values). The most likely explanation for this is related to the behaviour of the Greeks sovereign CDS during the same period. After being suspended in late 2011, this instrument was re-introduced into the market with a lower value (1/3 of the original 2011 value). This mechanically reduces the model-estimated level of stress. In contrast, as previously discussed, when the second wave of the Greek crisis escalated in 2014, our measure once again outperforms the one generated by the alternative model.

In addition to the above RMSE-based comparison, which treats predictions errors in a symmetric fashion, we also evaluate the predictive performance of our model using an alternative, asymmetric statistical measure of goodness of fit.[5] Namely, we focus on the Mean Signed Deviation (MSD), which is defined as the average of the predictions errors, split conditional on their signs. This measure is not symmetric and, therefore, could be utilised to distinguish between predictions, depending on whether they are underestimating or

[5]We would like to thank an anonymous referee for suggesting this asymmetric comparison.

overestimating future CDS spreads. The results from this alternative comparison are summarized in Figure 14.

FIGURE 14 ABOUT HERE

The results reveal that our measures remain close to the actual CDS values up until Q1-2011, while the forecasts obtained using a model based only on past CDS spreads tends to under-predict. In the middle of the sample, all forecast tend to under-predict CDS spreads, but the network model performs better, having a MSD closer to zero up to Q1-2013. Subsequently, the model based only on past CDS spreads perform better, but only because the multivariate measure over-estimates future CDS spreads. Nevertheless, towards the end of the sample, our proposed model outperforms the simple model once again.

5 Summary and policy implications

In this paper, we propose a technique aimed at summarising multi-layer networks into a single adjacency matrix and a resulting methodology for transforming univariate, market based, CDS spreads into multivariate CDS that take into account contagion effects arising from changing default probabilities of other countries and from changes in flow compositions. We apply our measure to a multi-layer international banking network derived from the BIS international banking statistics. We show that the resulting measure predicts CDS spreads better than an alternative measure based on (unadjusted) past values of CDS spreads. This is the case, especially during crisis times, when the non-linear network effects tend to be more important.

We believe our methodology can be rather useful for policymakers, as it gives an early warning measure of a national banking system's distress levels, which incorporates information on its foreign exposures. Our measure can also be extended to any multi-layer financial network, such as an interbank network.

Furthermore, the methodology we propose could potentially be utilised in a bottom-up stress test. More precisely, our proposed methodology could generate estimates of an institution's expected losses, while incorporating all relevant information on (direct and indirect) exposures, linkages and contagion probabilities. As noted in EBA (2017), in the context of banking supervision, a stress test should take into account that risks at the institution-wide level may not be well reflected by simple aggregation of stress tests on portfolios, individual risk areas or business units of the group. This opens room for possible vulnerabilities coming from contagion effects, as discussed in Greenwood et al. (2015), in the context of a simple fire sale model for the banking sector, based on the interbank networks. These concepts can be extended from banks to national banking systems, with the tensor approach overcoming the problem of a simple linear aggregation of risks (or exposures), and our proposed centrality measure becoming useful to investigate how changes in exposures and/or in CDS spreads affect the soundness of national banking systems.

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Appendix

Formally, following Kolda and Bader (2009), an N-th order tensor is an element of the tensor product of N vector spaces, each of which has its own coordinate system. In a tensor representation, vectors (tensors of order one) are denoted by boldface lowercase letters, e.g., $\mathbf{x}$. Matrices (tensors of order two) are denoted by boldface capital letters, e.g., $\mathbf{X}$. Higher-order tensors (order three or higher) are denoted by boldface Euler script letters, e.g., $\mathcal{X}$. Scalars are denoted by lowercase letters, e.g., x. The i-th entry of a vector $\mathbf{X}$ is denoted by x_i; the element (i, j) of a matrix $\mathbf{X}$ is denoted by x_{ij} , and the element (i, j, k) of a third-order tensor $\mathcal{X}$ is denoted by x_{ijk}. The n-th element in a sequence is denoted by a superscript in parentheses, e.g., $\mathbf{X}^n$ denotes the n-th matrix in a sequence. Fibers are the higher-order analogue of matrix rows and columns, defined by fixing every index but one. A matrix column is a mode-1 fiber and a matrix row is a mode-2 fiber. Third-order tensors have column, row, and tube fibers, denoted by $x_{:jk}$, $x_{i:k}$, and $x_{ij:}$, respectively. Slices are two-dimensional sections of a tensor, defined by fixing all but two indices: $\mathbf{X}_{i::}$, $\mathbf{X}_{:j:}$, and $\mathbf{X}_{::k}$, denotes horizontal, lateral, and frontal slides of a third-order tensor $\mathcal{X}$. Alternatively, the k-th frontal slice of a third-order tensor, $\mathbf{X}_{::k}$, may be denoted more compactly as $\mathbf{X}_k$. The norm of a tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is the square root of the sum of the squares of all its elements,

$$\parallel \mathcal{X} \parallel = \sqrt{\sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \dots \sum_{i_N=1}^{I_N} x_{i_1,i_2,\dots i_N}^2}, \tag{9}$$

in analogy with the matrix Frobenius norm. The inner product of two same-sized tensors $\mathcal{X}, \mathcal{Y} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is the sum of the products of their entries, i.e.,

$$\langle \mathcal{X}, \mathcal{Y} \rangle = \sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \dots \sum_{i_N=1}^{I_N} x_{i_1,i_2,\dots i_N} y_{i_1,i_2,\dots i_N} \tag{10}$$

An N-way tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is of rank one if it can be written as the outer product of N vectors, i.e.,

$$\mathcal{X} = \mathbf{a}^{(1)} \circ \mathbf{a}^{(2)} \circ \dots \circ \mathbf{a}^{(N)}, \tag{11}$$

where the symbol "$\circ$" represents the vector outer product. Matricization, also known as unfolding or flattening, is the process of reordering the elements of an N-way array into a matrix. The mode-matricization of a tensor $\mathcal{X} \in \mathbb{R}^{I_1} \times I_2 \times \cdots \times I_N$ is denoted by $\mathbf{X}_{(n)}$ and arranges the mode-n fibers to be the columns of the resulting matrix. A tensor element $(i_1, i_2, ..., i_N)$ maps to a matrix element (i_n, j), where

$$j = 1 + \textstyle\sum_{k=1,k\neq n}^{N} (i_k - 1)\, J_k \quad \text{with} \quad J_k = \prod_{m=1,m\neq n}^{k-1} I_m \tag{12}$$

Several matrix products are important in the following sections, so we briefly define them here. The Kronecker product of matrices $\mathbf{X} \in R^{I \times J}$ and $\mathbf{Y} \in R^{K \times L}$ is denoted by $\mathbf{X} \otimes \mathbf{Y}$: the result is a matrix of size $(IK) \times (JL)$. The Khatri–Rao product is the "matching column-wise" Kronecker product. Given two matrices $\mathbf{X} \in R^{I \times J}$ and $\mathbf{Y} \in R^{K \times L}$, their Khatri–Rao product is denoted by $\mathbf{X} \odot \mathbf{Y}$; the result is a matrix of size $(IJ) \times K$. The Hadamard product is the elementwise matrix product, and it is denoted by "$*$".

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Tables and Figures

Foreign claims on Greece, by lending bank nationality[1] — Table 1

	2008-Q2	2009-Q3	2012-Q2	2015-Q4
Ireland	11,157	8,717	119	60
Italy	10,874	8,573	977	673
Portugal	7,736	10,453	6,910	138
Spain	1,088	1,157	767	312
France	85,958	78,571	40,071	1,144
Germany	44,440	43,236	5,512	8,738
United Kingdom	14,625	12,492	5,565	3,259
Japan	8,265	8,777	536	215
United States	8,020	19,448	3,461	2,729
Other	117,817	109,560	8,364	5,186
All reporting countries	309,980	300,984	72,282	22,454

[1] End-of period outstanding stocks; in millions of US dollars.

Sources: BIS consolidated banking statistics on an ultimate risk basis.

Table 1: Exposures to Greece and PIIGS countries: banking sector foreing claims to all-sectors Greece on quarterly base.

	Mean Univar Cds	Max Univar Cds	Std Univar Cds	Skew Univar Cds	Kurt Univar Cds
GR	0.0373	0.0902	0.0738	2.1279	5.8092
PT	0.0030	0.0113	0.0031	1.3229	3.9996
IT	0.0016	0.0044	0.0013	0.7881	2.6014
ES	0.0017	0.0042	0.0014	0.7659	2.4496
IE	0.0025	0.0103	0.0027	1.3948	3.8835
US	0.0006	0.0009	0.0004	0.2854	2.2760
GB	0.0006	0.0014	0.0004	0.0317	1.8118
FR	0.0007	0.0022	0.0006	0.9970	3.4068
JP	0.0006	0.0014	0.0004	0.1537	2.5473
DE	0.0005	0.0013	0.0004	0.5605	2.9022

Table 2: Aggregate Statistics CDS spreads, univariate

	Mean Mult. Borr. Cds	Max Mult. Borr. Cds	Std Mult. Borr. Cds	Skew Mult. Borr. Cds	Kurt Mult. Borr. Cds
GR	0.1120	0.2706	0.2212	2.1280	5.8093
PT	0.0091	0.0342	0.0093	1.3268	4.0103
IT	0.0062	0.0153	0.0038	0.6501	2.5821
ES	0.0062	0.0137	0.0041	0.6338	2.4424
IE	0.0080	0.0314	0.0083	1.3724	3.8503
US	0.0072	0.0112	0.0043	1.5216	5.6945
GB	0.0053	0.0097	0.0029	0.8027	3.5346
FR	0.0041	0.0089	0.0022	0.3871	2.3965
JP	0.0025	0.0045	0.0012	0.1146	2.9704
DE	0.0039	0.0071	0.0019	0.5929	3.1892

Table 3: Aggregate Statistics CDS spreads, multivariate borrowing

	Mean Mult. Lend. Cds	Max Mult. Lend. Cds	Std Mult. Lend. Cds	Skew Mult. Lend. Cds	Kurt Mult. Lend. Cds
GR	0.1119	0.2705	0.2213	2.1279	5.8091
PT	0.0091	0.0342	0.0093	1.3278	4.0138
IT	0.0056	0.0143	0.0037	0.7090	2.6848
ES	0.0062	0.0148	0.0045	0.8280	2.7027
IE	0.0080	0.0315	0.0083	1.3637	3.8342
US	0.0054	0.0093	0.0030	0.5002	2.4662
GB	0.0066	0.0117	0.0040	1.0253	3.8756
FR	0.0060	0.0124	0.0038	0.9761	3.2264
JP	0.0042	0.0075	0.0022	0.5665	3.0674
DE	0.0030	0.0052	0.0013	-0.2392	2.5492

Table 4: Aggregate Statistics CDS spreads, multivariate lending

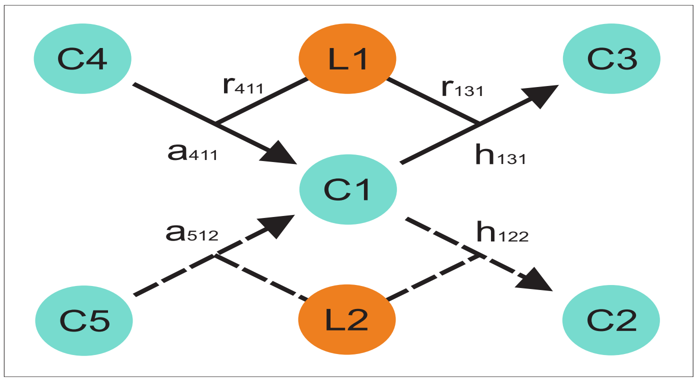

Figure 1: Multi-layer network example: the network has two layers L_1 and L_2, and five countries. In L_1 country C_1 borrows from country C_4 with probability a_{411}, moreover C_1 lends to C_3 with probability h_{131}. In L_2 country C_1 borrows from country C_1 with probability a_{512} and lends with probability h_{122} to C_2. The authority score of C_1 is $u_{C_1} = a_{411}v_{C_4}w_{L_1} + a_{512}v_{C_5}w_{L_2}$. The hub score of C_1 is $v_{C_1} = h_{131}v_{C_3}w_{L_1} + h_{122}v_{C_2}w_{L_2}$. Now consider layer L_1. The type score of L_1 is: $w_{L_1} = r_{411}v_{C_4}u_{C_1} + r_{131}v_{C_1}u_{C_3}$.

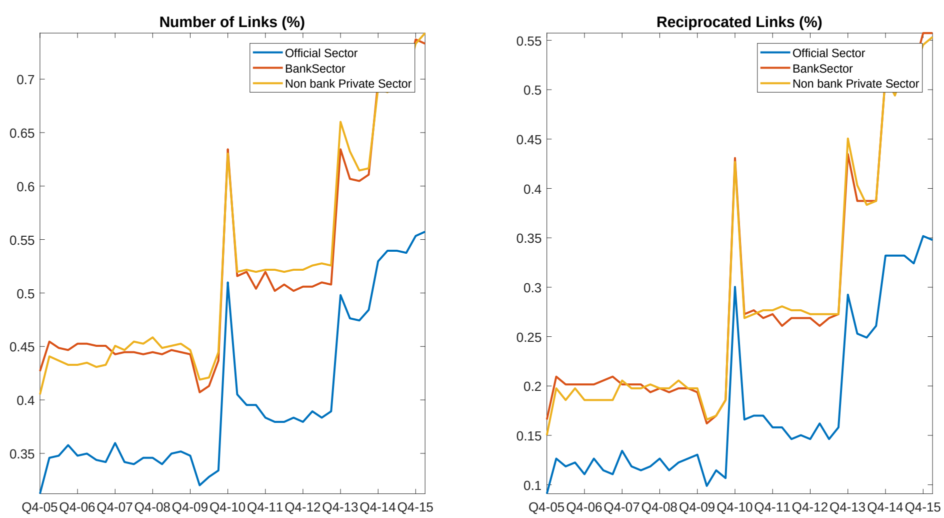

Figure 2: Network statistics: the left panel shows the network density (the fraction of existing links over all possible links). The right panel displays the percentage of reciprocated links.

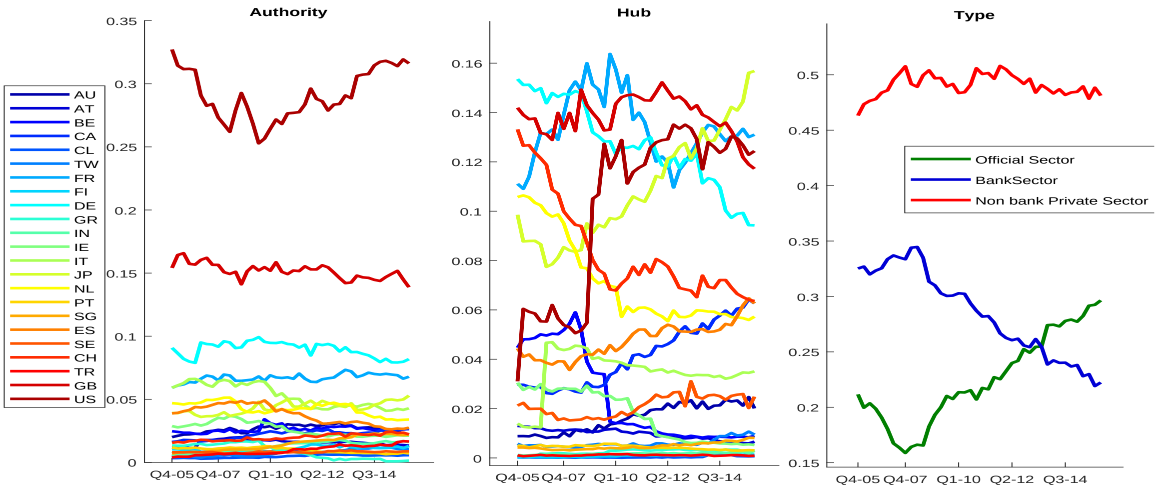

Figure 3: Hub, Authority and Type Scores: the left panel shows the authority score associated to each country, the central panel displays the hub score whereas the right panel reports the type score associated with each network layer.

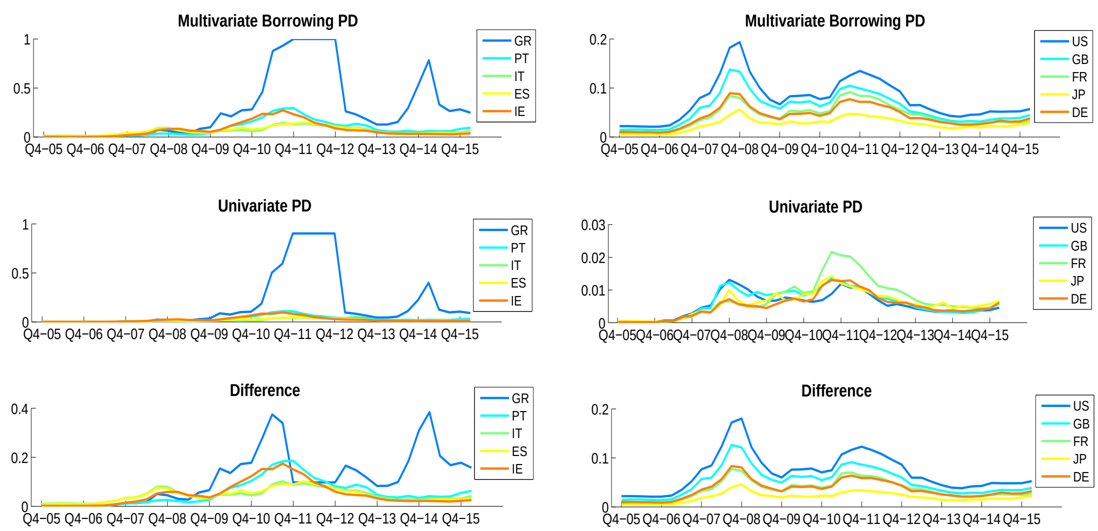

Figure 4: Modified Borrowing PD: for the sake of clarity we present the results only for the euro area periphery countries in the left panels and for the largest five national banking systems in the right panels. In particular, for the two groups the upper panels show the modified borrowing PD; the central panels display the original PD an the bottom panels report the difference between the two measures.

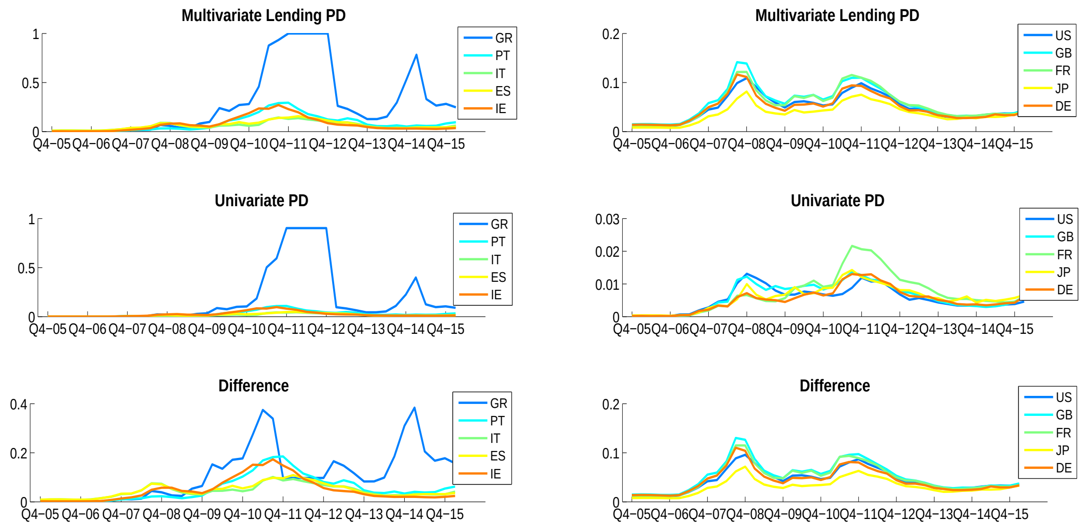

Figure 5: Modified Lending PD: for the sake of clarity we present the results only for the euro area periphery countries in the left panels and for the largest five national banking systems in the right panels. In particular, for the two groups the upper panels show the modified lending PD; the central panels display the original PD an the bottom panels report the difference between the two measures.

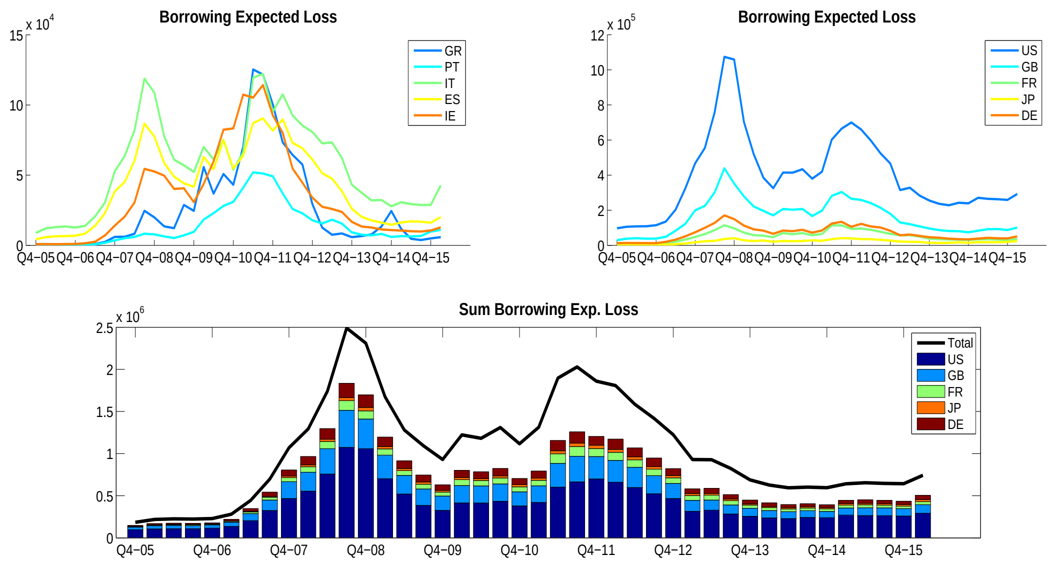

Figure 6: Borrowing EL: the upper panels report the borrowing expected losses for the two groups, namely for the euro area periphery countries on the left and for the largest five national banking systems on the right. The bottom panel displays the total borrowing expected losses together with the contribution of the largest five national banking systems.

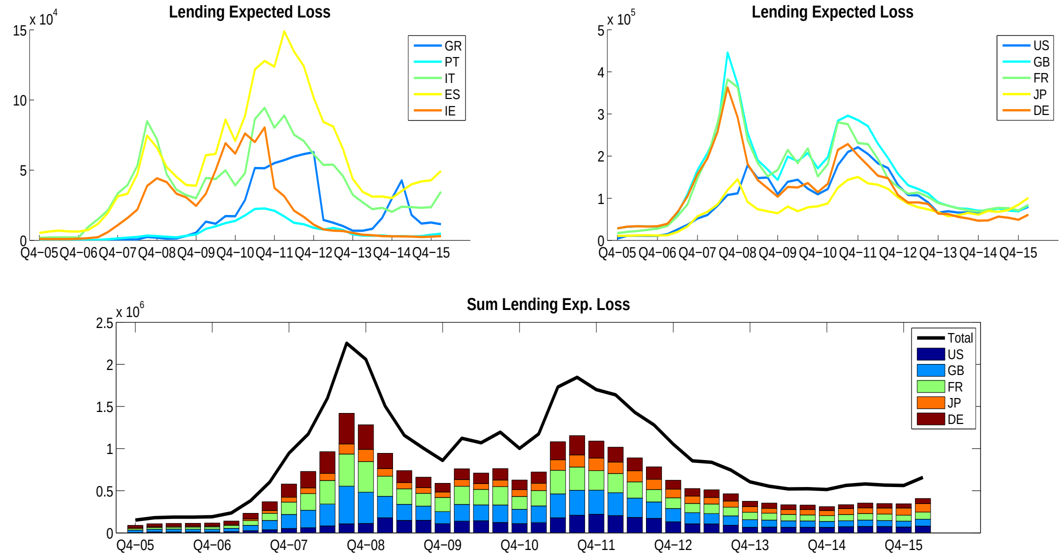

Figure 7: Lending EL: the upper panels report the lending expected losses for the two groups, namely for the euro area periphery countries on the left and for the largest five national banking systems on the right. The bottom panel displays the total lending expected losses together with the contribution of the largest five national banking systems.

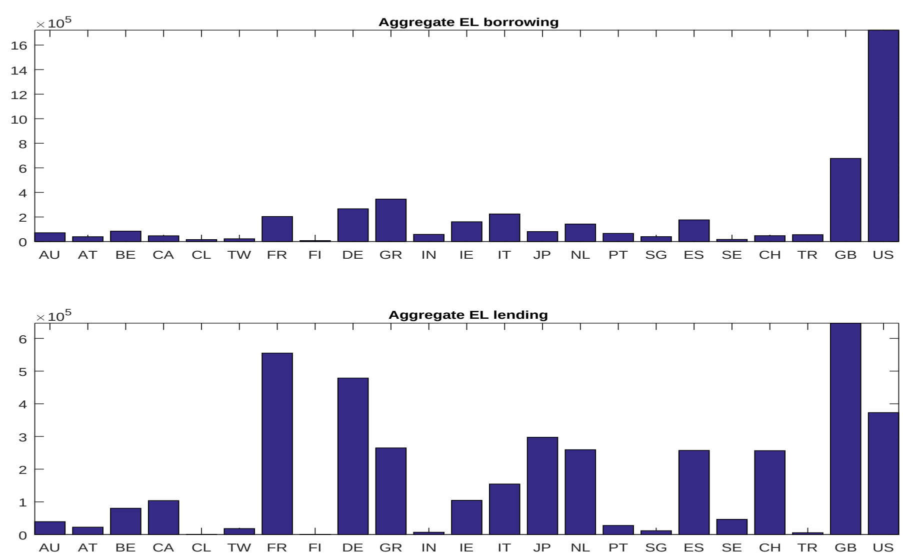

Figure 8: Borrowing vs Lending Expected Losses all sectors: the all sectors' borrowing expected losses cumulated over time are shown in the upper panel while the all sectors' lending expected losses are reported in the lower panel.

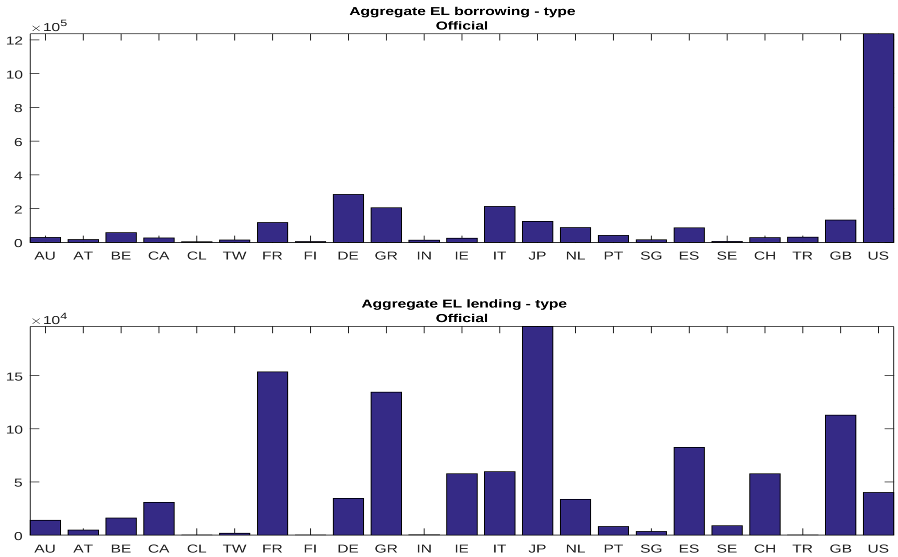

Figure 9: Borrowing vs Lending Expected Losses official sector: the official sector's borrowing expected losses cumulated over time are shown in the upper panel while the official sector's lending expected losses are reported in the lower panel.

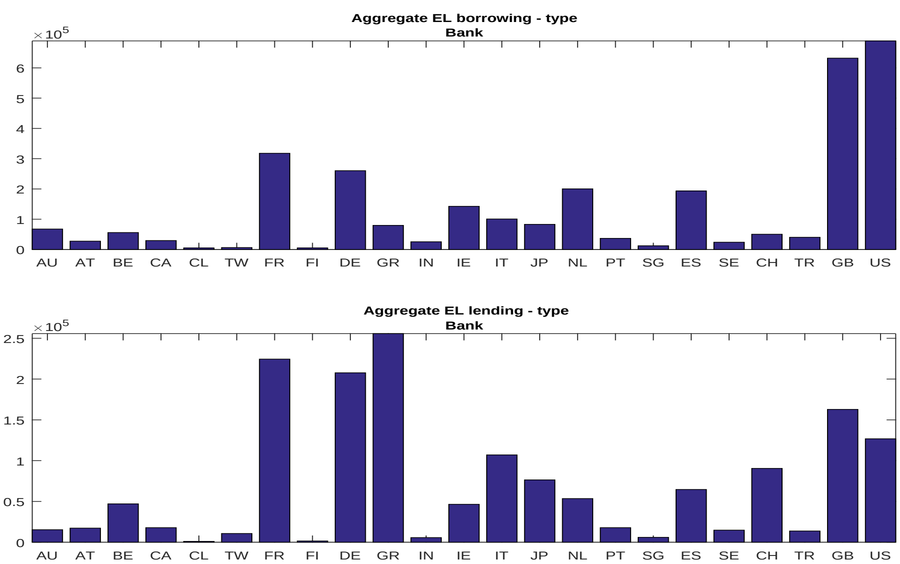

Figure 10: Borrowing vs Lending Expected Losses banking sector: the banking sector's borrowing expected losses cumulated over time are shown in the upper panel while the banking sector's lending expected losses are reported in the lower panel.

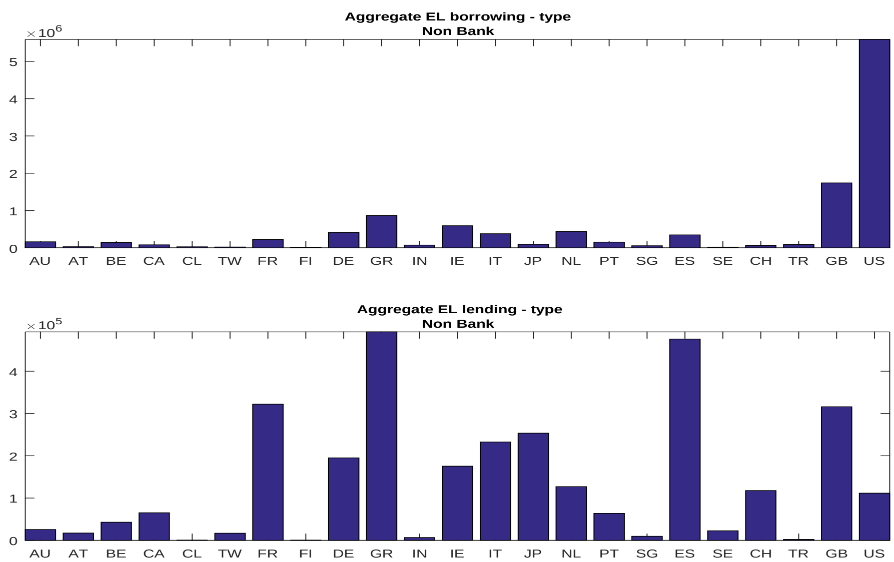

Figure 11: Borrowing vs Lending Expected Losses non bank private sector: the non bank private sector's borrowing expected losses cumulated over time are shown in the upper panel while the non bank private sector's lending expected losses are reported in the lower panel.

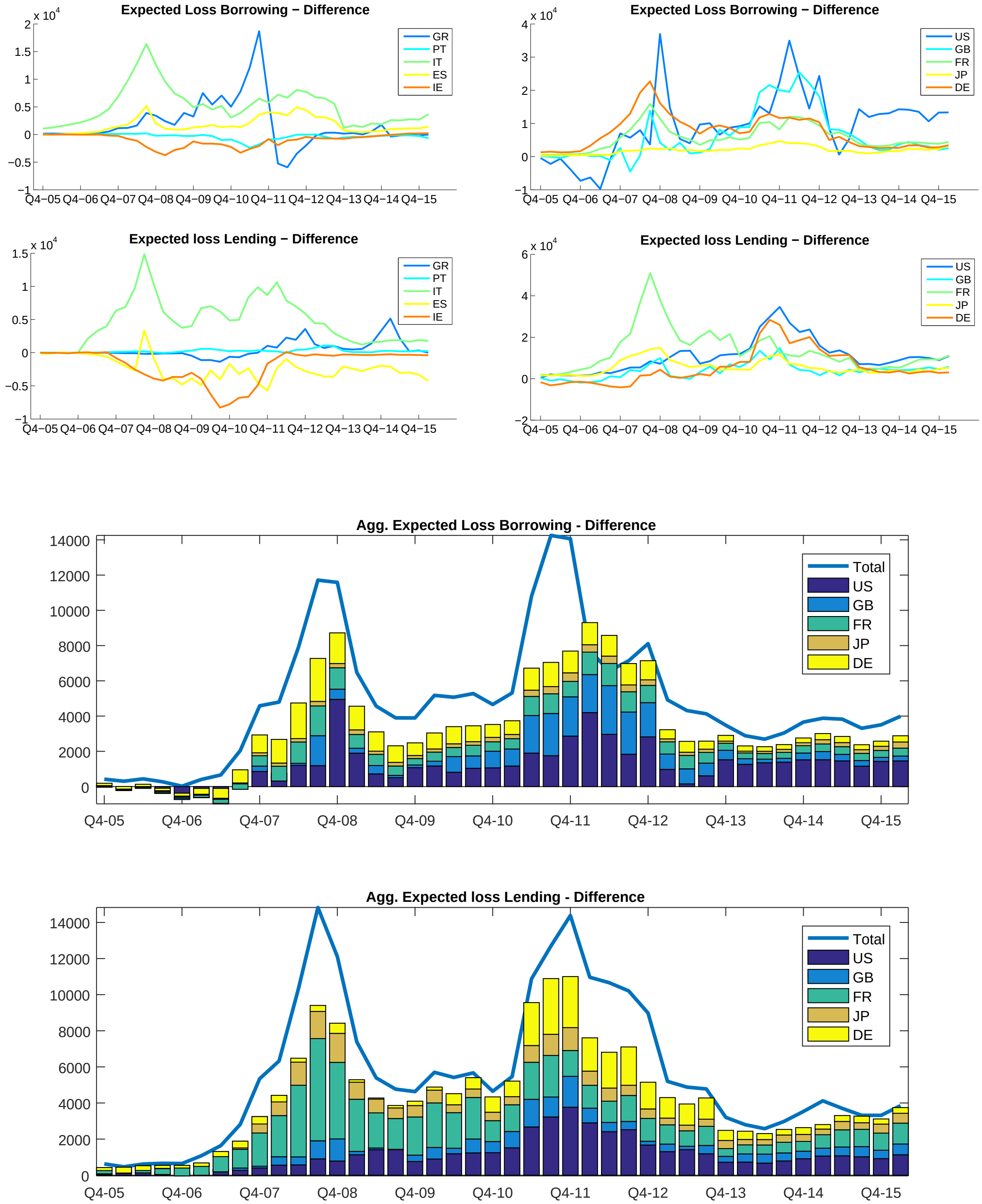

Figure 12: Non Linear Effects: the plots show the difference in terms of borrowing and lending expected losses between the tensor decomposition methods and the sum of the expected losses, computed separately for each sector using the matrix decomposition technique. The four upper panels show the difference disaggregated for the two groups, the two lower panels report the results for the complete economy together with the contribution of the five main advanced countries.

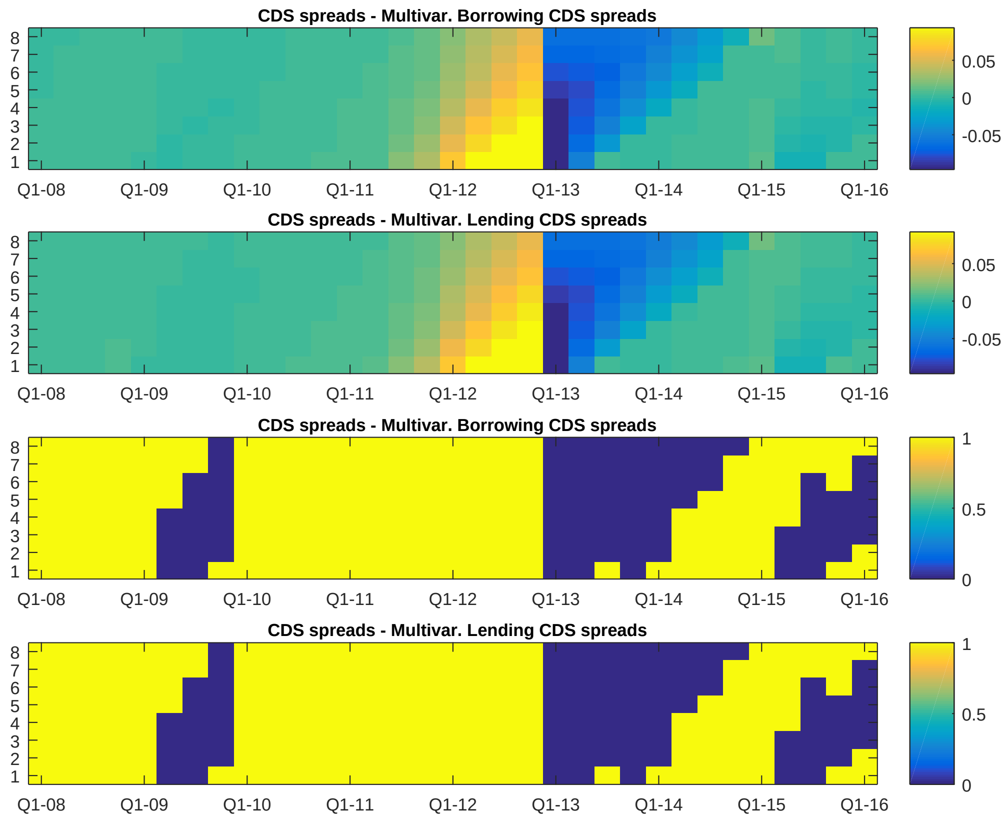

Figure 13: Predictive performance: the differences between the RMSE obtained using a model based only on past spreads values and a model based on network contagion, are shown in the first two panels (for the borrowing and for the lending side respectively). Positive values imply the network contagion model is superior in predicting one-step ahead value of the CDS spread. The lower panels show the binarised version of the results, the yellow colour indicates a superior performance of the network model.

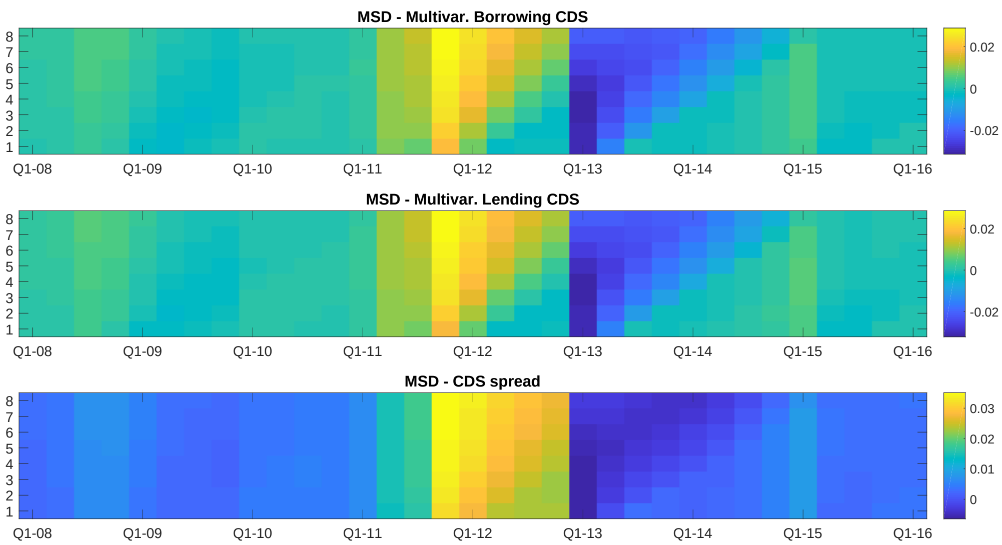

Figure 14: Predictive performance: MSD of the network based model obtained using the borrowing and lending multivariate CDS spreads (upper and central panels) and MSD obtained using a model based only on past market CDS spreads values (lower panel). Positive values imply underestimating the future CDS values while negative values imply an overestimation.